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INTERCEPTING A MANEUVERING TARGET IN A MULTIDIMENSIONAL STATIONARY ENVIRONMENT USING A WAVE EQUATION POTENTIAL FIELD STRATEGY

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ABSTRACT

This paper presents a potential-based technique for intercepting a maneuvering target that is moving amidst known stationary obstacles. The method is a generalization of the Harmonic Potential approach which is used to plan a path to a stationary target. It employs a time dependent potential field that is generated using the linear Wave Equation. It then construct a first order time dependent nonlinear differential equation to generate a trajectory leading from an initial point to the target. Proofs of the ability of the technique to converge to the target as well as its ability to avoid obstacles are supplied. Simulation results and comparisons with other approaches are also provided.

I. INTRODUCTION

Since its humble start from the simple idea of a repulsor and an attractor [1,2], the potential field approach to path planning has progressed into an active and promising area of research. An important class of these techniques uses harmonic potential fields by first solving the Laplace equation subject to the proper boundary conditions (BC), then using the gradient flow of the resulting field to steer motion along a safe path to the target. The approach was found to effectively guide motion in a complex environment. It can tell whether a path exist connecting a point in the environment to the target (the degeneration of the gradient field in the local neighborhood of the point of interest is an indication that no path to the target exist). Also, efficient numerical algorithms exist for solving Laplace equation. The Harmonic potential approach was first proposed by conolly et. al. [3] who solved Laplace equation subject to a constant positive potential on the obstacles boundaries and a zero potential on the location of the target. Such a setting of BC results in a fast decay of the potential field and a rapidly vanishing gradient field. The problem was alleviated by changing the boundary conditions on the obstacles to the homogeneous neumann conditions while forcing a positive potential on the point from which motion starts and a zero potential on the location of the target. Decuyper and Keymeulen arrived at such a setting for BC by modeling the natural flow of an incompressible fluid in a container that has a source, a sink, and rigid objects in the locations of the obstacles [4,5]. They mentioned that the same setting of BC governs the flow of electric current in a conducting sheet between a point with positive potential and that with a zero potential, where the obstacles are modeled as perfect insulators. They also reported that the path marked by such a flow is the shortest path to the target. It ought to be mentioned that the same relation governs the flow of the heat flux in a thermal conductor with the obstacles modeled as perfect thermal insulators. The same results were also reported by Blake and Tarasenko [6], and Kim and Khosla [7]. Lei [8] suggested an implementation of the approach using neural nets. The approach was recently generalized by Masoud and Bayoumi [9,10] who used vector potential to enhance the steering capabilities of the method.

A more general case involves targets that are moving in a stationary environment. Such a problem can be of considerable difficulty, especially, if the method is to engage a target that is intelligently maneuvering to evade capture in a familiar complex environment (e.g a maze), and is well informed about the movements of its pursuer. In this work we suggest a novel and complete (i.e. if there is a way to intercept the target the planner will find it) path planner to solve the above problem. The planner assumes the form of the dynamical system

$$\dot{x} = f(V(x(t),x_p(t),\Gamma)) \qquad x(0) = x_s \in \Omega$$

and is required to have

$$\lim_{t\to\infty} |x(t) - x_p(t)| \longrightarrow 0$$

and

$$x(t) \in \Omega \quad \text{for all } t \tag{1}$$

where Ω is called the workspace and is a subset of the R^n space ($\Omega \subset R^n$), $\Gamma = \partial\Omega$, $x_p(t)$ is the target point ($x_p \in R^n$) and is restricted to operate inside Ω all the time, $x \in R^n$, $f: R \longrightarrow R^n$, $V: R^n x R^n x R^m \longrightarrow R$, m is the dimensionality of Γ ($m \le n-1$), and X_s is an initial starting point. Synthesizing both f and V is done by extending the harmonic approach to the moving target case. The navigation field (V) is constructed using the linear transient wave equation, and the path to the target is laid by a first order time dependent differential operator (f). The planner should be able to successfully intercept the target despite the following implicit assumptions

1- The target is intelligent.
2- The target has an accurate model of the environment and good information about the movements of its pursuer.
3- The geometry of the environment may be multidimensional and complex.
4- The pursuer lacks the knowledge about the psychological profile of the target, its tendencies and habits which are usually used to construct a statistical model of behavior that aids in its capture.
5- The pursuer lacks the time to thoroughly study the situation and derive a suitable plan for capturing the target.

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On the other hand, the pursuer is assumed to have a good model of the environment, and is well aware of the location of the target. Also, the pursuer assumes that the target has limited power; therefore, it can't instantly change position or orientation (i.e. $x_p \in C^1$). Our work focuses on providing the guidance field that will lead to the capture of the target. If this field is to be converted to a control field that suits the propulsion mechanism of the pursuing robot, the result of Utkin et. al. can be used [18,19]. In this approach sliding mode theory is used to force the system (robot) trajectories to coincide with the flow lines of the guidance field. The sliding surface that is used has the form

$$\sigma(x,\dot{x}) = \dot{x} - \xi(t)f(V(\ )) \quad (2)$$

where ξ is an arbitrary function of time.

Although an approach was suggested to carry out such an extension using the Diffusion equation [11], it totally relied on an analogy with no proof of convergence or obstacle avoidance. Same thing can be said about the traditional approach for dealing with the problem which uses Laplace equation in a quasi-stationary manner. An example is provided where the target can engage in a simple maneuver to successfully evade a pursuer that is utilizing these techniques.

This paper is organized as follows: Section II suggests a general structure for a Potential-based navigation technique. Section III presents the proposed approach. In section IV the proof of convergence and ability to avoid obstacles are provided. Simulation results and comparisons are given in section V, and conclusions are placed in section VI.

II. A PROPOSED STRUCTURE

All potential-based techniques for path planning start by encoding the workspace geometry and the target location in a field (scalar or vector) that is spanning the whole workspace. Such a field function as a medium to communicate the location of the obstacles and the target to every point in the space. The field is constructed in a manner that would enable a receiver to decode the message by sensing the field at and in the local neighborhood of the present location. After the message is extracted a decision is made that is subsequently followed by a motor action to yield a step in what is perceived to be the right direction. This is iteratively repeated till, hopefully, convergence is safely achieved. Existing techniques simultaneously perform the act of sensing, decision making, and course alteration in a reflexive manner. Therefore, the three acts are lumped in one unit that is given the name TRACKING MECHANISM. We believe that a potential-based planning technique can be represented by the block diagram in Figure-1.

III. THE PROPOSED TECHNIQUE

As can be seen from the previous section a potential-based method can be divided into two stages: A stage that generates the navigation field, and a stage that utilizes that field for tracking. In the following a structure for each stage is supplied.

FIELD GENERATION :

Scalar fields which describe changes in both time and space can be generated using the Wave Equation

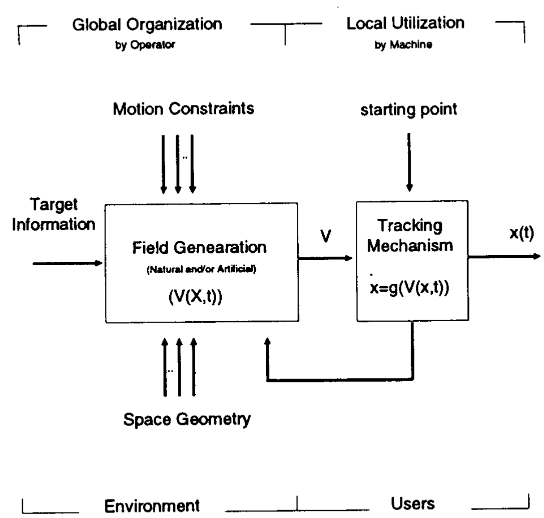


Figure 1: Basic Structure of Potential-Based. Path planning techniques.

$$\nabla^2 V = \frac{1}{a^2} \cdot \frac{\partial^2 V}{\partial t} . \quad (3)$$

The reason that the wave equation is chosen (for example, instead of the diffusion equation) for constructing the navigation field, is due to the nature of its solution. From the method of separation of variables the solution of the wave equation can be written in the form $V(X,t)=R(X)T(t)$, where R is dependent on position only, and T is dependent on time only. It can be shown that the solution can be reduced to solving the following two Helmholtz equations

$$\nabla^2 R + \lambda^2 R = 0 \qquad \text{(N-D Helmholtz eqn.)}$$
$$\frac{\partial^2 T}{\partial t^2} + (a\lambda)^2 T = 0 \qquad \text{(1-D Helmholtz eqn.)} \quad (4)$$

It is known that the fundamental solution of the Helmholtz equation in N-D provides N orthogonal basis functions [12]. Therefore, the above equations yield N+1 orthogonal basis, which are enough to represent (using the Generalized Fourier Series) an arbitrary smooth function of time and space. Field generation is carried out by solving the above wave equation subject to the following BC (Figure-2)

$$V(\Gamma,t) = C \qquad C > 0$$
$$V(X_p(t),t) = 0 \qquad X_p(t) \in \Omega \quad (5)$$

where $X_p(t)$ is the differentiable trajectory of the target. The initial conditions $(V(X,0))$ can be obtained by solving

$$\nabla^2 V = 0$$

subject to :

$$V(\Gamma) = C \qquad C > 0 \quad (6)$$
$$V(X_p(0)) = 0 \qquad X_p(0) \in \Omega$$

The existence and uniqueness of the solution of the above time dependent boundary value problem were proven in [13,14,15].

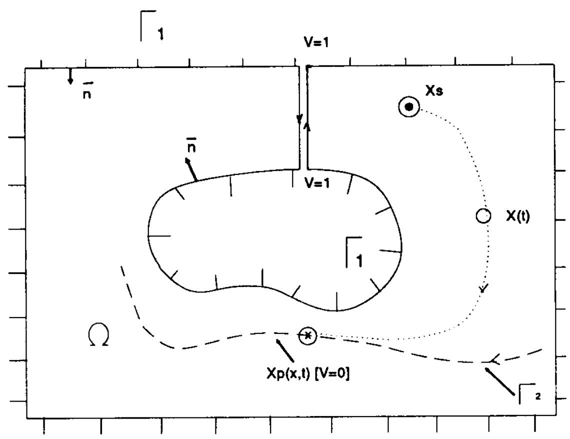


Figure 2 : The boundary conditions.

THE TRACKING MECHANISM

In the static case a path to the target is established by tracking the gradient field of the potential

$$\dot{X} = -\nabla V(X).$$

for the time varying case, the field assumes a more general form that involves a time varying vector potential $\mathbf{A}$

$$\dot{X} = -(\nabla V + \frac{\partial \mathbf{A}}{\partial t}) . \qquad (7)$$

In the sequel the following structure for $\partial\mathbf{A}/\partial t$ is shown to achieve absolute convergence (asymptotic stability)

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{\nabla V(X,t)}{\|\nabla V(X,t)\|^2}\cdot\frac{\partial V(X,t)}{\partial t} . \qquad (8)$$

The above mechanism has a singularity at the target location $X=X_p$ ($\nabla V(X_p,t)=0$). To remove this singularity the absolute convergence requirement is relaxed to that of convergence (stability) only. In this case the object is required to get an arbitrary small distance ρ close to the target. A guiding mechanism that can achieve this and is free of singularities has the form

$$\dot{X} = -(\nabla V(X,t) + \frac{\nabla V(X,t)}{\beta(\|\nabla V(X,t)\|^2)}\cdot\frac{\partial V}{\partial t}(X,t))$$

where : (9)

and $$\beta(X) = \begin{cases} X & X \geq \rho \\ \eta(X) & X < \rho \end{cases} .$$

The function $\eta(X)$ is a monotonically increasing function that satisfies the following properties

$$\eta(0) = \varepsilon , \quad \eta(\rho) = \rho \quad \rho > \varepsilon > 0$$

$$\left.\frac{d\eta(X)}{dX}\right|_{X=\rho} = 1 , \quad \left.\frac{d\eta(X)}{dX}\right|_{X=0} = 0 .$$

A form for $\eta(X)$ that satisfies the above conditions is

$$\eta(x) = \varepsilon + \frac{2\rho-3\varepsilon}{\rho^2}\cdot x^2 + \frac{2\varepsilon-\rho}{\rho^3}\cdot x^3 \qquad 0<\varepsilon<\rho .$$

It can be shown that the above dynamical system which can be placed in the general form $\dot{X}(t) = g(X(t),t)$ satisfies the global Lipschitz condition [16]. Therefore, it has one and only one solution over $t\in[0,\infty)$. This in turn guarantee that

i -X is differentiable almost every where ($\dot{X}$ exist).

ii -The above equation holds for t where $\dot{X}$ is defined.

iii-And X(t) satisfies

$$X(t) = X_0 + \int_0^t g(X(\tau),\tau)d\tau. \qquad (10)$$

IV. MOTION ANALYSIS

The ability of the path to converge to the target and to avoid obstacles is analyzed using Liapunov direct method.

THE WAVE POTENTIAL IS A LIAPUNOV FUNCTION CANDIDATE

It is shown here that a potential function that is generated using the Boundary Value Problem in section III is a Liapunov Function Candidate (LFC) [17].

It is well known that the solution of the wave equation is analytic. This in turn meets the requirement of a LFC to be differentiable or at least continuous. Since at a frozen instant of time (t_i) the potential $V(X,t_i)$ is a harmonic function which has its global maxima and minima at the boundary of Ω ($\Gamma \cup X_p$), the global minima of V occurs at X_p ($V(X_p(t)=0$, note that $V(\Gamma)=C$) which in turns results in the following for every $t_i \in t$

1- $V(X,t_i) = 0$ at and only at $X=X_p$.

2- $V(X,t_i) \geq 0$ for every $X\in\Omega$.

This satisfies the remaining requirements for the wave potential to be a LFC.

CONVERGENCE ANALYSIS

A point X_p is said to be an equilibrium point if

$$\dot{X} = g(X_p,t) = 0 \qquad \forall\ t \geq t0. \quad (11)$$

Such an equilibrium is globally asymptotically stable ($X \longrightarrow X_p$ as $t \longrightarrow \infty$ for every $X(0)\in\Omega$) if $\exists$ a continuously differentiable positive definite function V(X,t) such that

$$V(X,t) \geq 0 \qquad X \in R^N,\ \forall\ t \geq t_0,$$

and $$V(X,t) = 0 \qquad \text{only at} \quad X = X_p$$

$$\dot{V}(X,t) = \frac{d}{dt}V(X,t) \leq 0 \qquad X \in R^N,\ \forall\ t \geq t_0,$$

and $$\dot{V}(X,t) = 0 \qquad \text{only at} \quad X = X_p$$

$$\dot{V}(X,t) = \frac{dV(X,t)}{dt} = \frac{\partial V(X,t)}{\partial t} + \nabla V(X,t)^t\cdot\dot{X} \quad (12)$$

by substituting the above expression for $\dot{X}$ in $\dot{V}(X,t)$ we have

$$\dot{V}(X,t) = \frac{\partial V(X,t)}{\partial t} - \nabla V(X,t)^t\cdot(\nabla V(X,t) + \quad (13)$$

$$\frac{\nabla V(X,t)}{\|\nabla V(X,t)\|^2}\cdot\frac{\partial V(X,t)}{\partial t}) = -\|\nabla V(X,t)\|^2$$

which satisfies the conditions on $\dot{V}(X,t)$. To avoid the problem of the singul- arity at $X = X_p$ the requirement of asymptotic stability is relaxed. Instead of achieving asymptotic stability

$$\lim_{t\to\infty} X \longrightarrow X_p \qquad \text{for every } X(0) \in \Omega$$

we are going to require stability only. For such a case it is enough that X enters the neighborhood of X_p; ie

$$\lim_{t \to \infty} X \longrightarrow B\rho(X)$$

where $B\rho(X) : \{ X : \|X - X_p\| < \rho \}$. (14)

Therefore, the condition that V(X,t) be strictly negative definite can be relaxed, allowing it to be indefinite in $X \in B\rho(X)$.

AVOIDANCE OF OBSTACLES

The ability of the planning technique to avoid forbidden regions {O} (or equivalently $X(t) \in \Omega \ \forall \ t$) is of equal importance as its ability to converge to the target

$$X(t) \notin \{O\} \qquad t = [0,\infty)$$

and

$$\lim_{t \to \infty} X(t) \longrightarrow X_p(t) \qquad (15)$$

Our approach to analyze the behavior of the technique towards the obstacle focuses on studying the motion in a small region surrounding the forbidden regions $(\Gamma^{\delta}(X))$. Proving that the guiding mechanism can steer the path away from Γ in Γ^{δ} is enough to guarantee that the path will not intersect {O} (Figure-3).

Let us first begin by measuring the distance from Γ to the current position of the path (X(t)). This distance is denoted by the variable $X_n(t)$

$$X_n = X^t \cdot n$$

where n denotes a unit vector normal to Γ. Since n is not a function of time (stationary obstacles), the rate of change of X_n ($\dot{X}_n$) can be calculated as

$$\dot{X}_n = \dot{X} \cdot n = -\left(1 + \frac{1}{\|\nabla V\|^2}\frac{\partial V}{\partial t}\right)\nabla V \cdot n = \qquad (16)$$

$$-\left(1 + \frac{1}{\|\nabla V\|^2}\frac{\partial V}{\partial t}\right)\frac{\partial V}{\partial n}.$$

We are going to assume that motion starts outside Γ ($X_n > 0$). This means that if we can prove that a measure of the length of X_n is always increasing in Γ^{δ} (Liapunov theory of instability is used), we can prove that the path will never intersect {O}. Let us define Va as a measure of the length of X_n

$$V_a = \|X_n\|^2 = X_n^2. \qquad (17)$$

Its derivative with respect to time ($\dot{V}_a$) is

$$\dot{V}_a = 2 \cdot X_n \cdot \dot{X}_n = -2 \cdot X_n \cdot \left(1 + \frac{1}{\|\nabla V\|^2}\frac{\partial V}{\partial t}\right)\frac{\partial V}{\partial n} \qquad (18)$$

since we are constraining V(X,t) on Γ to a constant

$$\frac{\partial V}{\partial t} = 0 \qquad X \in \Gamma. \qquad (19)$$

Since V(x,t) is the solution of the wave equation and it is differentiable in both time and space, for a sufficiently small δ we can make the following approximation

$$\frac{\partial V}{\partial t} \simeq 0 \qquad X \in \Gamma^{\delta} \qquad (20)$$

Since at any frozen instant of time V is harmonic with its global maxima at Γ we have

$$\frac{\partial V}{\partial n} < 0 \qquad X \in \Gamma^{\delta}. \qquad (21)$$

Therefore, $\dot{V}_a$ can be approximated in Γ^{δ} as

$$\dot{V}_a = -2 \cdot X_n \cdot \frac{\partial V}{\partial n}. \qquad (22)$$

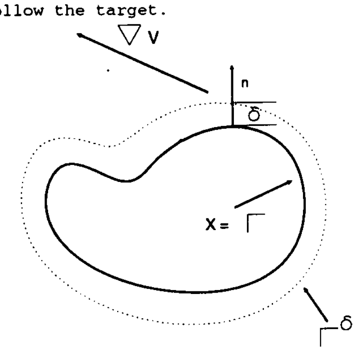
Since for $X \in \Gamma^{\delta}$, $X_n > 0$, we have

$$\dot{V}_a > 0 \qquad X \in \Gamma^{\delta} \qquad (23)$$

This means that Va is always increasing with time. Therefore, the magnitude of X_n is always increasing, and X is being steered away from Γ.

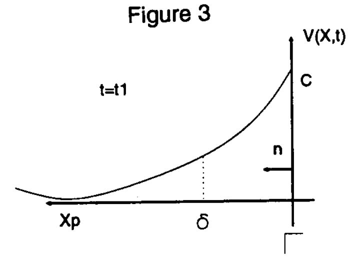
V RESULTS

In this section the tracking and obstacle avoidance capabilities of the proposed method are tested for different patterns of motion. The performance is also compared to that of the quasi-stationary laplace method and the diffusion equation strategy. In figure 4 a stationary target is used. The broken lines represent the boundary of the obstacles and the thick solid lines represent the path that is generated by the techniques. It is observed that all techniques exhibit equivalent capabilities in terms of converging to the target and avoiding the obstacles. The minor differences in the shape of the path are probably due to the implementation and the particular choice of parameters. Same observations were recorded for slowly moving targets. Figure 5 show the different techniques tracking a target moving along the X-axis. Although the performance of the different techniques in terms of their tracking abilities is comparable for slowly moving targets, the disparity greatly widens when a fast moving, rapidly fluctuating target is considered. In figure 6 the linear motion along the x-axis is supplemented with high sinusoidal oscillations along the y-axis. As can be seen these fluctuations confused the quasi-stationary laplace scheme leaving it undecided whether to proceed right or left of the corridor exit. As for the diffusion strategy, it was able to proceed in the general direction of the target; however, it wasn't able to keep up with its rapid fluctuations. On the other hand, the wave equation method was able to closely follow the target.



a. the region in which obstacle avoidance is investigated.

Figure 3



b. the behavior of the potential along the normal of the obstalce surface.

Figure 4a: Laplace (fixed target).

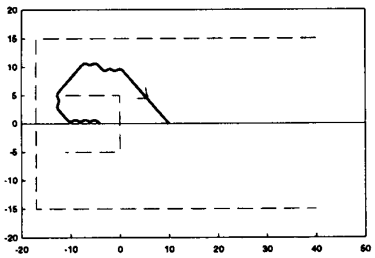


Figure 5a: Laplace (Motion along X).

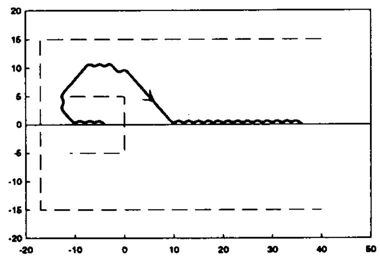


Figure 6a: Laplace (oscillating).

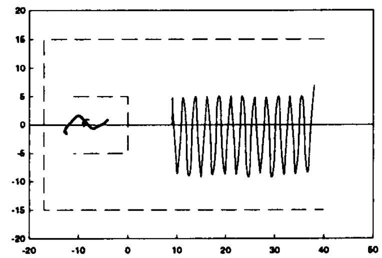


Figure 4b: Diffusion (fixed target).

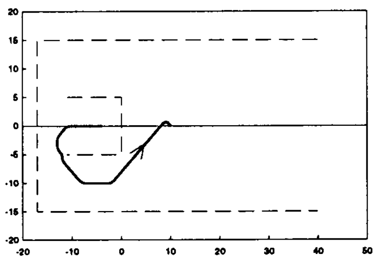


Figure 5b: Diffusion (Motion along X)

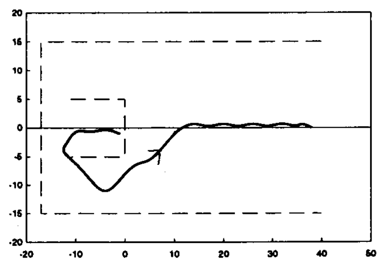


Figure 6b: Diffusion (oscillating).

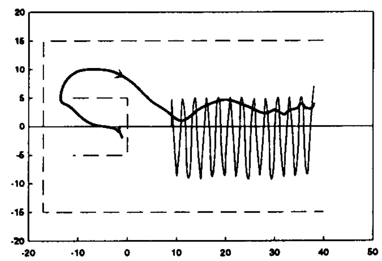


Figure 4c: Wave (Fixed target)

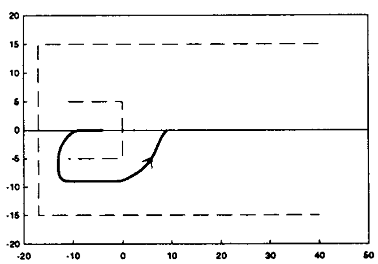


Figure 5c: Wave (motion along X)

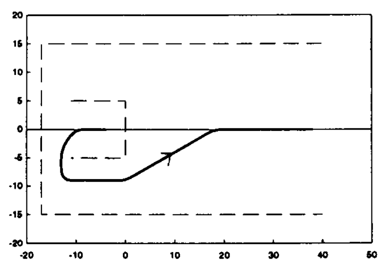
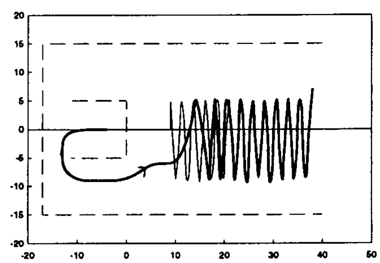


Figure 6c: Wave (oscillating)



IV CONCLUSIONS

In this paper a method of verifiable capabilities is suggested for safely tracking a target that is moving amidst known obstacles. The solution begins by suggesting a canonical form for potential-based navigation techniques. There are many questions that are yet to be answered about the behavior of the proposed technique. For example, will the method always succeed regardless of the intelligence of the maneuver used by the target?! What is the effect of delays on the ability of the technique to capture the target? What is the rate of convergence (i.e. how fast can the method capture the target); and what is the effect of giving the pursuer and the target finite masses instead of assuming that they have a zero mass? To begin answering these questions, a thorough experimentation has to be done first. This requires a robust, efficient, interactive implementation of the method which is our plan for immediate future.

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