2.7. Two impedances, $Z_1 = 0.8 + j5.6\ \Omega$ and $Z_2 = 8 - j16\ \Omega$, and a single-phase motor are connected in parallel across a 200-V rms, 60-Hz supply as shown in Figure 8. The motor draws 5 kVA at 0.8 power factor lagging.

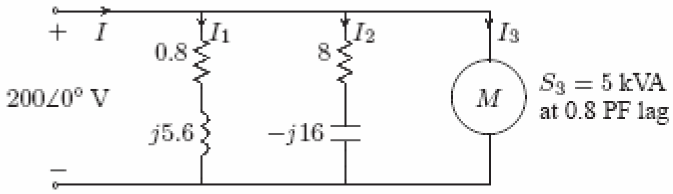

FIGURE 8
Circuit for Problem 2.7.

(a) Find the complex powers S_1, S_2 for the two impedances, and S_3 for the motor.
(b) Determine the total power taken from the supply, the supply current, and the overall power factor.
(c) A capacitor is connected in parallel with the loads. Find the kvar and the capacitance in μF to improve the overall power factor to unity. What is the new line current?

(a) The load complex power are

$$S_1 = \frac{|V|^2}{Z_1^*} = \frac{(200)^2}{0.8 - j5.6} = 1000 + j7000 \ \ \text{VA}$$
$$S_2 = \frac{|V|^2}{Z_2^*} = \frac{(200)^2}{8 + j16} = 1000 - j2000 \ \ \text{VA}$$
$$S_3 = 5000\angle 36.87^\circ = 4000 + j3000 \ \ \text{VA}$$

Therefore, the total complex power is

$$S_t = 6 + j8 = 10\angle 53.13^\circ \ \text{kVA}$$

(b) From $S = VI^*$, the current is

$$I = \frac{10000\angle{-53.13^\circ}}{200\angle 0^\circ} = 50\angle{-53.13} \ \ \text{A}$$

and the power factor is $\cos 53.13^\circ = 0.6$ lagging.

(c) For overall unity power factor, $Q_C = 8000$ Var, and the capacitive impedance is

$$Z_C = \frac{|V|^2}{S_C{}^*} = \frac{(200)^2}{j8000} = -j5 \ \ \Omega$$

and the capacitance is

$$C = \frac{10^6}{(2\pi)(60)(5)} = 530.5 \ \ \mu\text{F}$$

The new current is

$$I = \frac{6000\angle 0^\circ}{200\angle 0^\circ} = 30\angle 0 \ \ \text{A}$$

3.12. A 40-MVA, 20-kV/400-kV, single-phase transformer has the following series impedances:

$Z_1 = 0.9 + j1.8\ \Omega \quad \text{and} \quad Z_2 = 128 + j288\ \Omega$

Using the transformer rating as base, determine the per unit impedance of the transformer from the ohmic value referred to the low-voltage side. Compute the per unit impedance using the ohmic value referred to the high-voltage side.

The transformer equivalent impedance referred to the low-voltage side is

$$Z_{e1} = 0.9 + j1.8 + \left(\frac{20}{400}\right)^2 (128 + j288) = 1.22 + j2.52\ \Omega$$

The low-voltage base impedance is

$$Z_{B1} = \frac{(20)^2}{40} = 10\ \Omega$$

$$Z_{pu1} = \frac{1.22 + j2.52}{10} = 0.122 + j0.252 \text{ pu}$$

The transformer equivalent impedance referred to the high-voltage side is

$$Z_{e2} = \left(\frac{400}{20}\right)^2 (0.9 + j1.8) + (128 + j288) = 488 + j1008\ \Omega$$

The high-voltage base impedance is

$$Z_{B2} = \frac{(400)^2}{40} = 4000\ \Omega$$

$$Z_{pu2} = \frac{488 + j1008}{4000} = 0.122 + j0.252 \text{ pu}$$

We note that the transformer per unit impedance has the same value regardless of whether it is referred to the primary or the secondary side.

3.13. Draw an impedance diagram for the electric power system shown in Figure 26 showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

G_1 :	90 MVA	20 kV	$X = 9\%$
T_1 :	80 MVA	20/200 kV	$X = 16\%$
T_2 :	80 MVA	200/20 kV	$X = 20\%$
G_2 :	90 MVA	18 kV	$X = 9\%$
Line:		200 kV	$X = 120\ \Omega$
Load:		200 kV	$S = 48 \text{ MW} + j64 \text{ Mvar}$

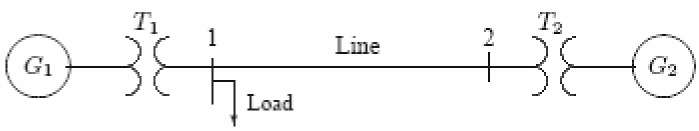

FIGURE 26
One-line diagram for Problem 3.13

The base voltage V_{BG1}on the LV side of T_1 is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20\left(\frac{200}{20}\right) = 200 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{BG2} = 200\left(\frac{20}{200}\right) = 20 \quad \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69)and (3.70) are

$$G: \quad X = 0.09\left(\frac{100}{90}\right) = 0.10 \text{ pu}$$

$$T_1: \quad X = 0.16\left(\frac{100}{80}\right) = 0.20 \text{ pu}$$

$$T_2: \quad X = 0.20\left(\frac{100}{80}\right) = 0.25 \text{ pu}$$

$$G_2: \quad X = 0.09\left(\frac{100}{90}\right)\left(\frac{18}{20}\right)^2 = 0.081 \text{ pu}$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \Omega$$

The per unit line reactance is

$$Line: \quad X = \left(\frac{120}{400}\right) = 0.30 \text{ pu}$$

The load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(200)^2}{48 - j64} = 300 + j400 \ \Omega$$

The load impedance in per unit is

$$Z_{L(pu)} = \frac{300 + j400}{400} = 0.75 + j1.0 \text{ pu}$$

The per unit equivalent circuit is shown in Figure 27.

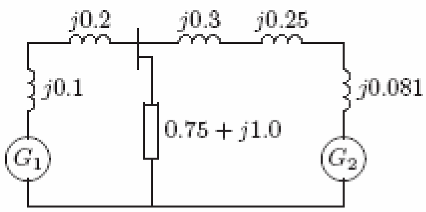

FIGURE 27
Per unit impedance diagram for Problem 3.11.

3.15. The three-phase power and line-line ratings of the electric power system shown in Figure 30 are given below.

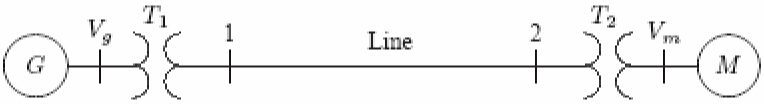

FIGURE 30
One-line diagram for Problem 3.15

G_1 :	60 MVA	20 kV	$X = 9\%$
T_1 :	50 MVA	20/200 kV	$X = 10\%$
T_2 :	50 MVA	200/20 kV	$X = 10\%$
M :	43.2 MVA	18 kV	$X = 8\%$
Line:		200 kV	$Z = 120 + j200\ \Omega$

(a) Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

(b) The motor is drawing 45 MVA, 0.80 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

The base voltage V_{BG1} on the LV side of T_1 is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20\left(\frac{200}{20}\right) = 200 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{Bm} = 200\left(\frac{20}{200}\right) = 20 \quad \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69)and (3.70) are

$$G: \quad X = 0.09\left(\frac{100}{60}\right) = 0.15 \text{ pu}$$

$$T_1: \quad X = 0.10\left(\frac{100}{50}\right) = 0.20 \text{ pu}$$

$$T_2: \quad X = 0.10\left(\frac{100}{50}\right) = 0.20 \text{ pu}$$

$$M: \quad X = 0.08\left(\frac{100}{43.2}\right)\left(\frac{18}{20}\right)^2 = 0.15 \text{ pu}$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \Omega$$

The per unit line impedance is

$$Line: \quad Z_{line} = (\frac{120 + j200}{400}) = 0.30 + j0.5 \text{ pu}$$

The per unit equivalent circuit is shown in Figure 31.

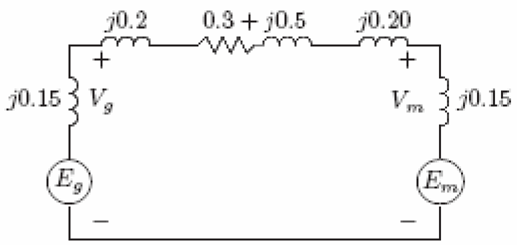

FIGURE 31
Per unit impedance diagram for Problem 3.15.

(b) The motor complex power in per unit is

$$S_m = \frac{45\angle 36.87^\circ}{100} = 0.45\angle 36.87^\circ \text{ pu}$$

and the motor terminal voltage is

$$V_m = \frac{18\angle 0^\circ}{20} = 0.90\angle 0^\circ \text{ pu}$$

$$I = \frac{0.45\angle{-36.87°}}{0.90\angle{0°}} = 0.5\angle{-36.87°} \text{ pu}$$

$$V_g = 0.90\angle{0°} + (0.3 + j0.9)(0.5\angle{-36.87°} = 1.31795\angle{11.82°} \text{ pu}$$

Thus, the generator line-to-line terminal voltage is

$$V_g = (1.31795)(20) = 26.359 \text{ kV}$$

$$E_g = 0.90\angle{0°} + (0.3 + j1.05)(0.5\angle{-36.87°} = 1.375\angle{13.88°} \text{ pu}$$

Thus, the generator line-to-line internal emf is

$$E_g = (1.375)(20) = 27.5 \text{ kV}$$

3.16. The one-line diagram of a three-phase power system is as shown in Figure 32. Impedances are marked in per unit on a 100-MVA, 400-kV base. The load at bus 2 is $S_2 = 15.93$ MW $-j33.4$ Mvar, and at bus 3 is $S_3 = 77$ MW $+j14$ Mvar. It is required to hold the voltage at bus 3 at $400\angle{0°}$ kV. Working in per unit, determine the voltage at buses 2 and 1.

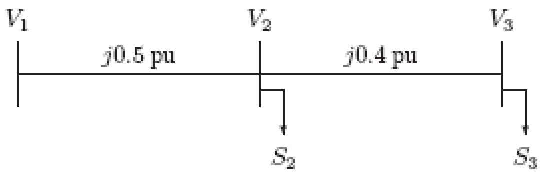

FIGURE 32
One-line diagram for Problem 3.16

$$S_2 = 15.93 \text{ MW} - j33.4 \text{ Mvar} = 0.1593 - j0.334 \text{ pu}$$
$$S_3 = 77.00 \text{ MW} + j14.0 \text{ Mvar} = 0.7700 + j0.140 \text{ pu}$$
$$V_3 = \frac{400\angle{0°}}{400} = 1.0\angle{0°} \text{ pu}$$
$$I_3 = \frac{S_3^*}{V_3^*} = \frac{0.77 - j0.14}{1.0\angle{0°}} = 0.77 - j0.14 \text{ pu}$$
$$V_2 = 1.0\angle{0°} + (j0.4)(0.77 - j0.14) = 1.1\angle{16.26°} \text{ pu}$$

Therefore, the line-to-line voltage at bus 2 is

$$V_2 = (400)(1.1) = 440 \text{ kV}$$
$$I_2 = \frac{S_2^*}{V_2^*} = \frac{0.1593 + j0.334}{1.1\angle{-16.26°}} = 0.054 + j0.332 \text{ pu}$$
$$I_{12} = (0.77 - j0.14) + (0.054 + j0.332) = 0.824 + j0.192 \text{ pu}$$
$$V_1 = 1.1\angle{16.26°} + (j0.5)(0.824 + j0.192) = 1.2\angle{36.87°} \text{ pu}$$

Therefore, the line-to-line voltage at bus 1 is

$$V_1 = (400)(1.2) = 480 \text{ kV}$$