USING LOCAL STRUCTURE FOR THE RELIABLE REMOVAL OF NOISE FROM THE OUTPUT OF THE LoG EDGE DETECTOR

Ahmad A. Masoud, Mohammed M. Bayoumi
Robotics Laboratory
Electrical Engineering Department
Queen's University, Kingston,Ontario, Canada K7L-3N6

ABSTRACT

In this paper a method is suggested for enhancing the performance of the Laplacian-of-Gaussian edge detector by reliably removing false edges that are created by noise from its output. The discrimination between false and valid edges is based on the distance between successive edge contours. Statistical analysis of the performance and simulation results are,also, provided.

Introduction :

Edge detection has caught the attention of researchers for over two decades. The prime motivation behind such an interest is due to several results [1,2] conjecturing that the edges of an image and the structure of intensity surrounding them contains much of the essential information about the scene; therefore, offering a compact and, if properly performed, rich representation of the image. The problem is by no mean simple, with difficulties starting as early as defining what an edge is [3]. Its importance is central to areas like machine vision, navigation, coding and restoration of images, and scene representation and understanding. Understandably, the literature tackling this problem is huge, and the principles employed for the solution are greatly diverse. In a recent article surveying the state of art in edge detection [4] it was noted that " edge detection techniques are reaching a performance plateau in which increased sophistication is not producing a commensurate improvement in performance". It was, also, concluded that " further hammering of the problem at the signal level will prove largely fruitless". While we strongly agree with the first claim, we believe, as is shown in this work, that significant improvements in terms of reducing complexity and enhancing performance can still be achieved by tackling the problem at the signal level.

One of the most popular techniques for edge detection utilizes the Laplacian-of-Gaussian operator (LoG) proposed by Marr [5]. In this technique, edges (E(x)) are located as the zero crossings (ZC) of the convolved signal (I(x)) with the Laplacian of the function $G(x,\sigma)$ given by:

$$G(x,\sigma) = \exp(-x^2/2\pi\sigma^2) \qquad (1)$$

$$E(x) = \{x: \nabla^2 G(x,\sigma) * I(x) = 0\}$$

Among the properties of this technique Efficient implementation [6], accurate localization [7,8], orientation, invariance high sensitivity, and ability to produce closed edge contours [9]. Although high sensitivity is useful in detecting weak edges, it causes noise amplification rendering the output virtually unusable. One approach to alleviate this problem is to choose the scale (σ) large enough to smooth the signal. Unfortunately, increasing σ increases the computational load, reduces the resolution, increases the chance of Phantom edges happening [10], and most of all it causes a dislocation in the position of the estimated edges [11] preventing the measurement of valuable information about the intensity structure around the edge contours. Another approach tracks the evolution of the ZC through scale space [12]. This approach was later modified to track the evolution of whole zero contours through scale space [13]. A simple and popular method to remove noise start by constructing a measure of the strength of the edge, then uses a threshold to discriminate between false and valid edges. The slope of the convolved signal at a ZC (equal to the third derivative of the signal) was used to construct such a measure [14]. A more robust measure utilizes the magnitude of the signal convolved by the Gradient of Gaussian (GoG) [15]. Testing the validity of a ZC based on the magnitude of the GoG has serious drawbacks. The difficulties are mainly due to the lack of knowledge of the variance of the noise and magnitude of the edge at a particular location. Contributing to that is the conflicting requirement of noise removal which requires a sufficiently high threshold, and the requirement of retaining weak edges which requires a relatively low threshold. If a static threshold is chosen, only a compromise can be achieved. Moreover, all the valid edges falling under the threshold are lost regardless of the noise intensity at that particular location. This prompted canny [16] to use optimal filtering to locally estimate the noise and the signal. Accordingly, an adaptive threshold is used to reduce the loss of valid edges. Although, an adaptive threshold improves the performance, it is,computationally expensive.Moreover, its performance in detecting minute jumps in intensity lying in natural scenes is not satisfactory. In a later work Canny's method was modified to make the scale adaptive too [17].

Our approach for discriminating between the valid and noisy zeros from the LoG detector makes use of a threshold. However, the feature on which this threshold is applied is carefully selected to be weakly coupled to the energy of the edge and at the same time provide a reliable measure of the confidence in the ZC. It, also, has to exhibit an acceptable level of stationary throughout the entire signal, and it should be inexpensive to compute. In this paper we show that the interval between the successive zero crossings of the signal from the LoG operator provides the desired feature. We expect this approach to allow a discrimination mechanism based on a simple static threshold to efficiently function in terms of reliably retaining valid ZC's and effectively removing false ones. Since the burden of noise removal is transferred to the discrimination mechanism, the operator scale (σ) can be kept small for high resolution and accurate localization.

This paper is organized as follows, section II discusses the proposed approach, in section III the performance of the proposed detector is analyzed and compared to the existing method of thretholding. Section IV contains simulation results, and conclusions are placed in section V.

II. THE PROPOSED APPROACH

Our goal is to construct a procedure to discriminate between a ZC generated by a valid edge ($I_1(x) = Au(x)+n(x)$), and a ZC generated by noise only ($I_2(x)=n(x)$). The edge is assumed to be an ideal step jump ($u(x)$) with a magnitude (A). The noise is assumed to be a stationary Additive White Gaussian Noise (AWGN) with zero mean and variance σ_n^2. Let $L_i(x)$ and $D_i(x)$ be :

$$L_i(x)=I_i(x)*\nabla^2 G(x),\quad D_i(x)=I_i(x)*\nabla G(x)$$

For the 1-D case, we have : (2)

$$\nabla^2 G(x)*u(x)=-\nabla G(x),\quad \nabla G(x)*u(x)=G(x)$$

Figure-1a,b shows the signal and noise components of D_i and L_i receptively. If an edge is present, L_i consist of the sum of both the GoG scaled by A, and the filtered noise (Figure-1b). The ZC of the GoG marks the location of the edge (the edge is displaced because of noise). Since the GoG has its highest slope at its ZC with a magnitude peaking around it, we expect with high probability a local region that is depleted of ZC's to exist on both sides of the ZC contours marking the true edges. This can, also, be concluded from the work of Tagare and deFigueiredo [7] where they derived the relative density of ZC's for $L_i(x)$ as a function of x ($\mu_r(x)$) :

$$\mu_r(x)=\exp[-(x\cdot\exp(-\frac{x^2}{\sigma^2}))^2/(2\sigma_n^2)] \tag{3}$$

In Figure-2 μ_r is drawn for different signal to noise ratios (SNR).It can be seen that on the average the distance between successive ZCs of L_i when only noise exist is less than the average distance between a valid ZC and the first false zero (this is discussed in the next section). Therefore, the validity of a ZC (Z_i) can be tested by computing $d_i^- = dist(Z_i, Z_{i-1})$, and $d_i^+ = dist(Z_{i+1}, Z_i)$; then using the following rule to accept or reject Z_i (Figure-3)

$$\text{if}[(d_i^- \geq th) \text{ and } (d_i^+ \geq th)] \quad Z_i \text{ is Valid}$$
$$\text{else} \quad Z_i \text{ is False} \qquad (4)$$

where th is a selected threshold. For the 2-D case d_i^- and d_i^+ are computed along both sides of the normal to the edge contours.

Using the intervals between the ZC's of L_i for discrimination has the advantage of reducing the dependency of the decision process on the magnitude of the edge (A), therefore reducing the need for adaptation to enhance the chance of detecting weak edges. It can be shown that for a relatively wide range of SNR the scale (σ) is the decisive factor in determining $d_{\mp}$. This let them exhibit significantly less fluctuations across the signal than a gradient-based measure does. Therefore, a static threshold on the intervals between the ZC's of L_i achieves far better performance in terms of removing the false ZC and retaining the valid ones than what a threshold on D_i can achieve.

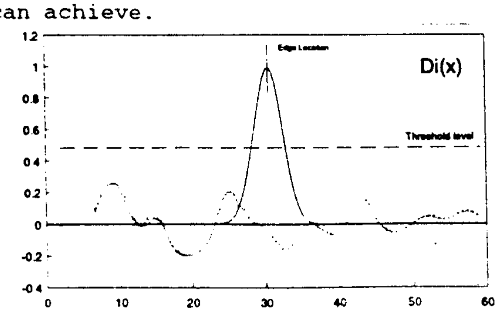

Figure 1a

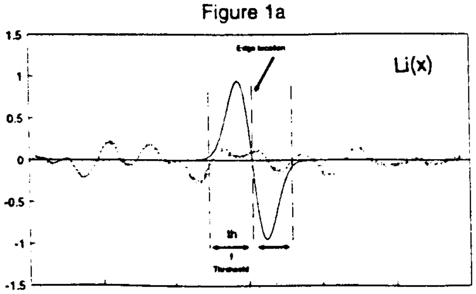

Figure 1b

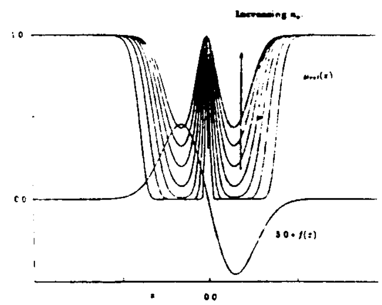
Figure 2

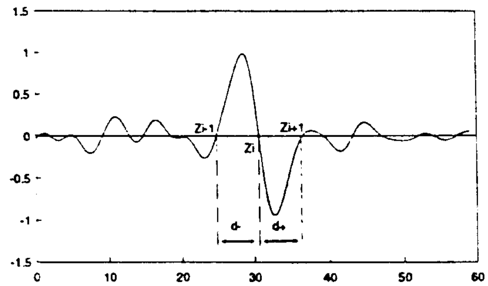

Figure 3

III. ANALYSIS

The performance of the proposed detector is analyzed by deriving the misdetection probability ($1-P_{cs}$), and the false detection probbility ($1-P_{cn}$) P_{cs} is the probability of a valid ZC decided as an edge, and P_{cn} is the probability that a false ZC is rejected. The same quantities are computed for the GoG-based scheme. The performance of both methods is compared using their probability of error (Pe) :

$$P_e = P_1(1-P_{cs}) + P_0(1-P_{cn}) \qquad (5)$$

where P_1 is the probability that a valid ZC occur, and P_0 is the probability that a false ZC occur. Among the factors on which P_1 and P_0 depend: the signal richness in edges, the characteristics of the noise, and the scale of the LoG (σ). Since P_1 and P_0 are not a priori known, they are assumed to be equal ($P_1=P_0=0.5$). The autocorrelation of the noise at the LoG input ($R_i(\tau)$) and output ($R_o(\tau)$) are:

$$R_i(\tau) = \sigma_n^2 \delta(\tau) \qquad (6)$$

$$R_o(\tau) = \frac{\sigma_n^2}{\sigma^7}\sqrt{\frac{\pi}{2}}[3\sigma^4 - 6\sigma^2\tau^2 + \tau^4]e^{-\tau^2/2\sigma^2}$$

$\delta(\tau)$ is the Kronecker delta function. The variances of the noise at the input and output are:

$$\sigma_1^2 = \sigma_n^2,\ \sigma_o^2 = 3\sqrt{\frac{\pi}{2}}[\sigma_n^2/\sigma^3] \quad (7)$$

1. The ZC-based Method :

We need to compute the following Conditional Probability Distribution Functions (PDF's). The first is $P_{no}(\tau)$ which is the conditional probability that the first ZC after time t occur between $t+\tau$ and $t+\tau+d\tau$ given a ZC at time t when the input is I_2. The second is $P_{so}(\tau)$ which is the same except it is computed when the input to the LoG is I_1. For a smooth zero mean Gaussian noise McFadden [18,19] derived an approximation to P_{no} as :

$$P_{no}(\tau) = \frac{\rho''(\tau)(1-\rho(\tau)^2)+\rho(\tau)\rho'(\tau)^2}{2\sqrt{\rho''(0)}\ [1-\rho(\tau)^2]^{3/2}} \quad (8)$$

where $\rho(\tau)$ is the normalized auto correlation function, and ρ', ρ'' are the first and second derivative of ρ with respect to τ. Assuming independence of the intervals between successive ZC's of the output noise. The probability of validating a false ZC is :

$$[\ 1 - \int_0^{th} P_{no}(\tau)d\tau\ ]^2 \quad (9)$$

To compute $P_{so}(\tau)$ we shall first approximate the GoG as follow:

$$G_x(x) = \frac{2A}{\sigma^2}xe^{-\frac{x^2}{\sigma^2}} \approx \begin{cases} A_0\sin(\frac{2\pi}{T_0}x) & |x|\le T_0 \\ 0 & |x|\ge T_0 \end{cases} \quad (10)$$

$T_0=4x_m$, $x_m=\sigma/\sqrt{2}$, and $A_0=\sqrt{2}\cdot e^{-1/2}A/\sigma$

x_m is the distance at which G_x reaches its peak, and A_o is the corresponding magnitude. Figure-4 shows the GoG and its approximation. The fit is good for $|x|\le x_m$. However, for $|x|\ge x_m$ the approximation has a lower magnitude which is acceptable since it provide a lower bound on P_{so}. Using this approximation we can use the result by COBB [20] for the distribution of intervals between the ZC's of a sinwave in AWGN :

$$P_{so}(\tau) = \frac{A_o/\sigma_n^2}{2\sqrt{\pi}\ \omega_o\sqrt{\rho(\tau)-\cos(\omega_o\tau)}} \cdot \left[\frac{\rho'(\tau)}{1+\rho(\tau)}\cos(\frac{\omega_o\tau}{2}) + \omega_o\sin(\frac{\omega_o\tau}{2})\right] \cdot \exp(-\frac{(A_o/\sigma_n^2)^2}{1+\rho(\tau)}\cos(\frac{\omega_o\tau}{2})) \quad (11)$$

where $\omega_o=2\pi/T_o$. This distribution is accurate for relatively high SNR, and tend to P_{no} as SNR goes to zero. For a moderate SNR we shall assume that d+ and d- are strongly dependent. By Bayes theorem [21] we have:

$$P(d+>th, d->th) = P(d+>th/d->th)P(d->th) \approx 1\cdot P(d->th) = P(d+>th)$$

therefore, we can compute P_{cs} as :

$$P_{cs} = 1 - \int_0^{th} P_{so}(\tau)d\tau \quad (12)$$

2. The GoG-based scheme :

Here, P_{cs} and P_{cn} are computed for the following decision rule :

$$\text{if } |D_i(x)| \ge th \quad Z_i \text{ is valid; else } Z_i \text{ is false} \quad (13)$$

To do this we need to compute the follwoing conditional PDF's: $P_{so}(a)$ which is: $P(a<D_i(x)\le a+da/L_i(x)=0)$ when the input is I_1, and P_{no} which is the same, but the input is I_2. R_o at the output of the GoG and its variance are :

$$R_o(\tau) = \frac{\sigma_n^2}{\sigma^3}\sqrt{\frac{\pi}{2}}[\sigma^2-\tau^2]e^{-\tau^2/2\sigma_n^2}$$

$$\sigma_o^2 = \frac{\sigma_n^2}{\sigma}\sqrt{\frac{\pi}{2}} \quad (14)$$

Since the noise is Gaussian and the GoG is a linear operator, the output noise is, also, Gaussian. Assuming a relatively high SNR and a small σ, the dislocation in the position of the edge can be disregarded and P_{so}, and P_{no} are approximated as :

$$P_{so}(a) = \frac{1}{\sigma_o\sqrt{(2\pi)}}\ e^{-(a-A)^2/(2\sigma_o^2)}$$

$$P_{no}(a) = \frac{1}{\sigma_o\sqrt{(2\pi)}}\ e^{-a^2/(2\sigma_o^2)} \quad (15)$$

P_{cn}, P_{cs} can be computed as :

$$P_{cn} = \int_{-th}^{th} P_{no}(a)da, \quad P_{cs} = 1-\int_{-th}^{th} P_{son}(a)da \quad (16)$$

In Figure-5 the minimum achievable P_e that is obtained by optimally setting th is plotted as a function of A for a fixed σ and σ_n^2.

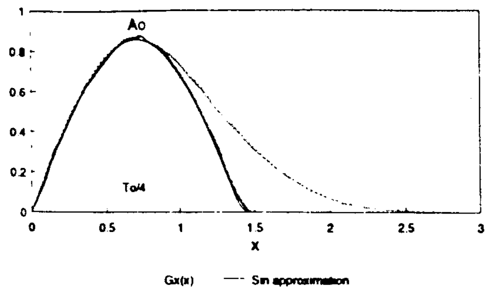

Figure 4

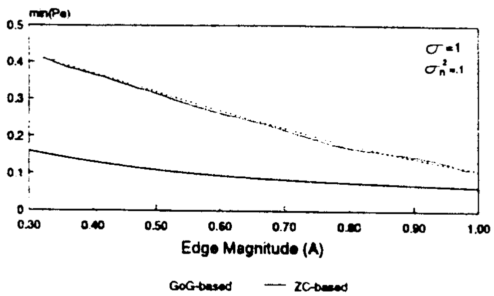

Figure 5

IV. RESULTS

The test image Figure-6 is chosen to include rough and smooth regions, and edges of widely varying contrasts. The size of the image is 256x256 pixels, and the size of the operator is 5x5 with σ=1. Figure-7 shows the edge contours from the LoG operator alone. As can be seen the output is highly contaminated with noise. Figures-8,9 show the edge contours that are filtered using the proposed method with a window of 3x3 and a 5x5 with edges with $|GoG|<.002\cdot max|GoG|$ eliminated. At least one pixel above and below the contours are required to be free of zeros to decide the validity of the contour. As can be seen many false edges were removed, and very faint edges were detected. In Figure-10 false ZC's were removed using a threshold on the magnitude of the corresponding GoG. The threshold is set to $th=.05\cdot max|D(i,j)|$. It can be seen that even a small threshold can lead to the loss of significant low intensity edges. In Figure-11 the edges are detected using a rather involved technique [22] that obtain the edges by minimizing an energy cost functional using the steepest descent technique.

Figure 6: Original Image.

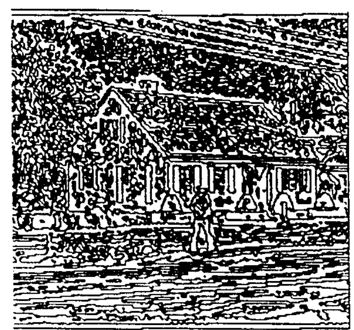

Figure 7: Edges, LoG only.

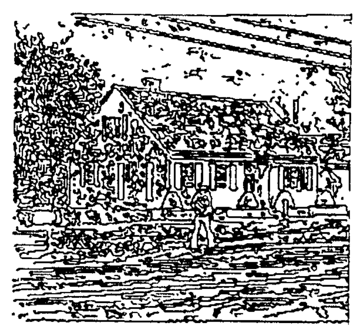

Figure 8: Edges, ZC-based, 3x3.

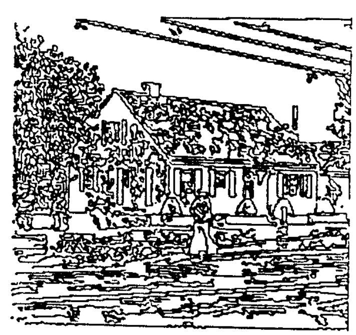

Figure 9: Edges, ZC-based, 5x5.

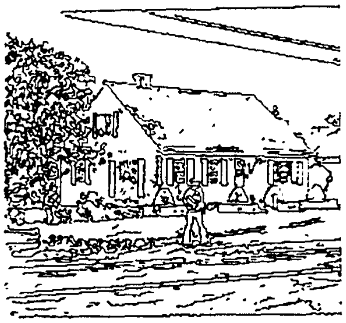

Figure 10: Edges, GoG-Based.

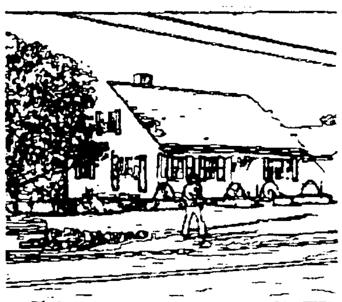

Figure 11:Edges, Energy Functional.

V. CONCLUSIONS

This paper suggests a method for enhancing the performance of the LoG edge detector. The approach propose to augment the LoG operator with a noise removal mechanism that operate to determine the validity of a ZC. This mechanism is based on the proximity of the ZC under consideration to the surrounding ZC's.Basing discrimination on a feature that is weakly coupled to the signal energy such as the distance between successive ZC's prove to have several significant merits. Moreover, by transferring the task of noise removal to the discrimination mechanism, the combined detector is permitted to maintain the attractive properties of a small scale LoG detector. Despite its simplicity, the performance of the proposed detector can compete with that of more sophisticated techniques.

REFERENCES :

[1] H. Barrow, J. Tenebaum, A. Hansa, E. Riseman Academic Press, 1978.

[2] V. Kovalevsky, 8'th Int. Conf. on Pat. Recog.,Paris,27-31, 1986.

[3] J. Mslik, P. Perona, Academic Press,1992, pp.315-344.

[4] K. Boyer, S. Sarkar,SPIE: Mach. Vii. and Robotics, Florida, 22-24, April,92, pp. 353-62.

[5] D. Marr,H. Hildreth,Proc. Roy. soc. Lond.,B207,1980, pp.187-217.

[6] G. Sotak, K. Boyer, Comp. Vis. Graph. and Image Proc., Vol.10, No.5, Sept. 1988, pp.147-189.

[7] H. Tagare, R. DeFigueiredo,SPIE: Image Processing Algorithms and Techniques, 12-14 Feb. 1990, California, pp.408-16.

[8] S. Sakar,K. Boyer,IEEE. Trans. on Patt. Anal. and Mach. Intel., Vol. 13, Nov. 13, 1991.

[9] A. Yuille,T. Poggio, IEEE Trans. on Pat. Anal.,Vol.8,No.1,Jan. 88, pp. 15-25.

[10] J. Clark, IEEE Trans. on Pat. Anal. and Mach. Intel. Vol.10, No.5, Sept. 88, pp. 720-727.

[11] Y. Leclerc, S. Zucker,IEEE Trans. Pat. Anal. and Mach. Intel.,Vol.9 No.3, May 87, pp. 341-355.

[12] Y. Lu, R. Jain, IEEE Tran. Pat. Anal. and Mach. Intell., Vol.14, No.4 April 92, pp. 450-467.

[13] S. Raman,S. Sakar,K. Boyer, IEEE Trans. on Medical Imaging,Vol.10, No.2 ,June 91 pp.109-21

[14] A. Asefi,M.Sc. Thesis, Elec. Eng. and Comp. Sci., Univ. of California, Davis, 1990.

[15] P. Kube,IEEE Comp. Soci. Conf. on Comp. Visi. and Pat. Recog. June 15-18, illinois, 92, pp.586-591.

[16] J. Canny,IEEE Tran. on Pat. Anal. and Mach. Intell., Vol. 8, No. 6, 1986, pp. 679-698.

[17] H. Heong, C. Kim,IEEE Trans. Pat. Anal. and Mach. Intel., Vol.14, No.5, May 92, pp. 579-585.

[18] J. McFadden, IRE Trans. on Infor. Theory, Dec. 1956, pp.146-150.

[19] J. McFadden, IRE Trans. on Infor. Theory, March 1958, pp.14-24.

[20] S. Cobb, IEEE Trans. on Infor. Theory, April 1965, pp.220-233.

[21] A. Papoulis, McGraw-Hill, Second Edition, 1984.

[22] D. Geiger,A. Yuille,Int. J. Comp. Vision,Vol.6,No.3, 91,pp. 227-43.