10.3. The operator a is defined as $a = 1\angle 120°$; show that

(a) $\frac{(1+a)}{(1+a^2)} = 1\angle 120°$

(b) $\frac{(1-a)^2}{(1+a)^2} = 3\angle -180°$

(c) $(a-a^2)(a^2-a) = 3\angle 0°$

(d) $V_{an}^1 = \frac{1}{\sqrt{3}} V_{bc}^1 \angle 90°$

(e) $V_{an}^2 = \frac{1}{\sqrt{3}} V_{bc}^2 \angle -90°$

(a) Since $1 + a + a^2 = 0$, we have

$$\frac{(1+a)}{(1+a^2)} = \frac{1+a}{-a} = -\frac{1}{a} - 1$$
$$= 0.5 + j0.866 - 1 = 1\angle 120°$$

(b)

$$\frac{(1-a)^2}{(1+a)^2} = \frac{(1-a)^2}{(-a^2)^2} = \frac{1-2a+a^2}{a} = \frac{1}{a} - 2 + a =$$
$$= -0.5 - j0.866 - 2 - 0.5 + j0.866 = 3\angle 180°$$

(c)

$$(a-a^2)(a^2-a) = 2a^3 - a^2 - a^4 = 2a^3 - (a^2+a)$$
$$= 2(1) - (-1) = 3$$

(d)

$$V_{bc}^1 = V_{bn}^1 - V_{cn}^1 = a^2 V_{an}^1 - a V_{an}^1 = (a^2 - a) V_{an}^1$$
$$= (-0.5 - j0.866 + 0.5 - j0.866) V_{an}^1$$
$$= \sqrt{3}\angle -90° V_{an}^1$$

or

$$V_{an}^1 = \frac{1}{\sqrt{3}} V_{bc}^1 \angle 90°$$

(e)

$$V_{bc}^2 = V_{bn}^2 - V_{cn}^2 = a V_{an}^2 - a^2 V_{an}^2 = (a - a^2) V_{an}^2$$
$$= (-0.5 + j0.866 + 0.5 + j0.866) V_{an}^2$$
$$= \sqrt{3}\angle 90° V_{an}^2$$

or

$$V_{an}^2 = \frac{1}{\sqrt{3}} V_{bc}^2 \angle -90°$$

10.8. The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 600\angle 36.87°$, $V_{bc} = 800\angle 126.87°$, and $V_{ca} = 1000\angle -90°$. A Y-connected load with a resistance of 37 Ω per phase is connected to the supply. Determine
(a) The symmetrical components of voltage.
(b) The phase voltages.
(c) The line currents.

We use the following statements

```
Vabbcca=[600   36.87                  % Unbalanced line voltages
         800   126.87
         1000  -90];
VL012=abc2sc(Vabbcca); % Sym. comp. line voltages, rectangular
VL012p=rec2pol(VL012)        % Sym. comp. line voltages, polar
Va012=[0
       VL012(2)/(sqrt(3)*(0.866+j*.5))
       VL012(3)/(sqrt(3)*(0.866-j*.5))];  % Sym. components of
                                 % phase voltages, rectangular
Va012p=rec2pol(Va012)    % Sym. comp. of phase voltages, polar
Vabc=sc2abc(Va012);              % Phase voltages, rectangular
Vabcp=rec2pol(Vabc)                    % Phase voltages, polar
Iabc=Vabc/37;                     % Line currents, rectangular
Iabcp=rec2pol(Iabc)                      % Line currents, polar
```

which result in

```
   VL012p =
             0.0006   -179.9999
           237.0762    169.9342
           781.3204     24.0621

   Va012p =
             0          0
           136.8790    139.9335
           451.1055     54.0628

   Vabcp =
           480.7542     70.5606
           333.3386    163.7411
           569.6111    -73.6857

   Iabcp =
            12.9934     70.5606
             9.0092    163.7411
            15.3949    -73.6857
```

10.9. A generator having a solidly grounded neutral and rated 50-MVA, 30-kV has positive-, negative-, and zero-sequence reactances of 25, 15, and 5 percent, respectively. What reactance must be placed in the generator neutral to limit the fault current for a bolted line-to-ground fault to that for a bolted three-phase fault?

The generator base impedance is

$$Z_B = \frac{(30)^2}{50} = 18 \ \ \Omega$$

The three-phase fault current is

$$I_{f\,3\phi} = \frac{1}{0.25} = 4.0 \ \ \text{pu}$$

The line-to-ground fault current is

$$I_{f\,LG} = \frac{3}{0.25 + 0.15 + 0.05 + 3X_n} = 4.0 \ \ \text{pu}$$

Solving for X_n, results in

$$\begin{aligned} X_n &= 0.1 \quad \text{pu} \\ &= (0.1)(18) = 1.8 \ \ \Omega \end{aligned}$$

10.14. The zero-, positive-, and negative-sequence bus impedance matrices for a three-bus power system are

$$\mathbf{Z}_{bus}^0 = j \begin{bmatrix} 0.20 & 0.05 & 0.12 \\ 0.05 & 0.10 & 0.08 \\ 0.12 & 0.08 & 0.30 \end{bmatrix} \quad \text{pu}$$

$$\mathbf{Z}_{bus}^1 = \mathbf{Z}_{bus}^2 = j \begin{bmatrix} 0.16 & 0.10 & 0.15 \\ 0.10 & 0.20 & 0.12 \\ 0.15 & 0.12 & 0.25 \end{bmatrix} \quad \text{pu}$$

Determine the per unit fault current and the bus voltages during fault for
(a) A bolted three-phase fault at bus 2.
(b) A bolted single line-to-ground fault at bus 2.
(c) A bolted line-to-line fault at bus 2.
(d) A bolted double line-to-ground fault at bus 2.

(a) The symmetrical components of fault current for a bolted balanced three-phase fault at bus 2 is given by

$$I_2^{012}(F) = \begin{bmatrix} 0 \\ \frac{1}{Z_{22}^1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{j0.20} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix}$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5\angle -90^\circ \\ 5\angle 150^\circ \\ 5\angle 30^\circ \end{bmatrix}$$

For balanced fault we only have the positive-sequence component of voltage. Thus, from (10.98), bus voltages during fault for phase a are

$$\begin{aligned} V_1(F) &= 1 - Z_{12}^1 I_2(F) = 1 - j0.10(-j5) = 0.5 \\ V_2(F) &= 1 - Z_{22}^1 I_2(F) = 1 - j0.20(-j5) = 0.0 \\ V_3(F) &= 1 - Z_{32}^1 I_2(F) = 1 - j0.12(-j5) = 0.4 \end{aligned}$$

(b) From (10.90), the symmetrical components of fault current for a single line-to-ground fault at bus 2 is given by

$$\begin{aligned} I_2^0(F) = I_2^1(F) = I_2^2(F) &= \frac{1.0}{Z_{22}^1 + Z_{22}^2 + Z_{22}^0} \\ &= \frac{1.0}{j0.20 + j0.20 + j0.10} = -j2 \end{aligned}$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j2 \\ -j2 \\ -j2 \end{bmatrix} = \begin{bmatrix} 6\angle -90^\circ \\ 0\angle 0^\circ \\ 0\angle 0^\circ \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(-j2) \\ 1 - j0.10(-j2) \\ 0 - j0.10(-j2) \end{bmatrix} = \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(-j2) \\ 1 - j0.20(-j2) \\ 0 - j0.20(-j2) \end{bmatrix} = \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(-j2) \\ 1 - j0.12(-j2) \\ 0 - j0.12(-j2) \end{bmatrix} = \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix} = \begin{bmatrix} 0.50\angle 0^\circ \\ 0.9539\angle -114.79^\circ \\ 0.9539\angle +114.79^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix} = \begin{bmatrix} 0.0\angle 0^\circ \\ 0.9165\angle -109.11^\circ \\ 0.9165\angle +109.11^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix} = \begin{bmatrix} 0.36\angle 0^\circ \\ 0.9625\angle -115.87^\circ \\ 0.9625\angle +115.87^\circ \end{bmatrix}$$

(c) From (10.92) and (10.93), the symmetrical components of fault current for line-to-line fault at bus 2 are

$$I_2^0 = 0$$

$$I_2^1 = -I_2^2 = \frac{V_2(0)}{Z_{22}^1 + Z_{22}^2} = \frac{1}{j0.20 + j0.20} = -j2.5$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2.5 \\ j2.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -4.33 \\ 4.33 \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.10(-j2.5) \\ 0 - j0.10(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.20(-j2.50) \\ 0 - j0.20(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.12(-j2.5) \\ 0 - j0.12(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6614\angle{-139.11^\circ} \\ 0.614\angle{+130.11^\circ} \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.50\angle 180^\circ \\ 0.50\angle{+180^\circ} \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6083\angle{-145.285^\circ} \\ 0.6083\angle{+145.285^\circ} \end{bmatrix}$$

(d) From (10.94)–(10.96), the symmetrical components of fault current for a double line-to-ground fault at bus 2 is given by

$$I_2^1 = \frac{V_2(0)}{Z_{22}^1 + \frac{Z_{22}^2(Z_{22}^0)}{Z_{22}^2 + Z_{22}^0}} = -\frac{1}{j0.20 + \frac{j0.20(j0.10)}{j0.20 + j0.10}} = -j3.75$$

$$I_2^2 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^2} - \frac{1 - j0.20(-j3.75)}{j0.20} = j1.25$$

$$I_2^0 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^0} = -\frac{1 - j0.20(-j3.75)}{j0.20} = j2.5$$

The phase currents at the faulted bus are

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j2.5 \\ -j3.75 \\ j1.25 \end{bmatrix} = \begin{bmatrix} 0 \\ 5.7282\angle 139.11^\circ \\ 5.7282\angle 40.89^\circ \end{bmatrix}$$

and the total fault current is

$$I_2^b + I_2^c = 5.7282\angle 139.11^\circ + 5.7282\angle 40.89^\circ = 7.5\angle 90^\circ$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(j2.5) \\ 1 - j0.10(-j3.75) \\ 0 - j0.10(j1.25) \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(j2.5) \\ 1 - j0.20(-j3.75) \\ 0 - j0.20(j1.25) \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(j2.5) \\ 1 - j0.12(-j3.75) \\ 0 - j0.12(j1.25) \end{bmatrix} = \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 0.875\angle 0^\circ \\ 0.50\angle{-120^\circ} \\ 0.50\angle{+120^\circ} \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.75\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 0.90\angle 0^\circ \\ 0.3775\angle{-113.413^\circ} \\ 0.3775\angle{+113.413^\circ} \end{bmatrix}$$

10.15. The reactance data for the power system shown in Figure 85 in per unit on a common base is as follows:

FIGURE 85
The impedance diagram for Problem 10.15.

Obtain the Thévenin sequence impedances for the fault at bus 1 and compute the fault current in per unit for the following faults:

(a) A bolted three-phase fault at bus 1.
(b) A bolted single line-to-ground fault at bus 1.
(c) A bolted line-to-line fault at bus 1.
(d) A bolted double line-to-ground fault at bus 1.

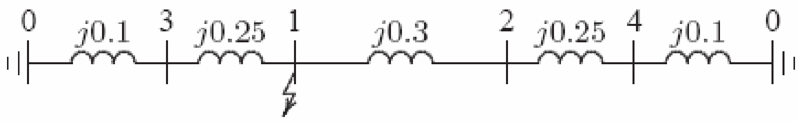

FIGURE 86
Positive-sequence impedance network for Problem 10.15.

The positive-sequence impedance network is shown in Figure 86, and impedance to the point of fault is

$$Z^1 = j\frac{(0.35)(0.65)}{0.35 + 0.65} = j0.2275 \text{ pu}$$

Since negative-sequence reactances are the same as positive-sequence reactances, $X^2 = X^1 = 0.2275$. The zero-sequence impedance network is shown in Figure 87, and impedance to the point of fault is

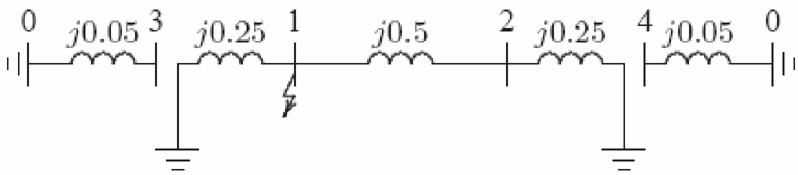

FIGURE 87
zero-sequence impedance network for Problem 10.15.

$$Z^0 = j\frac{(0.25)(0.75)}{0.25 + 0.75} = j0.1875 \text{ pu}$$

(a) For a bolted three-phase fault at bus 1, the fault current is

$$I_f = \frac{1}{j0.2275} = 4.3956\angle{-90^\circ} \text{ pu}$$

(b) For a bolted single-line to ground fault at bus 1, the fault current is

$$I_f = 3I_a^0 = \frac{3}{j(0.2275 + 0.2275 + 0.1875)} = 4.669\angle{-90^\circ} \text{ pu}$$

(c) For a bolted line-to-line fault at bus 1, the fault current in phase b is

$$I_a^1 = \frac{1}{j(0.2275 + 0.2275)} = -j2.1978 \text{ pu}$$

$$I_b(F) = -j\sqrt{3}I_a^1 = -3.8067 \text{ pu}$$

(d) For a bolted double line-to-line fault at bus 1, we have

$$I_a^1 = \frac{1}{j0.2275 + +j\frac{(0.2275)(0.1875)}{0.2275+0.1875}} = -j3.02767 \text{ pu}$$

$$I_a^0 = \frac{1 - (0.2275)(-j3.02767)}{j0.1875} = j1.65975 \text{ pu}$$

$$I(F) = 3I_a^0 = 4.979\angle 90^\circ \text{ pu}$$