8.1. A sinusoidal voltage given by $v(t) = 390\sin(315t + \alpha)$ is suddenly applied to a series RL circuit. $R = 32\ \Omega$ and $L = 0.4$ H.
(a) The switch is closed at such a time as to permit no transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.
(b) The switch is closed at such a time as to permit maximum transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.
(c) What is the maximum value of current in part (b) and at what time does this occur after the switch is closed?

$$v(t) = 390\sin(315t + \alpha)$$
$$i(t) = I_m \sin(315t + \alpha - \gamma) - I_m e^{-t/\tau}\sin(\alpha - \gamma)$$

(a) For no transient $\alpha = \gamma$

$$\gamma = \tan^{-1}\frac{(315)(0.4)}{32} = 75.75^\circ \quad \Rightarrow \quad \alpha = 75.75^\circ$$
$$Z = 32 + j(315)(0.4) = 32 + j126 = 130\angle 75.75^\circ \quad \Omega$$
$$I = \frac{390}{130} = 3 \text{ A} \quad \Rightarrow \quad i(t) = 3\sin 315t$$

(b) For maximum transient current $\alpha - \gamma = -90^\circ$. Therefore, $\alpha = 75.75 - 90 = -14.25^\circ$, and $\tau = \frac{L}{R} = 0.0125$ sec, and the current is

$$i(t) = 3\sin(315t - \frac{\pi}{2}) + 3e^{-80t}$$

(c)

$$\frac{di(t)}{dt} = (3)(315)\cos(315t - \frac{\pi}{2}) - 240e^{-80t} = 0$$

Use the command `[Imax, k] = max(i), tmax = t(k)` to find the maximum value of current, and the corresponding time.

$$i_{max} = 4.371 \ \text{A}$$
$$t_{max} = 0.0096 \ \text{sec}$$

Save the following statements in a file named **ch8p1.m**, and run to obtain the plots.

```
disp('Problem 8.1')
R = 32; L = 0.4; tau= L/R;w=315; X=w*L;
Z=R+j*X; Vm = 390;
gama=angle(Z);
gamad=gama*180/pi;
Im=Vm/130;
disp('(a)')
alpha1= gama; alpha = alpha1*180/pi
t=0:.0001:.08;
i1=Im*sin(315*t+alpha1 -gama);
subplot(211), plot(t, i1), grid
xlabel('t, sec'), ylabel('i(t)')
[I1max,k] = max(i1)
tmax= t(k)
disp('(b), (c)')
alpha2=gama - pi/2;  alpha = alpha2*180/pi
i2=Im*sin(315*t+alpha2 -gama)-Im*exp(-t/tau).*sin(alpha2-gama);
subplot(212), plot(t, i2), grid
xlabel('t, sec'), ylabel('i(t)')
[I2max,k] = max(i2)
tmax= t(k)
```

9.1. The system shown in Figure 65 is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reactances are marked on the diagram. All resistances are neglected. The line impedance is $j160\ \Omega$. A three-phase balanced fault occurs at the receiving end of the transmission line. Determine the short-circuit current and the short-circuit MVA.

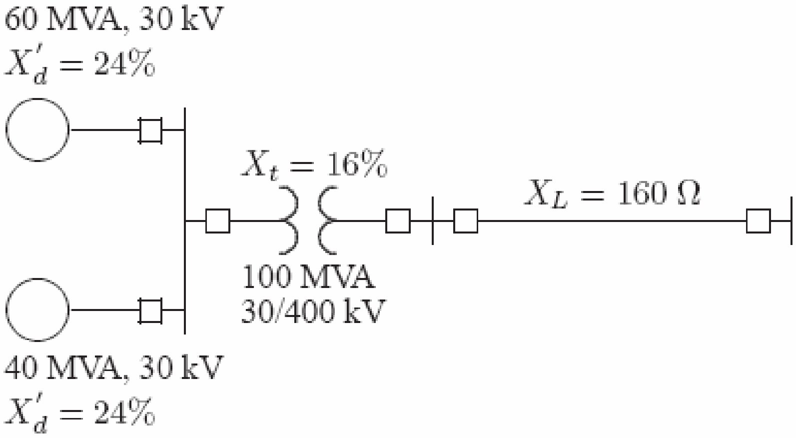

FIGURE 65
One-line diagram for Problem 9.1.

The base impedance for line is

$$Z_B = \frac{(400)^2}{100} = 1,600\ \ \Omega$$

and the base current is

$$I_B = \frac{100,000}{\sqrt{3}(400)} = 144.3375 \text{ A}$$

The reactances on a common 100 MVA base are

$$X'_{dg1} = \frac{100}{60}(0.24) = 0.4 \text{ pu}$$
$$X'_{dg2} = \frac{100}{40}(0.24) = 0.6 \text{ pu}$$
$$X_t = \frac{100}{100}(0.16) = 0.16 \text{ pu}$$
$$X_{line} = \frac{160}{1600} = 0.1 \text{ pu}$$

The impedance diagram is as shown in Figure 66.

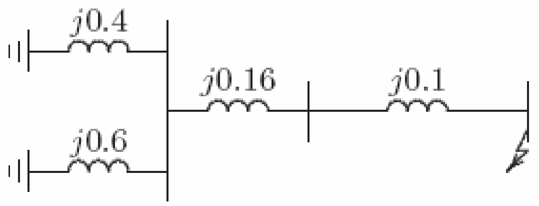

FIGURE 66
The impedance diagram for Problem 9.1.

Impedance to the point of fault is

$$X = j\frac{(0.4)(0.6)}{0.4+0.6} + j0.16 + j0.1 = j0.5 \text{ pu}$$

The fault current is

$$I_f = \frac{1}{j0.5} = 2\angle{-90^\circ} \text{ pu}$$
$$= (144.3375)(2\angle{-90^\circ}) = 288.675\angle{-90^\circ} \text{ A}$$

The Short-circuit MVA is

$$\text{SCMVA} = \sqrt{3}(400)(288.675)(10^{-3}) = 200 \text{ MVA}$$

9.3. The one-line diagram of a simple power system is shown in Figure 69. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 1 through a fault impedance of $Z_f = j0.08$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

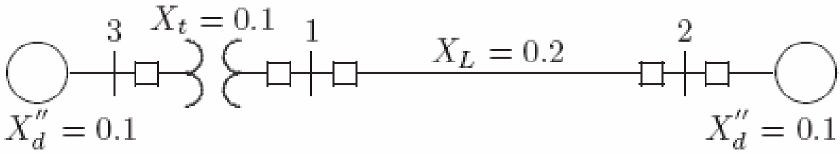

FIGURE 69
One-line diagram for Problem 9.3.

The impedance diagram is as shown in Figure 70.
(a) Impedance to the point of fault is

$$X = j\frac{(0.2)(0.3)}{0.2 + 0.3} = j0.12 \ \text{pu}$$

The fault current is

$$I_f = \frac{1}{j0.12 + j0.08} = 5\angle{-90^\circ} \ \text{pu}$$

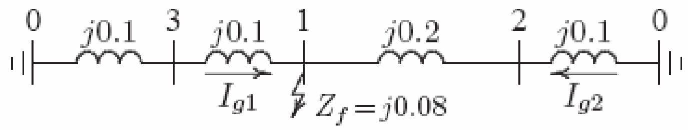

FIGURE 70
The impedance diagram for Problem 9.3.

(b)

$$V_1 = (j0.08)(-j5) = 0.4 \text{ pu}$$
$$I_{g1} = \frac{j0.3}{j0.5}(5)\angle{-90^\circ} = 3\angle{-90^\circ} \text{ pu}$$
$$I_{g2} = \frac{j0.2}{j0.5}(5)\angle{-90^\circ} = 2\angle{-90^\circ} \text{ pu}$$
$$V_2 = 0.4 + (j0.2)(-j2) = 0.8 \text{ pu}$$
$$V_3 = 0.4 + (j0.1)(-j3) = 0.7 \text{ pu}$$

9.4. The one-line diagram of a simple three-bus power system is shown in Figure 71 Each generator is represented by an emf behind the subtransient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

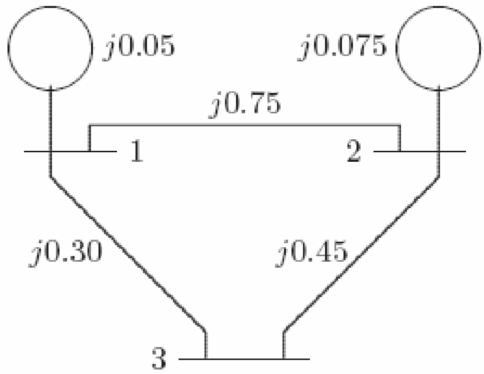

FIGURE 71
One-line diagram for Problem 9.4.

Converting the Δ formed by buses 123 to an equivalent Y as shown in Figure 72(a), we have

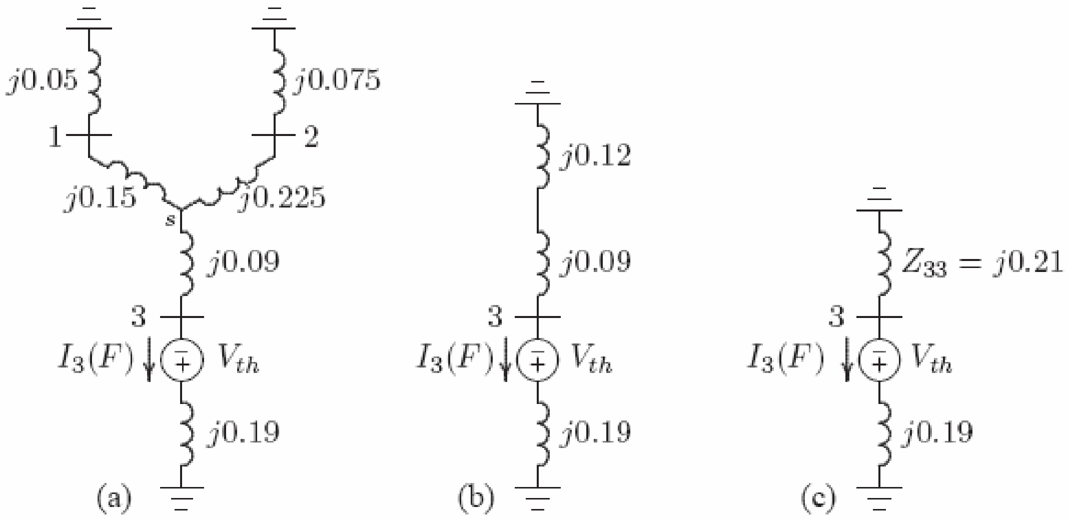

FIGURE 72
Reduction of Thévenin's equivalent network.

$$Z_{1s} = \frac{(j0.3)(j0.75)}{j1.5} = j0.15 \qquad Z_{2s} = \frac{(j0.75)(j0.45)}{j1.5} = j0.225$$

$$Z_{3s} = \frac{(j0.3)(j0.45)}{j1.5} = j0.09$$

Combining the parallel branches, Thévenin's impedance is

$$\begin{aligned} Z_{33} &= \frac{(j0.2)(j0.3)}{j0.2 + j0.3} + j0.09 \\ &= j0.12 + j0.09 = j0.21 \end{aligned}$$

From Figure 72(c), the fault current is

$$I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \quad \text{pu}$$

With reference to Figure 72(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.3}{j0.2 + j0.3} I_3(F) = -j1.5 \quad \text{pu}$$

$$I_{G2} = \frac{j0.2}{j0.2 + j0.3} I_3(F) = -j1.0 \quad \text{pu}$$

For the bus voltage changes from Figure 72(a), we get

$$\begin{aligned} \Delta V_1 &= 0 - (j0.05)(-j1.5) \;\; = -0.075 \;\; \text{pu} \\ \Delta V_2 &= 0 - (j0.075)(-j1) \;\; = -0.075 \;\; \text{pu} \\ \Delta V_3 &= (j0.19)(-j2.5) - 1.0 = -0.525 \;\; \text{pu} \end{aligned}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$\begin{aligned} V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.075 = 0.925 \text{ pu} \\ V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.075 = 0.925 \text{ pu} \\ V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.525 = 0.475 \text{ pu} \end{aligned}$$

The short circuit-currents in the lines are

$$\begin{aligned} I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \quad \text{pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \quad \text{pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \quad \text{pu} \end{aligned}$$

9.6. Using the method of building algorithm find the bus impedance matrix for the network shown in Figure 76.

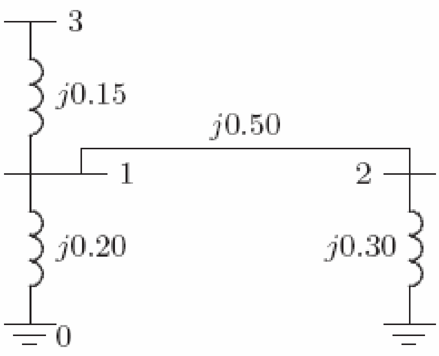

FIGURE 76
One-line diagram for Problem 9.6.

Add branch 1, $z_{10} = j0.2$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.20$$

Next, add branch 2, $z_{20} = j0.3$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 \\ 0 & j0.3 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.15$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.5$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\begin{aligned}\mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix} \\ &= \begin{bmatrix} j0.2 & 0 & j0.2 & -j0.2 \\ 0 & j0.3 & 0 & j0.3 \\ j0.2 & 0 & j0.35 & -j0.2 \\ -j0.2 & j0.3 & -j0.2 & Z_{44} \end{bmatrix}\end{aligned}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.5 + j0.2 + j0.3 - 2(j0) = j1.0$$

and

$$\begin{aligned}\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} &= \frac{1}{j1.0}\begin{bmatrix} -j0.2 \\ j0.3 \\ -j0.2 \end{bmatrix}\begin{bmatrix} -j0.2 & j0.3 & -j0.2 \end{bmatrix} \\ &= \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix}\end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned}\mathbf{Z}_{bus} &= \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix} \\ &= \begin{bmatrix} j0.16 & j0.06 & j0.16 \\ j0.06 & j0.21 & j0.06 \\ j0.16 & j0.06 & j0.31 \end{bmatrix}\end{aligned}$$

9.7. Obtain the bus impedance matrix for the network of Problem 9.3.

Add branch 1, $z_{20} = j0.1$ between node $q = 2$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{22} = z_{20} = j0.10$$

Next, add branch 2, $z_{30} = j0.1$ between node $q = 3$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{22} & 0 \\ 0 & Z_{33} \end{bmatrix} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.1$ between the new node $q = 1$ and the existing node $p = 3$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{33} + z_{13} & 0 & Z_{33} \\ 0 & Z_{22} & 0 \\ Z_{33} & 0 & Z_{33} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix}$$

Add link 4, $z_{12} = j0.2$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$

$$= \begin{bmatrix} j0.2 & 0 & j0.1 & -j0.2 \\ 0 & j0.1 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & -j0.1 \\ -j0.2 & j0.1 & -j0.1 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.2 + j0.2 + j0.1 - 2(j0) = j0.5$$

and

$$\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} = \frac{1}{j0.5}\begin{bmatrix} -j0.2 \\ j0.1 \\ -j0.1 \end{bmatrix}\begin{bmatrix} -j0.2 & j0.1 & -j0.1 \end{bmatrix}$$
$$= \begin{bmatrix} j0.08 & -j0.04 & j0.04 \\ -j0.04 & j0.02 & -j0.02 \\ j0.04 & -j0.02 & j0.02 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.2 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix} - \begin{bmatrix} j0.08 & -j0.04 & j0.04 \\ -j0.04 & j0.02 & -j0.02 \\ j0.04 & -j0.02 & j0.02 \end{bmatrix}$$
$$= \begin{bmatrix} j0.12 & j0.04 & j0.06 \\ j0.04 & j0.08 & j0.02 \\ j0.06 & j0.02 & j0.08 \end{bmatrix}$$

9.9. The bus impedance matrix for the network shown in Figure 77 is given by

$$Z_{bus} = j\begin{bmatrix} 0.300 & 0.200 & 0.275 \\ 0.200 & 0.400 & 0.250 \\ 0.275 & 0.250 & 0.41875 \end{bmatrix}$$

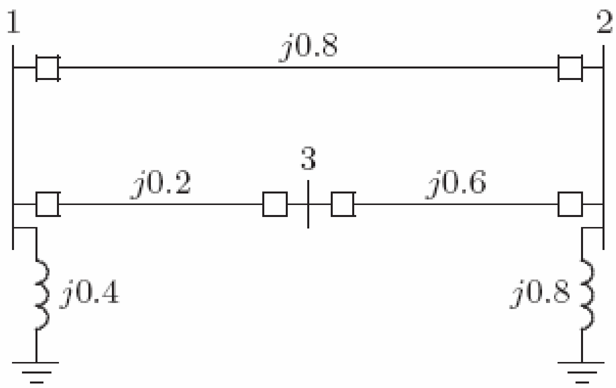

FIGURE 77
One-line diagram for Problem 9.9.

There is a line outage and the line from bus 1 to 2 is removed. Using the method of building algorithm determine the new bus impedance matrix.

The line between buses 1 and 2 with impedance $Z_{12} = j0.8$ is removed. The removal of this line is equivalent to connecting a link having an impedance equal to the negated value of the original impedance. Therefore, we add link $z_{12} = -j0.8$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$

Thus, we get

$$\mathbf{Z}_{bus}^{(1)} = \begin{bmatrix} j0.300 & j0.200 & j0.27500 & -j0.100 \\ j0.200 & j0.400 & j0.25000 & j0.200 \\ j0.275 & j0.250 & j0.41875 & -j0.025 \\ -j0.100 & j0.200 & -j0.02500 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = -j0.8 + j0.3 + j0.4 - 2(j0.2) = -j0.5$$

and

$$\begin{aligned} \frac{\Delta \mathbf{Z}\, \Delta \mathbf{Z}^T}{Z_{44}} &= \frac{1}{-j0.5} \begin{bmatrix} -j0.100 \\ j0.200 \\ -j0.025 \end{bmatrix} \begin{bmatrix} -j0.10 & j0.20 & -j0.025 \end{bmatrix} \\ &= \begin{bmatrix} -j0.020 & j0.040 & -j0.0050 \\ j0.040 & -j0.080 & j0.0100 \\ -j0.005 & j0.010 & -j0.0013 \end{bmatrix} \end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned} \mathbf{Z}_{bus} &= \begin{bmatrix} j0.300 & j0.200 & j0.27500 \\ j0.200 & j0.400 & j0.25000 \\ j0.275 & j0.250 & j0.41875 \end{bmatrix} - \begin{bmatrix} -j0.020 & j0.040 & -j0.00500 \\ j0.040 & -j0.080 & j0.01000 \\ -j0.005 & j0.010 & -j0.00125 \end{bmatrix} \\ &= \begin{bmatrix} j0.320 & j0.160 & j0.280 \\ j0.160 & j0.480 & j0.240 \\ j0.280 & j0.240 & j0.420 \end{bmatrix} \end{aligned}$$

9.10. The per unit bus impedance matrix for the power system of Problem 9.4 is given by

$$Z_{bus} = j \begin{bmatrix} 0.0450 & 0.0075 & 0.0300 \\ 0.0075 & 0.06375 & 0.0300 \\ 0.0300 & 0.0300 & 0.2100 \end{bmatrix}$$

A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault. Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a fault at bus 3 with fault impedance $Z_f = j0.19$ per unit, the fault current is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$V_1(F) = V_1(0) - Z_{13}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \text{ pu}$$
$$V_2(F) = V_2(0) - Z_{23}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \text{ pu}$$
$$V_3(F) = V_3(0) - Z_{33}I_3(F) = 1.0 - (j0.21)(-j2.5) = 0.475 \text{ pu}$$

From (9.25), the short circuit currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \text{ pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \text{ pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \text{ pu}$$

To check the result, we form the line data for the network of Problem 9.4, and we use the following commands

```
zdata=[0   1   0   0.05
       0   2   0   0.075
       1   2   0   0.75
       1   3   0   0.30
       2   3   0   0.45];
Zbus=zbuild(zdata)
symfault(zdata, Zbus)
```

The result is

```
Zbus =

     0 + 0.0450i      0 + 0.0075i      0 + 0.0300i
     0 + 0.0075i      0 + 0.0638i      0 + 0.0300i
     0 + 0.0300i      0 + 0.0300i      0 + 0.2100i
```

```
Enter Faulted Bus No. -> 3
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*.19

Balanced three-phase fault at bus No. 3
Total fault current =   2.5000 per unit

Bus Voltages during fault in per unit

     Bus      Voltage       Angle
     No.      Magnitude     degrees
      1        0.9250       0.0000
      2        0.9250       0.0000
      3        0.4750       0.0000

Line currents for fault at bus No.  3

     From       To      Current      Angle
     Bus        Bus     Magnitude   degrees
      G          1       1.5000    -90.0000
      1          2       0.0000    -90.0000
      1          3       1.5000    -90.0000
      G          2       1.0000    -90.0000
      2          3       1.0000    -90.0000
      3          F       2.5000    -90.0000
Another fault location? Enter'y' or'n' within single quote->'n'
```