

USING LOCAL INFORMATION FOR THE RELIABLE REMOVAL OF NOISE FROM THE OUTPUT OF THE LoG EDGE DETECTOR

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ABSTRACT

This paper suggests a method for enhancing the performance of the Laplacian of Gaussian (LoG) edge detector using local information from the neighborhood of the potential edge contours. This information is employed to separate valid edges from false ones in a reliable, efficient, low complexity manner. Statistical analysis, simulation, as well as, comparison with other techniques are provided.

I. INTRODUCTION

In a recent paper surveying the state of the art of edge detection techniques [1] it was noticed that the increased sophistication of edge detectors is not producing a commensurate improvement in performance. Based on that it was concluded that hammering the problem at the signal level will prove largely fruitless. In a previous work the authors [2] demonstrated that edge detection at the signal level is still a promising area of research with real possibilities for reducing complexity and enhancing performance. The proposed detector combines the popular LoG detector [3] with a decision rule on the local structure of the signal to discriminate between false and valid edges. Although several methods were proposed to filter out noise from the output of the LoG detector [4,5,6,7], the proposed scheme was shown to combine reliability with computational efficiency. In this paper the work in [2] is generalized to combine information from both local energy (magnitude of the signal convolved with the Gradient of Gaussian (GoG) [8,9] and local structure to enhance the performance.

This paper is organized as follows: In section II the results in [2] are briefly restated for convenience. Section III presents the proposed approach, and section IV outlines the statistical analysis of the detector. The results are presented in section V, and conclusions are placed in section VI.

II. PREVIOUS WORK

In the LoG technique the edges $E(x)$ are located as the zero crossings (ZC) of the signal $I(x)$ convolved with the Laplacian of $G(x, \sigma)$:

$$G(x, \sigma) = \exp(-x^2/2\pi\sigma^2), \quad E(x) = \{x: \nabla^2 G(x, \sigma) * I(x) = 0\}$$

The goal is to construct a procedure to discriminate between a ZC generated by a valid edge ($I_1(x) = Au(x) + n(x)$), and a ZC generated by noise only ($I_2(x) = n(x)$). The edge is assumed to be an ideal step jump ($u(x)$) with a magnitude (A). The noise is assumed to be a stationary Additive White Gaussian Noise (AWGN) with zero mean and variance σ_n^2 . Let $L_1(x)$ and $D_1(x)$ be:

$$L_1(x) = I_1(x) * \nabla^2 G(x), \quad D_1(x) = I_1(x) * \nabla G(x)$$

For the 1-D case, we have:

$$\nabla^2 G(x) * u(x) = -\nabla G(x), \quad \nabla G(x) * u(x) = G(x)$$

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Figures 1a,b show the signal and noise components of D_1 and L_1 receptively.

The performance of the detector is analyzed by deriving the misdetection probability ($1-P_{cs}$), and the false detection probability ($1-P_{cn}$). P_{cs} is the probability of a valid ZC decided as an edge, and P_{cn} is the probability that a false ZC is rejected. The probability of error (P_e) is:

$$P_e = P_1(1-P_{cs}) + P_0(1-P_{cn})$$

where P_1, P_0 are the probabilities that a valid and a false ZC occur respectively. There are several factors on which P_1 and P_0 depend; among them: the signal richness in edges, the characteristics of the noise, and the scale of the LoG (σ). However, since P_1 and P_0 are not a priori known, they are assumed to be equal ($P_1=P_0=0.5$).

1- ZC-based Scheme:

The validity of a ZC (Z_i) can be tested by computing $d_i = \text{dist}(Z_i, Z_{i-1})$, and $d_{i+1} = \text{dist}(Z_{i+1}, Z_i)$; then the following rule will be used to accept or reject Z_i .

$$\begin{aligned} \text{if } \{d_i \geq \text{th}\} \text{ and } \{d_{i+1} \geq \text{th}\} \quad & Z_i \text{ is Valid} \\ \text{else} \quad & Z_i \text{ is False} \end{aligned}$$

where th is a selected threshold. For the 2-D case d_i^x and d_i^y are computed along both sides of the normal to the edge contours. Using the intervals between the ZC of L_1 for discrimination has the advantage of reducing the need for adaptation to enhance the chance of detecting weak edges. The autocorrelation of the noise at the LoG input ($R_i(\tau)$) and output ($R_o(\tau)$) are:

$$\begin{aligned} R_i(\tau) &= \sigma_n^2 \delta(\tau), \\ R_o(\tau) &= [\sigma_n^2 / \sigma^7] \sqrt{\frac{\pi}{2}} [3\sigma^4 - 6\sigma^2 \tau^2 + \tau^4] e^{-\tau^2 / 2\sigma^2} \end{aligned}$$

$\delta(\tau)$ is the Kronecker-delta function. The variances of the noise at the input and output are:

$$\sigma_i^2 = \sigma_n^2, \quad \sigma_o^2 = 3\sqrt{\frac{\pi}{2}} [\sigma_n^2 / \sigma^3]$$

To compute the performance probabilities of this scheme, we need to compute the following Conditional Probability Distribution Functions (PDF's). The first is $P_{no}(\tau)$ which is the conditional probability that the first ZC after time t occurs between $t+\tau$ and $t+\tau+d\tau$ given a ZC at time t when the input to the LoG is I_2 . The second is $P_{so}(\tau)$ which is the same except that it is computed when the input to the LoG is I_1 . For a smooth zero mean Gaussian noise P_{no} is approximated as:

$$P_{no}(\tau) \approx \frac{\rho^*(\tau) (1-\rho(\tau)^2) + \rho(\tau) \rho'(\tau)^2}{2\sqrt{\rho^*(0)} [1-\rho(\tau)^2]^{3/2}}$$

where $\rho(\tau)$ is the normalized autocorrelation function, and ρ', ρ^* are the first and second derivative of ρ with respect to τ . Assuming independence of the intervals between successive ZC's of the output noise, the probability of validating a false ZC is:

$$\left[1 - \int_0^{\text{th}} P_{no}(\tau) d\tau \right]^2$$

The approximate PDF of intervals between the ZC's is:

$$P_{so}(\tau) \approx \frac{A_0/\sigma_n^2}{2\sqrt{\pi} \omega_0 \sqrt{\rho(\tau) - \cos(\omega_0 \tau)}} \cdot \left[\frac{\rho'(\tau)}{1+\rho(\tau)} \cos\left(\frac{\omega_0 \tau}{2}\right) + \omega_0 \sin\left(\frac{\omega_0 \tau}{2}\right) \right] \cdot \exp\left(-\frac{(A_0/\sigma_n^2)^2}{1+\rho(\tau)} \cos\left(\frac{\omega_0 \tau}{2}\right)\right)$$

where $\omega_0 = 2\pi/T_0$, $T_0 = 4\sigma/\sqrt{2}$, and $A_0 = \sqrt{2} \exp(-1/2) A/\sigma$. This distribution is accurate for relatively high SNR, and it tends to P_{no} as SNR goes to zero. For a moderate SNR we shall assume that d_+ and d_- are strongly dependent; therefore, P_{cs} can be approximated as:

$$P_{cs} \approx 1 - \int_0^{th} P_{so}(\tau) d\tau$$

2. The GoG-based scheme :

Here, P_{cs} and P_{cn} are computed for the following rule :

if $|D_i(x)| \geq th$ Z_i is valid
 else Z_i is false

We need to compute the following conditional PDF's: $P_{so}(a) = P(a < D_i(x) \leq a + da | I_i(x) = 0)$ when the input is I_1 , and P_{no} which is the same, but the input is I_2 . R_0 at the output of the GoG and its variance are :

$$R_0(\tau) = [\sigma_n^2/\sigma^3] \sqrt{\frac{\pi}{2}} [\sigma^2 - \tau^2] e^{-\tau^2/2\sigma_n^2}, \quad \sigma_0^2 = [\sigma_n^2/\sigma] \sqrt{\frac{\pi}{2}}$$

Since the noise is Gaussian and the GoG is a linear operator, the output noise is, also, Gaussian. Assuming a relatively high SNR and a small σ , the dislocation in the position of the edge can be disregarded and P_{so} , and P_{no} are approximated as :

$$P_{so}(a) \approx \frac{e^{-(a-A)^2/(2\sigma_0^2)}}{\sigma_0 \sqrt{2\pi}}, \quad P_{no}(a) \approx \frac{e^{-a^2/(2\sigma_0^2)}}{\sigma_0 \sqrt{2\pi}}$$

P_{cn} , P_{cs} can be computed as :

$$P_{cn} = \int_{-th}^{th} P_{no}(a) da, \quad P_{cs} = 1 - \int_{-th}^{th} P_{so}(a) da$$

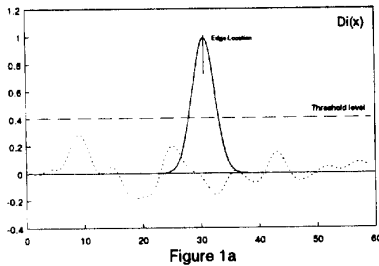


Figure 1a

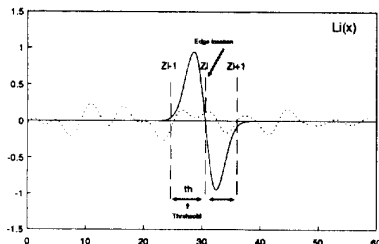


Figure 1b

III. THE PROPOSED APPROACH

To enhance the fidelity of a ZC in representing the physical edges of a signal, the scale of the operator is kept small for accuracy, sensitivity, and resolution. Additional information is extracted from the output signal in the form of indicators tying the ZC's to the underlying physical edges. In [2] two simple indicators (the energy at the ZC, and the local structure around it) were individually examined, compared, and utilized for

false ZC removal. The work showed the significant advantages a first passage time indicator has over an energy indicator.

It is well known that the amount of information used in filtering the noise sets an upper limit on the achievable performance. On the other hand, the method by which this information is utilized determines the achievable percentage of such a limit with complexity blowing up to infinity as the limit is approached. We believe that the combined information from both indicators offers a net increase in the amount of information that can be utilized by the detector. This does increase the limit on the potential of the achievable performance. Moreover, the higher ceiling on the potential performance makes it possible to improve the achievable performance while maintaining the same level of detector complexity [10]. The conclusion that combining both indicators offers a net increase of information available to the edge detector is based on the observation that in the presence of a physical edge both indicators are correlated in the sense that they tend to give a positive identification of the physical edge. They, also, tend to exhibit strong independence in the absence of a physical edge, which is deduced from the PDF of the ZC's which is only dependent on the autocorrelation function of the output signal and is not affected by the power of noise. On the other hand, the PDF of the GoG is mainly dependent on the power of the input noise and is totally unaffected by the autocorrelation of the output noise. Although efficient techniques exist to fuse the information from both indicators [11, 12], the following simple rule is utilized :

if $[(d_i \geq th_1) \text{ and } (d_i \leq th_1)]$ Z_i is Valid
 and $|D_i(x)| \geq th_2$
 else Z_i is False

IV. ANALYSIS

The probability that the above two-indicator decision rule makes an error is :

$$P_e = P_1[P_{s01} + P_{s10} + P_{s00}] + P_0[P_{n11}]$$

where the s subscript indicates that P_{sij} is derived when an edge is present while an n subscript indicates noise only. The digits ij indicate the combined outcome from the ZC-based and the GoG-based rules respectively. To derive the above quantities, the joint PDF of the intervals between the ZC's of the LoG and the signal from the Marked Zero Crossing problem [13], and has the following formulation: Suppose that $y(t)$ is a continuously differentiable process, and let T_1 be the first ZC of $y(t)$. Consider an n -variate continuously differentiable vector process $\mathbf{x}(t) = (x_1(t), \dots, x_n(t))$ and assume that y and \mathbf{x} are defined on the same probability space. Derive the first passage time T_1 and the values of the vector process \mathbf{x} at T_1 ($f(\tau, a)$). In our case $y(t) = L_i(t)$, and \mathbf{x} has one variable $\mathbf{x} = (D_i(t))$. Since L_i and D_i are generated by linearly transforming the same Gaussian input, they are jointly Gaussian [14, 15]. The joint PDF is :

$$P_{x,y} = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \cdot \exp\left\{-\frac{1}{2(1-\rho^2)} \left[\frac{(x-\bar{x})^2}{\sigma_x^2} - 2\rho \frac{(x-\bar{x})(y-\bar{y})}{\sigma_x\sigma_y} + \frac{(y-\bar{y})^2}{\sigma_y^2} \right]\right\}$$

where $\sigma_y^2 = 3\sqrt{\pi/2}(\sigma_n^2/\sigma^3)$, $\sigma_x^2 = \sqrt{\pi/2}(\sigma_n^2/\sigma)$,

$$\bar{x} = \begin{cases} G(t) & I=I1 \\ 0 & I=I2 \end{cases}, \quad \bar{y} = \begin{cases} -\sqrt{G(t)} & I=I1 \\ 0 & I=I2 \end{cases}$$

and

$$\rho = \begin{cases} (\sqrt{2/3\pi})(\sigma^2/\sigma_m)G(t)\nabla G(t) & I = I_1 \\ 0 & I = I_2 \end{cases}$$

The marked ZC distribution for this case is:

$$f(\tau, a) = E[J(y; \tau) | y'(\tau)] P_{x, y}(0, a)$$

where $J(y; \tau)$ is an indicator function which is taken as one if the sample path of y does not cross zero prior to τ , and is equal to zero otherwise. $E[\cdot]$ is the expected value. Since the indicator function is difficult to evaluate we will resort to an approximation to compute the above PDF. Let us write the above distribution in the following form :

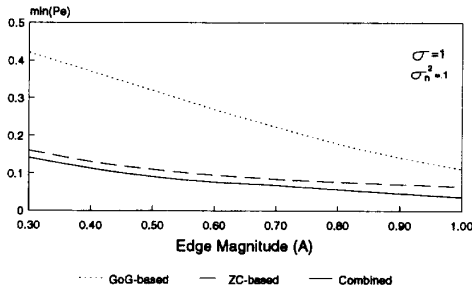
$$f(\tau, a) = f_T(\tau) \cdot f_{x(T)/T=\tau}(a)$$

This can be approximated as :

$$f(\tau, a) \approx \begin{cases} Pzso(\tau) \cdot PGso(a) & I = I_1 \\ Pzno(\tau) \cdot PGno(a) & I = I_2 \end{cases}$$

where Z indicates a pdf for the intervals between the ZC's, and G indicates a pdf for the output signal from the GoG at a ZC. Figure 2 shows the minimum achievable P_e 's obtained by optimally setting the threshold/s for the ZC-based, GoG-based, and the combined rules.

Figure 2: Minimum Probability of error.



V. RESULTS

The proposed scheme has been tested for the following 1-D discrete signal: $S(n)=u(n-25)+N(n)$. The size of the discrete LoG operator is 13 samples, and its scale is $\sigma=1$. The GoG threshold is set to 10% from the maximum. At least two samples before and after a ZC have to be free of zero crossings for that ZC to be considered valid. The experiment was repeated several times for different signal to noise ratios. Figures 3a,3b show respectively the average number of false components, and the percentage detection of the correct one for the LoG alone, the GoG-based, ZC-based, and combined schemes.

Figure 3a: Average # of False detections

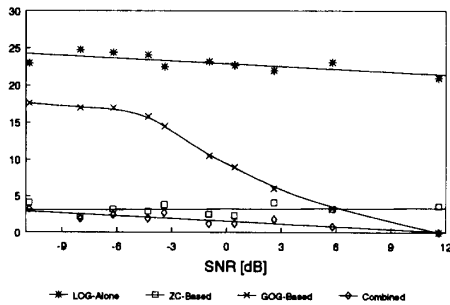
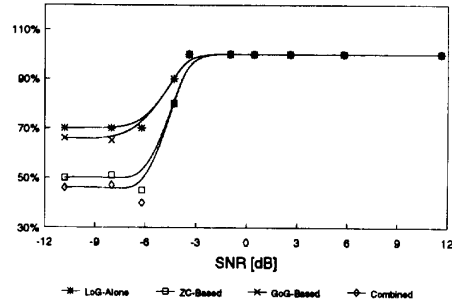


Figure 3b: Percentage Correct Detection.



The method is, also, tested for the 2-D case. The test image Figure 4a is chosen to include rough and smooth regions, and edges of widely varying contrasts. The size of the image is 256x256 pixels, and the size of the operator is 5x5 with $\sigma=1$. Figure 4b shows the edge contours from the LoG operator alone. As can be seen the output is highly contaminated with noise. Figures 4c,d show the edge contours that are filtered using the proposed method with a window of 3x3 and that of 5x5 with edges satisfying $|GoG| < .002 \cdot \max|GoG|$ eliminated respectively. At least one pixel above and below the contours should be free of zeros to decide the validity of the contour. As can be seen many false edges were removed, and very faint edges were detected. In Figure 4e false ZC's were removed based on the magnitude of the corresponding GoG. The threshold is set to $th = .05 \cdot \max|D(i, j)|$. It can be seen that even a small threshold can lead to the loss of significant low intensity edges. In Figure 4f the edges are detected using a rather involved technique [16] that minimizes an energy cost functional (E) using the steepest descent technique :

$$E = \iint_D (f(x) - g(x))^2 dx + \alpha \iint_{D-C} |\nabla f(x)|^2 dx + \gamma \int_C ds$$

where g is the image, f is the smoothed image, D is the domain, and C is the set of edges separating the domains.

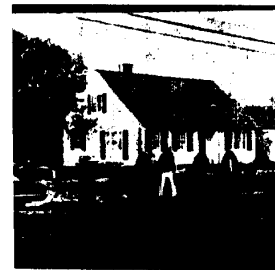


Figure 4a: Original Image.

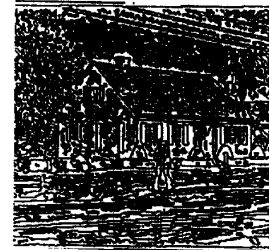


Figure 4b: Edges, LoG only.



Figure 4c: Edges ZC-based, 3x3.



Figure 4d: Edges, ZC-based, 5x5.



Figure 4e: Edges, GoG-based.



Figure 4f: Edges, Energy functional.

VI. CONCLUSIONS

In this work the LoG noise removal mechanism that is suggested in [2] is generalized to utilize information from both the local structure and energy of the potential edge contours. Our approach to the design highly relies on the fact that increasing the amount of information that is used to filter out noise improves the potential to enhance reliability while maintaining the detector complexity at an acceptable level. Although a simple decision rule was used to utilize the information from the indicators, we are aware that converting a potential to an achievable performance is dependent on the way this information is utilized (the information fusion technique). Therefore, future work will concentrate on improving the decision making process. We strongly believe that an information theoretic approach to the problem of edge detection does offer a promising framework in which the design can take into consideration both performance and complexity.

REFERENCES

- [1] K. Boyer, S. Sarkar, "Assessing the State of Art in Edge detection: 1992", Applications of Art. Intell. X: Mach. Visi. and Robotics, 22-24 April, Florida, 1992, pp. 353-361.
- [2] A. Masoud, M Bayoumi, "Using Local Structure for the Reliable Removal of Noise from the Output of the LoG Edge Detector", IEEE Winter Workshop on Nonlinear Digital Signal Processing, Jan 17-21, Tampere, Finland, 1993.
- [3] D. Marr, E. Hildreth, "Theory of Edge Detection", Proc. Roy. Soc. Lond., B207, 1980, pp. 187-217.
- [4] S. Mallat, W. Hwang, "Singularity Detection and Processing with Wavelets", IEEE Trans. on Inf. Theory, Vol.38, No.2, Mar 92, pp. 617-643.
- [5] Y. Lu, R. Jain, "Reasoning About Edges in Scale Space", IEEE Trans. Patt. Anal. Mach. Intell., Vol. 14, No. 4, April 92, pp. 450-467.
- [6] A. Asefi, "Implementation and Evaluation of Different Edge Detection Techniques", M.Sc. Thesis, Elec. Eng. and Comp. Sci., Univ. of California Davis, 90.
- [7] H. Heong, C. Kim, "Adaptive Determination of Filter scales for Edge Detection", IEEE Trans. Patt. Anal. and Mach. Vision, Vol. 14, No.5, May 92, pp. 579-85.
- [8] P. Kube, "Properties of Energy Edge Detectors", IEEE Conf. on Comp. Vision and Patt. Recog., June 15-18, 1992, Illinois, pp. 586-591.
- [9] A. Masoud, M. Bennamoun, M. Bayoumi, "An M-D Robust Edge Detector", IEEE Workshop on Visual Signal Processing and Communications, June 6-7, 1991, Hsinchu, Taiwan, pp. 222-225.
- [10] R. Gallager, "Information Theory and Reliable Communication", New York: Wiley, 1969.
- [11] I. Hoballah, R. Varshney, "An Information Theoretic Approach to the Distributed Detection Problem", IEEE Trans. on Inf. Theory, Vol.35, No.5, Sept.89, pp. 988-994.
- [12] I. Hoballah, R. Varshney, "Distributed Bayesian Signal Detection", IEEE Trans. on Inf. Theory, Vol.35, No.5, Sept.89, pp. 995-1000.
- [13] I. Rychlik, "New Bounds for the First Passage, Wavelength, and Amplitude Densities Stochastic Processes and Their Applications, Vol. 34, 1990, pp.313-339.
- [14] A. Papoulis, "Probability, Random Variables, and Stochastic Processes", McGraw-Hill, 1984.
- [15] W. Bennett, "Methods for Solving Noise Problems", Proc. IRE, Vol. 44, May 1956, pp. 609-638.
- [16] D. Geiger, A. Yuville, "A Common Framework for Image Segmentation", Int. Jour. of Comp. Vision, Vol.6, No.3, 1991, pp.227-243.