7.5. Find the minimum value of the function

$$f(x, y) = x^2 + y^2$$

subject to the equality constraint

$$g(x, y) = x^2 - 6x - y^2 + 17 = 0$$

Forming the Lagrangian function, we obtain

$$\mathcal{L} = x^2 + y^2 + \lambda(x^2 - 6x - y^2 + 17)$$

The resulting necessary conditions for constrained local maxima of $\mathcal{L}$ are

$$\frac{\partial \mathcal{L}}{\partial x} = 2x + \lambda(2x - 6) = 0$$
$$\frac{\partial \mathcal{L}}{\partial y} = 2y - 2\lambda y = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = x^2 - 6x - y^2 + 17 = 0$$

From the second condition $\lambda = 1$. Substituting in the first condition

$$2x + (1)(2x - 6) = 0 \quad \text{or} \quad x = 1.5$$

Substituting for x in the third condition results in $y = 3.20156$. Thus the minimum value of the function is

$$f(\hat{x}, \hat{y}) = (1.5)^2 + (3.20156)^2 = 12.5$$

7.7. The fuel-cost functions in \$/h for two 800 MW thermal plants are given by

$$C_1 = 400 + 6.0P_1 + 0.004P_1^2$$
$$C_2 = 500 + \quad \beta P_2 + \qquad \gamma P_2^2$$

where P_1 and P_2 are in MW.

(a) The incremental cost of power λ is \$8/MWh when the total power demand is 550 MW. Neglecting losses, determine the optimal generation of each plant.

(b) The incremental cost of power λ is \$10/MWh when the total power demand is 1300 MW. Neglecting losses, determine the optimal generation of each plant.
(c) From the results of (a) and (b) find the fuel-cost coefficients β and γ of the second plant.

$$\frac{dC_1}{dP_1} = 6 + 0.008P_1 = \lambda$$
$$\frac{dC_2}{dP_2} = \beta + 2\gamma P_2 = \lambda$$

(a) For $\lambda = 8$, and $P_D = 550$ MW, we have

$$P_1 = \frac{8-6}{0.008} = 250 \text{ MW}$$
$$P_2 = P_D - P_1 = 500 - 250 = 300 \text{ MW}$$

(b) For $\lambda = 10$, and $P_D = 1300$ MW, we have

$$P_1 = \frac{10-6}{0.008} = 500 \text{ MW}$$
$$P_2 = P_D - P_1 = 1300 - 500 = 800 \text{ MW}$$

(c) The incremental cost of power for plant 2 are given by

$$\beta + 2\gamma(300) = 8$$
$$\beta + 2\gamma(800) = 10$$

Solving the above equations, we find $\beta = 6.8$, and $\gamma = 0.002$

7.8. The fuel-cost functions in \$/h for three thermal plants are given by

$$C_1 = 350 + 7.20P_1 + 0.0040P_1^2$$
$$C_2 = 500 + 7.30P_2 + 0.0025P_2^2$$
$$C_3 = 600 + 6.74P_3 + 0.0030P_3^2$$

where P_1, P_2, and P_3 are in MW. The governors are set such that generators share the load equally. Neglecting line losses and generator limits, find the total cost in \$/h when the total load is

(i) $P_D = 450$ MW
(ii) $P_D = 745$ MW
(iii) $P_D = 1335$ MW

(i) For $P_D = 450$ MW, $P_1 = P_2 = P_3 = \frac{450}{3} = 150$ MW. The total fuel cost is

$$C_t = 350 + 7.20(150) + 0.004(150)^2 + 500 + 7.3(150) + 0.0025(150)^2 + 600 + 6.74(150) + 0.003(150)^2 = 4,849.75 \ \ \$/\text{h}$$

(ii) For $P_D = 745$ MW, $P_1 = P_2 = P_3 = \frac{745}{3}$ MW. The total fuel cost is

$$C_t = 350 + 7.20\left(\frac{745}{3}\right) + 0.004\left(\frac{745}{3}\right)^2 + 500 + 7.3\left(\frac{745}{3}\right) + 0.0025\left(\frac{745}{3}\right)^2 + 600 + 6.74\left(\frac{745}{3}\right) + 0.003\left(\frac{745}{3}\right)^2 = 7,310.46 \ \ \$/\text{h}$$

(iii) For $P_D = 1335$ MW, $P_1 = P_2 = P_3 = 445$ MW. The total fuel cost is

$$C_t = 350 + 7.20(445) + 0.004(445)^2 + 500 + 7.3(445) + 0.0025(445)^2 + 600 + 6.74(445) + 0.003(445)^2 = 12,783.04 \ \ \$/\text{h}$$