

# Evasion of Multiple, Intelligent Pursuers in a Stationary, Cluttered Environment: A Harmonic Potential Field Approach

By

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## Abstract

In this paper an intelligent controller is suggested for the evasive navigation of an agent that is engaging multiple pursuers in a stationary environment. The controller is required to generate a sequence of directions to guide the motion of the evader so that it will be able to escape the pursuers and avoid a set of forbidden regions (clutter). The focus here is on continuous evasion where the agent does not have the benefit of a target zone (e.g. a shelter) which upon reaching it can discontinue engaging the pursuers. The controller is constructed using an extension of the harmonic potential field approach to behavior synthesis. A preliminary demonstration of the controller's capabilities is provided using simulation.

## I. Introduction

Evasive navigation is an important tactical aid that is needed to enhance survivability of an agent operating in an adversarial environment [1]. It is also a non-determinate game in which an agent (evader) has to move from an initial location to a final one while avoiding an number of pursuers in an environment that may be populated by forbidden regions. The presence of clutter complicates the evasion strategy which is normally studied for one pursuer in an open environment [2]. Clutter (figure-1) excludes simple solutions to the evasion problem such as running along a straight line towards infinity using the highest possible speed. It also excludes commonly used maneuvers such as protean behavior [3] in which an evader turns in an unpredictable manner to confuse a faster, but less maneuverable, pursuer. In this case, simple reflexive control, most probably, will not work. A high level controller is needed to fuse the context in which the actors are operating, the strategy and intentions of the pursuers in the decision making process which the evader uses to guide its actions.

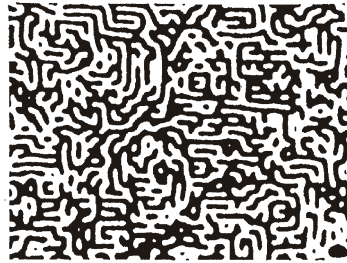


Figure-1: A cluttered environment.

Many aspects of pursuit evasion have been investigated. In [4] the problem of a group of pursuers searching a cluttered environment for an intruder using flash lights is investigated. On the other hand, in [5] the behavior of an agent trying to hide from a pursuer is evaluated in terms of the amount of protection provided by the environment. In [6] an algorithm is suggested for a group of robots to capture a fugitive agent moving on a grid. In [7] an intelligent controller is suggested for intercepting a known, well-informed target that is intelligently maneuvering in a cluttered environment to evade capture. An intensive literature survey on pursuit evasion may be found in [8].

In this paper the focus is on evasive navigation of an agent that is being tracked by multiple pursuers in a cluttered environment. However, here the evader does not have a target point present. A target point is equivalent to a shelter, or a safe-house which when reached the evader no longer have to engage the pursuers. The situation being considered here necessitate that the evader continuously engage the pursuers. This presents the evader with a considerable intellectual burden, especially when facing intelligent pursuers who may be cooperating and have the ability to learn regularities or patterns in the evader's behavior and evolve a capture strategy.

This paper presents work in progress on utilizing a modified version of the harmonic potential field (HPF) approach for behavior synthesis [9,10,11] for constructing an intelligent controller to guide an evader in a situation such as the one described above. The controller is required to be complete, i.e. if a behavioral pattern exists which the evader can use to escape capture, the controller will find it. Otherwise, capture indicates that escape was not possible to begin with. In its fully developed form, a theoretical proof of completeness will be supplied. At this stage, the performance of the controller is only examined using simulation. The paper is organized as follows: in section II the pursuit-evasion problem is formulated. Section III discusses the difficulties encountered in applying the HPF approach to the case of evasion when no target for the evader is present. In section IV a modification of the HPF approach is suggested. Also, the boundary value problem (BVP) used to generate the HPF, whose gradient field constitutes the evasion action, is presented. Section V contains simulation results, and conclusions are placed in section VI.

## II. Problem Formulation

The pursuers and evader are assumed to be operating in a multidimensional environment ( $\mathbb{R}^N$ ) that is populated by stationary forbidden regions ( $O, \Gamma=\partial O$ ). All the actors are required to restrict their activities to the subset,  $\Omega$ , of the multidimensional space known as the workspace ( $\Omega=\mathbb{R}^N-O$ ). The location of the  $i$ 'th pursuer is  $x_{p_i}$ . A group of  $L$  pursuers is assumed to be operating in  $\Omega$ . The location of the group is described using the vector  $XP=[x_{p_1} \ x_{p_2} \ \dots \ x_{p_L}]^t$ .

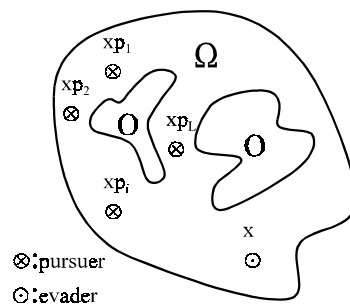


Figure-2: the pursuit-evasion environment

The evader is assumed to have full knowledge of the environment and the location of the pursuers. Likewise, the pursuers, who may be communicating, are assumed to have full knowledge of the environment and the location of the evader, figure-3.

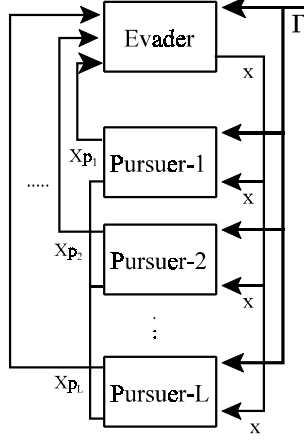


Figure-3: interaction between the evader and the pursuers

The high-level guidance mechanism which the evader is using aggregates the data about the environment ( $\Gamma$ ), and the locations of the pursuers ( $XP$ ) in order to advise the evader regarding the direction it needs to head along if it is to be safe and escape capture. Although the controller yields only a reference trajectory for the evader to follow, many techniques exist for translating a trajectory marked by the gradient field of an HPF into a control signal. A summary of some of these techniques may be found in [12].

Mathematically speaking, constructing the evasion control requires the construction of the gradient dynamical system:

$$\dot{x} = -\nabla V(x, XP(t), \Gamma) \quad (1)$$

such that:

$$x(t) \in \Omega \quad \forall t$$

and

$$\sum_{i=1}^L |xp_i - x| > 0 \quad \forall t,$$

where  $\nabla$  is the gradient operator, and  $V$  is the HPF.

### III. Difficulties

In the HPF approach, the navigation field is synthesized by first encoding the goal of the evader and the adversarial component of the environment (i.e. forbidden regions, and pursuers) in the differential properties of the potential field. The gradient field of the potential is then used for guiding the motion of the evader. The encoding scheme has the form of the BVP:

$$\nabla \cdot \nabla V(x) = \nabla^2 V(x) \equiv 0 \quad \forall x \in \Omega \quad (2)$$

$$V(x) = 1 \Big|_{x \in \Gamma}, \quad V(x) = 1 \Big|_{x = xp_i(t), i=1, \dots, L}, \quad V(x) = 0 \Big|_{x = x_t}$$

where  $x_t$  is the target point, the potential,  $V(x)$ , is valid for only the time instant  $t$ . The control at time  $t$  is derived from the negative gradient of  $V(x)$

$$u = -\nabla V(x). \quad (3)$$

It is obvious that  $V(x)$  is a harmonic potential. Harmonic functions assume their minima and maxima on

their boundaries (here  $\Gamma$ ,  $XP(t)$ , and  $x_t$ ). There are no stagnating points in  $\Omega$  where  $\nabla V$  vanishes. Therefore, the highest potential,  $V=1$ , will be at  $x = \Gamma$ , and  $x=xp_i(t)$ ,  $i=1,\dots,L$ ; while the lowest potential,  $V=0$ , will be at  $x=x_t$ . It is not hard to see that the negative gradient field will steer motion of the evader away from the forbidden regions and the pursuers and direct it towards the target. Figure-4a shows a rectangular forbidden region confining the motion of three pursuers and an evader whose goal is to reach the target without running into the pursuers or the walls of the workspace. Figures-4b, and c shows the HPF, and its negative gradient field respectively. It is clear from the vector plot in figure-4c that an evader utilizing this field to guide its actions will be steered away from danger and towards its goal.

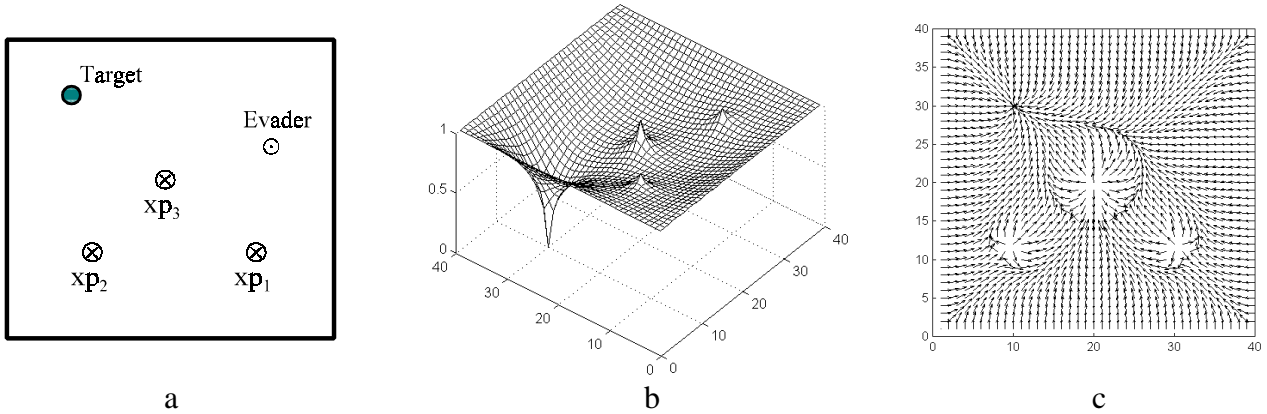


Figure-4: a: environment, b: potential field, c: negative gradient field

Removing the goal point,  $x_t$ , where the potential is fixed to zero, from the BVP in (2) makes the maxima and minima of  $V(x)$  equal to 1. In other words the value of  $V(x)$  is a constant equal to 1 for all points in  $\Omega$ . This causes the gradient field to degenerate every where in  $\Omega$  ( $\nabla V(x) \equiv 0, \forall x \in \Omega$ ) making it impossible to construct the evasion field, figures-6a,b.

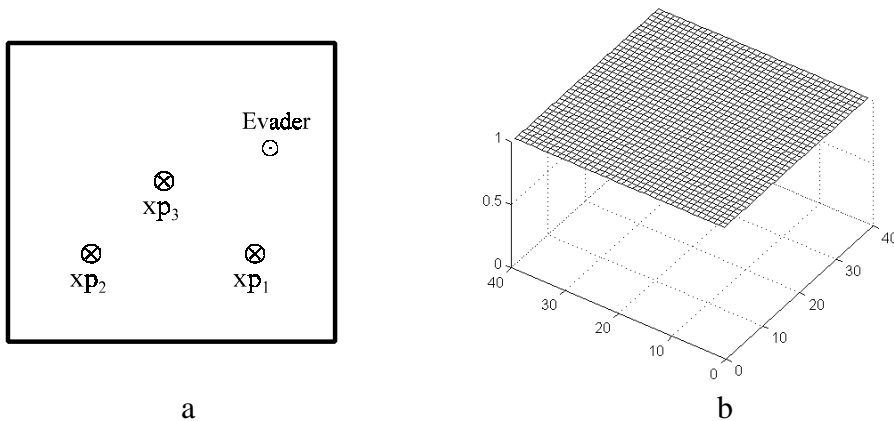


Figure-5: a: the environment, b: the potential field

#### IV. The Modified BVP

The removal of the goal point,  $x_t$ , from (2) causes the potential to become flat and the gradient field to degenerate. However, careful examination of  $\nabla V(x)$  reveals that while the magnitude field of  $\nabla V$ ,  $a(x)$ ,

degenerates, the phase field,  $\mathbf{G}(x)$ , remains stable and computable. This makes it possible to adapt the generating BVP to work for the case where no target point is explicitly specified. In the following the modified BVP is derived.

The gradient field of the potential may be expressed as:

$$\nabla V(x) = a(x)\mathbf{G}(x) . \quad (4)$$

Substituting this in the laplacian operator, we have

$$\nabla^2 V(x) = \nabla \cdot \nabla V(x) = \nabla \cdot (a(x)\mathbf{G}(x)) \equiv 0 . \quad (5)$$

Expanding the above expression yields

$$\nabla a(x) \cdot \mathbf{G}(x) + a(x)\nabla \cdot \mathbf{G}(x) \equiv 0 . \quad (6)$$

When the gradient field degenerates, its magnitude field tends to an arbitrarily small constant  $\epsilon$  ( $1 \gg \epsilon > 0$ ). Therefore, the above expression becomes

$$0 \cdot \mathbf{G}(x) + \epsilon \nabla \cdot \mathbf{G}(x) \equiv 0 ,$$

and Laplace equation reduces to:  $\nabla^2 V(x) = \nabla \cdot \mathbf{G}(x) \equiv 0 . \quad (7)$

The new BVP is: solve  $\nabla \cdot \mathbf{G}(x) \equiv 0 \quad \mathbf{X} \in \Omega \quad (8)$

subject to:  $\mathbf{G}(x) = n_\Gamma \big|_{x \in \Gamma} , \quad \mathbf{G}(x) = n_{\Gamma_i} \big|_{x \in \Gamma_i} , \quad \text{and} \quad |\mathbf{G}(x)| = 1, \quad i=1, \dots, L$

Where  $n_{\Gamma_i}$  is a unit vector field orthogonal to  $\Gamma_i$ ,

$$\Gamma_i = \{x: |x - x_{p_i}| = \delta, \delta > 0\} , \quad (9)$$

and  $n_\Gamma$  is a unit vector field orthogonal to  $\Gamma$ . The above vector BVP (VBVP) is solved for the environment shown in figure-6a. The corresponding evasion field is shown in figure-6b.

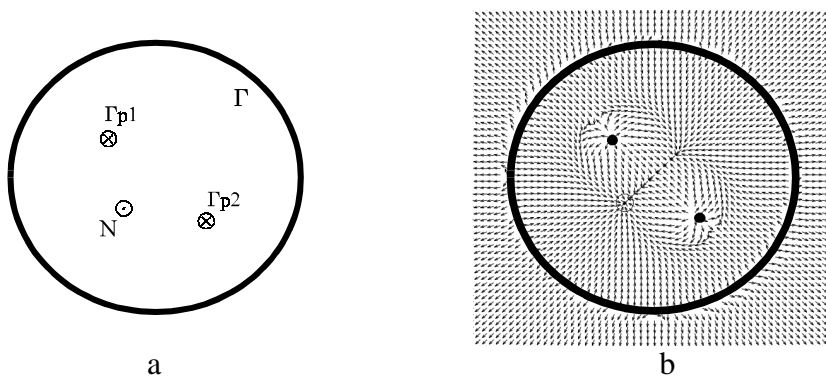


Figure-6: a. environment, b. evasion field

Although a target point was not specified in the modified BVP, a stable equilibrium point,  $N$  (i.e. a target point), spontaneously emerged in the synthesized field (a north pole, figure-6b). Unlike (2) where target location is a priori specified, in the modified VBVP, the target location is free to move in a manner

dependant on the environment and the locations of the pursuers. The target keeps adapting its location positioning itself as far as possible away from the pursuers and the forbidden regions. The intelligent, high-level controller suggested here for continuously steering the evader away from harm, accepts  $\Gamma$  and  $XP(t)$  as inputs, and generates  $N(t)$  as an output. The generated time sequence of locations,  $N(t)$ , is the one the evader has to follow in order to avoid harm and capture, figure-7.

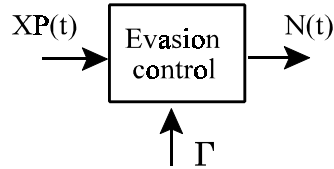


Figure-7: suggested evasion controller

## V. Results

Preliminary simulation results are presented to demonstrate the capabilities of the suggested method. In figure-8, an agent (evader) is engaging two pursuers. Although the stationary environment is known to the evader, the intentions of the pursuers, their tactics, and any strategic coalition are not known. The agent has to maneuver to avoid capture, and, at the same time, make sure that it will not collide with the obstacles in the environment. The location of the actors, and the corresponding evasion control field are shown. In the location graphs, the symbol  $\otimes_i$  is used to indicate the  $i$ 'th pursuer,  $\odot$  indicates the evader, a dotted arrow indicates a pursuer's next move, and a broken curved line trace the trajectory the evader moved along. In the evasion field plot, a solid circle marks the location of the pursuers, while a circle marks the stable equilibrium point of the field. The ability of the agent to evade capture while staying clear of harm is apparent from figure-8.

## VI. Conclusions

In this paper preliminary results are reported regarding the extension of the harmonic potential field approach to motion guidance to the case of continuous evasive navigation in a stationary, cluttered environment. Along with extending the capabilities of the HPF approach, the suggested method sheds some light on the origin of purpose in a behavior synthesis process. While the common belief is that an a priori specified goal is a must for the generation of a goal-oriented behavior, the suggested method demonstrates that it is possible for a goal (stable equilibrium in the evasion field) to emerge as a result of the interaction of the agent with its surroundings. A sizable work remains to be done in terms of theoretical analysis of the suggested approach, and exploration of its behavior using simulation.

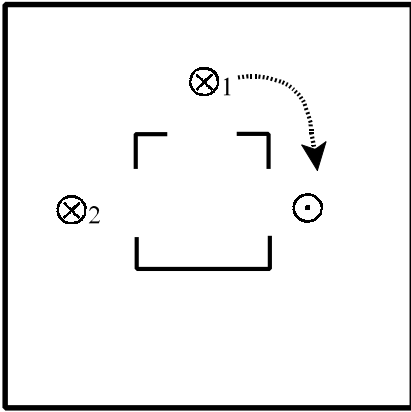


Figure-8.1a: situation, frame-1

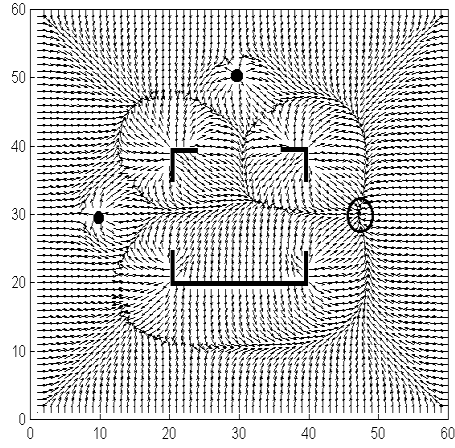


Figure-8.1b: Evasion field, frame-1

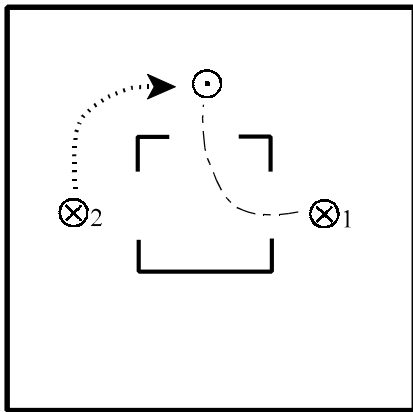


Figure-8.2a: situation, frame-2

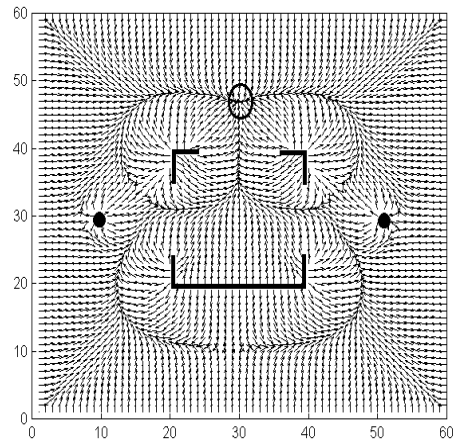


Figure-8.2b: evasion field, frame-2

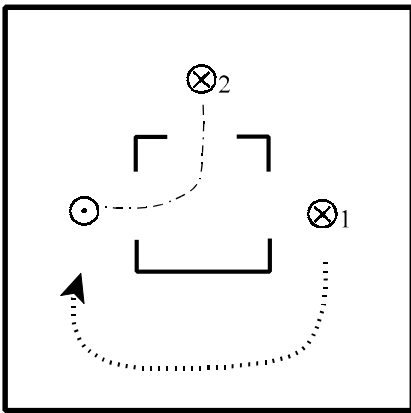


Figure-8.3a: situation, frame-3

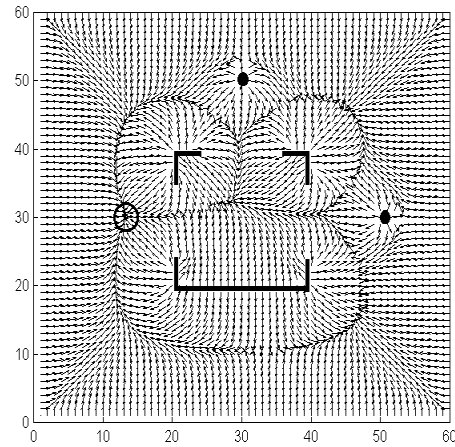


Figure-8.3b: evasion field, frame-3

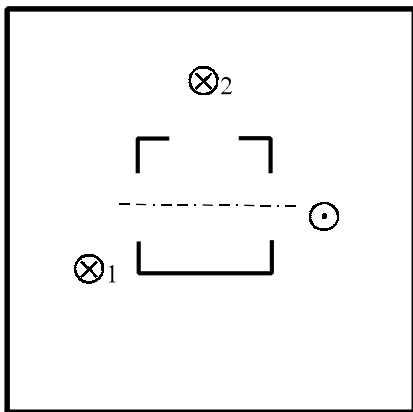


Figure-8.4a: situation, frame-4

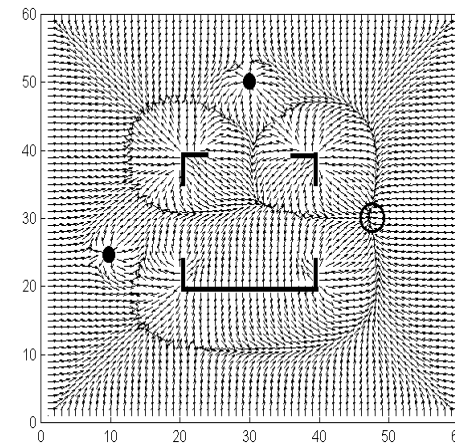


Figure-8.4b: evasion field, frame-4



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