A BOUNDARY VALUE PROBLEM FORMULATION OF PURSUIT-EVASION IN A KNOWN STATIONARY ENVIRONMENT: A POTENTIAL FIELD APPROACH

by
Ahmad A. Masoud
Robotics Laboratory

Electrical Engineering Department, Queen's University, Kingston, Ontario, Canada K7L-3N6

ABSTRACT

In this paper, pursuit-evasion in a known cluttered stationary environment is formulated as a Boundary Value Problem. The devised approach is a generalization of the Harmonic Potential approach used to plan a path to a stationary target. It employs a time dependent potential field that is generated using the linear Wave Equation. It then constructs a first order time dependent nonlinear differential equation to generate a trajectory leading from an initial point to the target. The planning process enjoys an objectivity that enables it to guarantee interception regardless of the intelligence of the maneuver that the target may employ to avoid being captured (a complete planner). It also has a causal implementation making it possible to lay a course for interception without apriori knowing the path of the target. Proofs of the ability of the technique to converge to the target as well as its ability to avoid obstacles are supplied. Simulation results and comparisons are also provided.

1. INTRODUCTION

The survival of any species depends on its ability to develop its navigation competence to handle a prey-hunter situation successfully. The developed capabilities are subjective in nature; that is, the structure of the capture scheme is built using cues that are "meaningful" to the "hunter" and are related to aspects of the personality of the prey. In a situation where an intelligent prey is being hunted, the prey may be aware of the hunter; even more, it may be aware of what the hunter expects from it. In such a situation, the prey may initiate a deception scheme that is masked by its expected behavior to engage in an interactive "message" exchange with the hunter. The structure of this exchange is based on the model that the prey has of the hunter's decision mechanism and is used to out maneuver him and evade capture. The prey could even acquire a "soft control" of the hunter that could reverse the role of each. Here, the nesting of actions (or equivalently messages) has the form of I KNOW THAT HE KNOWS THAT I KNOW etc. [1]. The deeper the nesting is, the more likely that the action of the concerned party reaches a successful conclusion.

To the best of my knowledge, all planning techniques upto now [2,3] do not consider active intelligent targets that can extend their domain of awareness to include that of the pursuer and reactively adjust their path to enhance their chance of survival. Even in the case of static targets, psychological aspects of the pursuer (e.g. humans use cues like edges, corners, corridors etc. to navigate themselves) may be used by the target to design an environment that will either (reflexively) guide the pursuer to it or bewilder and confuse him to keep him away (Figure-1a,b). Such a subjectivity seems inherent in the structure of this problem and has led most researchers to believe that a learning mechanism that adapt to the nature of the situation (e.g. neural nets, genetics algorithms,etc.) has to be incorporated in a solution to these types of problems. However, the concept of a complete path planner seems to challenge this notion. A complete path planner is an "objective" machine that does not have the psychological profile of individual situations "mental worlds" determine its ability to reach its target. Such an ability is tied to the "potential" of achieving a successful conclusion. The harmonic approach to path planning [4-6] has proven to be successful in such a regard. In view of this success as well as the sizable body of literature analyzing this approach, it is tempting to extend the Harmonic approach to design a complete planner for moving targets.

Designing a path planner to intercept moving targets can be of considerable difficulty, especially if the method is to engage a target that is intelligently maneuvering, to evade capture, in a familiar complex environment (e.g. a maze), and is well informed about the movements of its pursuer. In this work, a novel and complete (i.e. if there is a way to intercept the target the planner will find it) path planner to solve the above problem is suggested. The suggested planner assumes the form of the dynamical system

$$\dot{\mathbf{x}} = \mathbf{f}(V(\mathbf{x}(t),\mathbf{x}_p(t),\Gamma)) \qquad \mathbf{x}(0) = \mathbf{x}_s \in \Omega \qquad 1$$

and is required to have

$$\lim_{t\to\infty} |\mathbf{x}(t) - \mathbf{x}_p(t)| \longrightarrow 0$$

$$\text{and} \qquad \mathbf{x}(t) \in \Omega \quad \text{for all } t, \qquad 2$$

where Ω is the workspace and is a subset of the R^n space ($\Omega \subset R^n$), $\Gamma = \partial\Omega$, $\mathbf{x}_p(t)$ is the target point ($\mathbf{x}_p \in R^n$) and is restricted to operate inside Ω all the

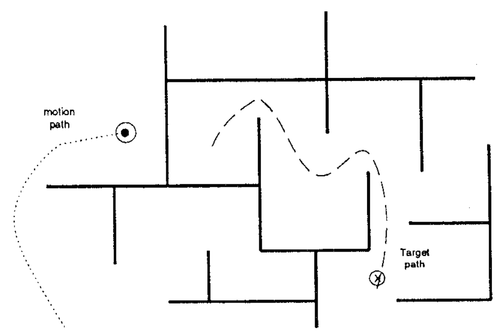

Figure-1a: An easy environment for a human to navigate.

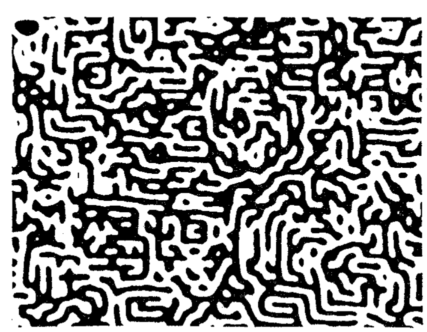

Figure-1b: A difficult environment for a human to navigate.

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time, $\mathbf{x} \in R^n$, $\mathbf{f}: R \longrightarrow R^n$, $V: R^n x R^n x R^m \longrightarrow R$, m is the dimensionality of Γ ($m \leq n-1$), and $\mathbf{x}_s$ is an initial starting point. Synthesizing both f and V is done by extending the harmonic approach to the moving target case. The navigation field (V) is constructed using the linear transient wave equation, and the path to the target is laid by a first order time dependent differential operator ($\mathbf{f}$). The planner should be able to intercept the target successfully despite the following implicit assumptions

1- The target is intelligent.
2- The target has an accurate model of the environment and good information about the movements of its pursuer.
3- The geometry of the environment may be multidimensional and complex.
4- The pursuer lacks the knowledge about the psychological profile of the target, its tendencies and habits which are usually used to construct a statistical model of behavior that aids in its capture.
5- The pursuer lacks the time to thoroughly study the situation and derive a suitable strategy for capturing the target.

On the other hand, the pursuer is assumed to have a good model of the environment, and is well aware of the location of the target. Also, the pursuer assumes that the target has limited power; therefore, it can not instantly change position or orientation (i.e. $\mathbf{x}_p \in C^1$). This work focuses on providing the guidance field that will lead to the capture of the target. If this field is to be converted to a control field that suits the propulsion mechanism of the pursuing robot, the result in [7] where sliding mode theory is used to force the system (robot) trajectories to coincide with the flow lines of the guidance field, can be used.

Although an approach was suggested to carry out such an extension using the Diffusion equation [8], it totally relied on an analogy with no proof of convergence or obstacle avoidance. The same thing can be said about the traditional approach which uses Laplace equation in a quasi-stationary manner. An example is provided where the target can engage in a simple maneuver to successfully evade a pursuer that is utilizing these techniques.

This paper is organized as follows: section 2 presents the proposed approach; in section 3, the proof of convergence and ability to avoid obstacles are provided; simulation results and comparisons are given in section 4; and conclusions are placed in section 5.

2. THE PROPOSED APPROACH

A metaphor that would intuitively capture the essence of the proposed approach is that of a point isotropic radiator (resembling the target) that is moving in an environment (Figure-2), with the obstacles taken as a perfect conductor (to shield those regions in order to prevent field lines from entering). As a result, the field lines that are emanating from the radiator will fill the workspace. Such a field can be seen as a medium to communicate the location of the target and the obstacles to an interceptor that would lock on one of these lines and traverse it back to the source (target). The manner in which such a scheme functions is equivalent to, first extracting the message which the target is sending to the present location of the interceptor, then making a decision (by the interceptor) and finally initiating a motor action that would propel the interceptor a step in what it perceive to be the "right" direction. This procedure is iteratively repeated till "hopefully" convergence is achieved. The model for potential-based

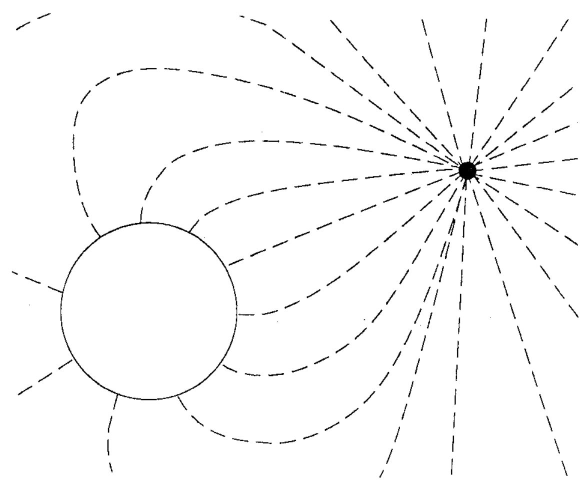

Figure-2: The radiator metaphor.

planning techniques (figure-3) that is suggested in [9] supports such a behavior. Here, it is combined with the above metaphor to construct a planner for intercepting a target moving in a stationary environment. The proposed model is divided into two stages: a stage to generated the potential field, and a stage to utilize that potential for navigation (Tracking Mechanism). In the following, I shall begin by considering a general, scalar, time-dependent potential ($V(\mathbf{x},t)$) that has not yet been conditioned to acquire navigational capabilities. This potential is first restricted to a class of fields that is suitable for navigation. A procedure that relies on boundary value problem formulation is then suggested for using $\mathbf{x}_p$ and Γ to transform the potential to a one that is capable of navigation. Finally, a reflexive, vector operator that is a function of $V(\mathbf{x},t)$ only is proposed for extracting a path that satisfies the conditions in (2).

2.1 FIELD GENERATION

Scalar fields which describe changes in both time and space can be generated using the Wave Equation

$$\nabla^2 V = \frac{1}{a^2} \cdot \frac{\partial^2 V}{\partial t^2}. \qquad 3$$

The reason the wave equation is chosen (i.e. describe the spread of information using a traveling wave mechanism instead, as the perception is, a diffusion mechanism) for constructing the navigation field is mainly due to the nature of its solution. From the method of the separation of variables, the solution of the wave equation can be written in the form

$$V(\mathbf{x},t) = R(\mathbf{x})T(t) \qquad 4$$

where R is dependent on position only, and T is dependent on time only. Therefore, the wave equation can be placed in the form

$$T\, \nabla^2 R - \frac{1}{a^2} \cdot R \cdot \frac{\partial^2 T}{\partial t^2} = 0\ . \qquad 5$$

The only way the above equation can be satisfied is for each of the terms dependent on the position only and the time only to be equal to the same constant; that is

$$\frac{\nabla^2 R}{R} = a^2 \cdot \frac{(\partial^2 T/\partial t^2)}{T} = -\lambda^2. \qquad 6$$

This leads to the solution of

$$\nabla^2 R + \lambda^2 R = 0 \qquad \text{(N-D Helmholtz equation)} \qquad 7$$

$$\frac{\partial^2 T}{\partial t^2} + (a\lambda)^2 T = 0 \quad \text{(1-D Helmholtz equation)}. \qquad 8$$

It is known that the fundamental solution of the Helmholtz equation in N-D provides N orthogonal basis function. Therefore, the above equations yield N+1 orthogonal basis, which are enough to represent (using the Generalized Fourier Series) a smooth function in both time and space while satisfying a given boundary condition. However, for the field to be usable for navigation, it has to satisfy other conditions which will be discussed later.

2.1.1 BVP1: The Dirichlet case

Here, field generation is carried out by solving the wave equation

$$\nabla^2 V = \frac{1}{a^2}\cdot\frac{\partial^2 V}{\partial t^2} \qquad 9$$

subject to the following BC (Figure-4)

$$V(\Gamma,t) = C,\ V(\mathbf{x}_p(t),t) = 0 \qquad C > 0,\ \mathbf{x}_p(t) \in \Omega \qquad 10$$

where $\mathbf{x}_p(t)$ is the differentiable trajectory of the target. The initial conditions $(V(\mathbf{x},0))$ can be obtained by solving

$$\nabla^2 V = 0 \qquad 11$$

subject to :

$$V(\Gamma) = C,\ V(\mathbf{x}_p(0)) = 0 \qquad C > 0,\ \mathbf{x}_p(0) \in \Omega\ .$$

2.1.2 BVP2: The Neumann case

Another method to generate the field may solve

$$\nabla^2 V = \frac{1}{a^2}\cdot\frac{\partial^2 V}{\partial t^2}. \qquad 12$$

subject to

$$V(\mathbf{x}_s,t) = C,\ \frac{\partial V(\Gamma,t)}{\partial n} = 0,\ V(\mathbf{x}_p(t),t) = 0 \qquad \mathbf{x}_s\in\Omega,\ \mathbf{x}_p\in\Omega.$$

The initial conditions $(V(\mathbf{x},0))$ can be obtained by solving

$$\nabla^2 V = 0 \qquad 13$$

$$\text{subject to :}\quad V(\mathbf{x}_s) = C,\ \frac{\partial V(\Gamma)}{\partial n} = 0,\ V(\mathbf{x}_p(0)) = 0 \quad .$$

Existence and uniqueness of the above BVP were proven in [10,11].

2.2 THE TRACKING MECHANISM

In the static case, a path to the target is traversed by tracking the current stream. This naturally lead to the use of the negative gradient of the potential (electric field) as a tracking mechanism:

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \alpha\cdot\nabla V(\mathbf{x})$$

which is derived from

$$\dot{\mathbf{x}} = \mathbf{E} = -\nabla V.$$

For the time varying case, the electric field has the more general form

$$\dot{\mathbf{x}} = \mathbf{E} = -(\nabla V + \frac{\partial \mathbf{A}}{\partial t}) \qquad 14$$

where the additional component is the result of the time variation of the vector magnetic potential. In the following section, a structure for $\mathbf{A}$ that is capable of producing global convergence is provided.

2.2.1 A proposed Tracking Mechanism

It is shown in the sequel that the following structure for $\partial\mathbf{A}/\partial t$ can achieve absolute convergence (asymptotic stability):

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{\nabla V(\mathbf{x},t)}{\|\nabla V(\mathbf{x},t)\|^2}\cdot\frac{\partial V(\mathbf{x},t)}{\partial t}\ . \qquad 15$$

The resulting tracking mechanism has the form

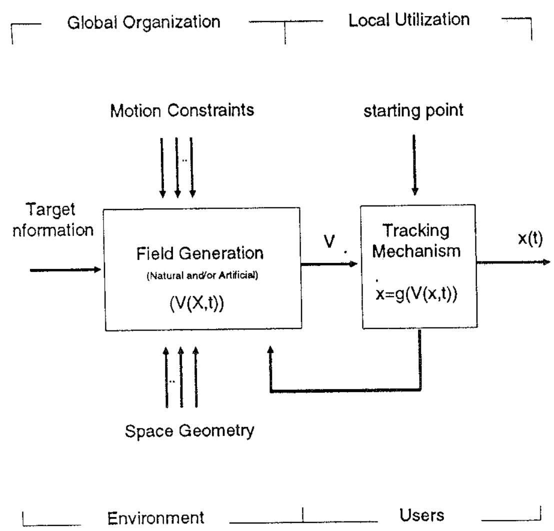

Figure-3: Basic Structure of reflexive Potential-Based Path planning techniques.

$$\dot{\mathbf{x}} = -[\nabla V(\mathbf{x},t) + \frac{\nabla V(\mathbf{x},t)}{\|\nabla V(\mathbf{x},t)\|^2}\cdot\frac{\partial V(\mathbf{x},t)}{\partial t}]\ . \qquad 16$$

It ought to be noticed that the partial, implicit dependence of $\partial V/\partial t$ on $d\mathbf{x}_p/dt$ by no means imply that the velocity of the target is needed for its computation. $\partial V/\partial t$ is always computed as

$$\frac{\partial V(\mathbf{x},t)}{\partial t} = \frac{V(\mathbf{x},(t),t) - V(\mathbf{x}(t-dt),t-dt)}{dt},$$

which only requires knowing $\mathbf{x}_p(t)$ and $\mathbf{x}_p(t-dt)$. The above mechanism has a singularity at the target location $\mathbf{x}=\mathbf{x}_p$ $(\nabla V(\mathbf{x}_p,t)=0)$. To remove this singularity, the absolute convergence requirement has to be relaxed to that of convergence (stability) only. In this case, the robot is only required to get an arbitrary small distance ρ close to the target. A guiding mechanism that can achieve this and is free of singularities has the form

$$\dot{\mathbf{x}} = -(\nabla V(\mathbf{x},t) + \frac{\nabla V(\mathbf{x},t)}{\beta(\|\nabla V(\mathbf{x},t)\|^2)}\cdot\frac{\partial V}{\partial t}(\mathbf{x},t)) \qquad 17$$

where

$$\beta(X) = \begin{cases} X & X \ge \rho \\ \eta(X) & X < \rho \end{cases}\ .$$

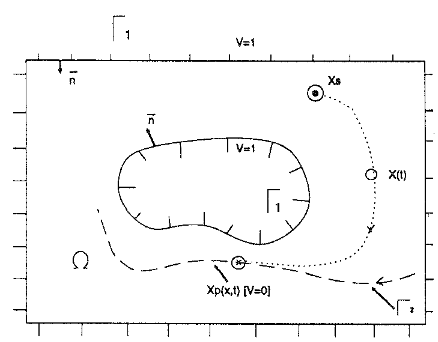

Figure-4: Boundary conditions for tracking (Dirichlet).

The function $\eta(X)$ is a monotonically increasing function that satisfies the following properties

$$\eta(0) = \varepsilon\ , \quad \eta(\rho) = \rho \qquad \rho > \varepsilon > 0$$

$$\left.\frac{d\eta(X)}{dX}\right|_{X=\rho} = 1\ , \quad \left.\frac{d\eta(X)}{dX}\right|_{X=0} = 0. \qquad 18$$

A form for $\eta(X)$ that satisfies the above conditions is

$$\eta(x) = \varepsilon + \frac{2\rho-3\varepsilon}{\rho^2}\cdot x^2 + \frac{2\varepsilon-\rho}{\rho^3}\cdot x^3 \qquad 0 < \varepsilon < \rho. \quad 19$$

2.2.2 The solution of the differential equation exist

The global Lipschitz condition [12] has to be satisfied if a solution for the nonlinear differential system (equation 17), which can be placed in the general form

$$\dot{\mathbf{x}}(t) = \mathbf{g}(\dot{\mathbf{x}}(t),t) \qquad \mathbf{x}(0) = \mathbf{x}_0 \quad t \geq 0\ , \qquad 20$$

is to globally exist and be unique ($\mathbf{x}(t) \in R^N$, and $\mathbf{g}$: $R_+ \times R^N \longrightarrow R^N$).

Suppose that for each $T \in [0,\infty)$ $\exists$ finite constants K_T, and h_T so that

$$\| \mathbf{g}(\mathbf{x},t) - \mathbf{g}(\mathbf{y},t) \| \leq K_T\cdot\| \mathbf{x} - \mathbf{y} \| \qquad 21$$

and

$$\| \mathbf{g}(\mathbf{x}_0,t) \| \leq h_T \qquad \forall\ t \in [0,T] \quad \forall\ \mathbf{x},\mathbf{y} \in R^N,$$

then the above dynamical system has exactly one solution over $[0,T]$, $\forall\ T \in [0,\infty)$. This condition is called the global Lipschitz condition. The requirement that the differential system in equation 17 satisfy this condition directly follows from the continuity and bounded nature of the spatial derivative of the system. Existence of a solution means that

i - $\mathbf{x}$ is differentiable almost everywhere ($\dot{\mathbf{x}}$ exist).
ii - The equation holds for all t where $\dot{\mathbf{x}}$ is defined.
iii- And $\mathbf{x}(t)$ satisfies

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_0^t \mathbf{g}(\mathbf{x}(\tau),\tau)d\tau\ . \qquad 22$$

3. MOTION ANALYSIS

The ability of the path to converge to the target and to avoid obstacles is analyzed using Liapunov direct method.

3.1 THE WAVE POTENTIAL IS A LIAPUNOV FUNCTION CANDIDATE

It is shown here that a potential function that is generated using the Boundary Value Problem in section 2.1 is a Liapunov Function Candidate (LFC) [13].

It is well known that the solution of the wave equation is analytic. This in turn meets the requirement of a LFC being differentiable or at least continuous. The fact that at a frozen instant of time (t_i) the potential $V(\mathbf{x},t_i)$ is a harmonic function makes X_p its global minimum. This in turn results in

1- $V(\mathbf{x},t_i) = 0$ at and only at $\mathbf{x}=\mathbf{x}_p$
2- $V(\mathbf{x},t_i) \geq 0$ for every $\mathbf{x} \in \Omega$

for every $t_i \in t$. This satisfies the remaining requirements for the wave potential to be a LFC.

3.2 Liapunov's Direct Method

A point $\mathbf{x}_p$ is said to be an equilibrium point of the above system if

$$\mathbf{g}(\mathbf{x}_p,t) = 0 \qquad \forall\ t \geq t_0.$$

Such an equilibrium is considered to be globally asymptotically stable ($\mathbf{x} \longrightarrow \mathbf{x}_p$ as $t \longrightarrow \infty$) if $\exists$ a continuously differentiable positive definite function $V(\mathbf{x},t)$ so that

$$V(\mathbf{x},t) \geq 0 \qquad \mathbf{x} \in R^N,\ \forall\ t \geq t_0,$$

and

$$V(\mathbf{x},t) = 0 \qquad \text{only at} \quad \mathbf{x} = \mathbf{x}_p$$

$$\dot{V}(\mathbf{x},t) = \frac{d}{dt}V(\mathbf{x},t) \leq 0 \qquad \mathbf{x} \in R^N,\ \forall\ t \geq t_0,$$

and

$$\dot{V}(\mathbf{x},t) = 0 \qquad \text{only at} \quad \mathbf{x} = \mathbf{x}_p\ . \quad 23$$

In the following, this method is used to prove that the proposed planner is capable of intercepting the target regardless of the maneuver that it may attempt. The Liapunov instability theory (same as above, but $\dot{V}(\mathbf{x},t) \geq 0$) is used to prove that the planner will also be able to avoid collision with the obstacles.

3.3 CONVERGENCE ANALYSIS (both Neumann and Dirichlet)

The time derivative of the wave potential is

$$\dot{V}(\mathbf{x},t) = \frac{dV(\mathbf{x},t)}{dt} = \frac{\partial V(\mathbf{x},t)}{\partial t} + \nabla V(\mathbf{x},t)^t\cdot\dot{\mathbf{x}}\ . \qquad 24$$

By substituting the above expression for $\dot{\mathbf{x}}$ (equation 16) in $\dot{V}(\mathbf{x},t)$, we have

$$\dot{V}(\mathbf{x},t) = \frac{\partial V(\mathbf{x},t)}{\partial t} - \nabla V(\mathbf{x},t)^t\cdot(\nabla V(\mathbf{x},t) + \frac{\nabla V(\mathbf{x},t)}{\|\nabla V(\mathbf{x},t)\|^2}\cdot\frac{\partial V(\mathbf{x},t)}{\partial t}) = -\|\nabla V(\mathbf{x},t)\|^2 \qquad 25$$

which satisfies the conditions on $\dot{V}(\mathbf{x},t)$.

To avoid the problem of the singularity at $\mathbf{x} = \mathbf{x}_p$, the requirement of asymptotic stability is relaxed. Instead of achieving asymptotic stability

$$\lim_{t\to\infty} \mathbf{x} \longrightarrow \mathbf{x}_p \qquad \text{for every } \mathbf{x}(0) \in \Omega$$

we are going to require stability only. For such a case, it is enough that $\mathbf{x}$ enters the neighborhood of $\mathbf{x}_p$; that is

$$\lim_{t\to\infty} \mathbf{x} \longrightarrow B\rho(\mathbf{x}) \qquad 26$$

where

$$B\rho(\mathbf{x}) : \{\ \mathbf{x}:\ \|\mathbf{x} - \mathbf{x}_p\| < \rho\ \}.$$

By using $\dot{\mathbf{x}}$ in equation 17, $\dot{V}(\mathbf{x},t)$ becomes strictly negative definite outside $B\rho(\mathbf{x})$

$$\dot{V}(\mathbf{x},t) = -\|\nabla V(\mathbf{x},t)\|^2 \qquad \mathbf{x} \notin B\rho(\mathbf{x}).$$

On the other hand, $\dot{V}$ is indefinite inside.

3.4 AVOIDANCE OF OBSTACLES (the Dirichlet case)

The ability of the planner to avoid forbidden regions {O} (or equivalently make $\mathbf{x}(t) \in \Omega\ \forall\ t$) is of equal importance as its ability to converge to the target

$$\mathbf{x}(t) \notin \{O\} \qquad t = [0,\infty)$$

and

$$\lim_{t\to\infty} \mathbf{x}(t) \longrightarrow \mathbf{x}_p(t)\ .$$

Our approach to analyze the behavior of the technique towards an obstacle focuses on studying the motion in a small region surrounding the forbidden regions ($\Gamma^\delta(\mathbf{x})$). Proving that the guiding mechanism can steer the path away from Γ in Γ^δ is enough to guarantee that the path will not intersect {O}.

Let us first begin by measuring the distance from Γ to the current position of the path ($\mathbf{x}(t)$). This distance is denoted by the variable $X_n(t)$

$$X_n = \mathbf{x}^t\cdot\mathbf{n} \qquad 27$$

where $\mathbf{n}$ denotes a unit vector normal to Γ. Since $\mathbf{n}$ is not a function of time (stationary obstacles), the rate of change of X_n ($\dot{X}_n$) can be calculated as

$$\dot{X}_n = \dot{\mathbf{x}}\cdot\mathbf{n}$$

$$= -(1+\frac{1}{\|\nabla V\|^2}\frac{\partial V}{\partial t})\nabla V\cdot\mathbf{n} = -(1+\frac{1}{\|\nabla V\|^2}\frac{\partial V}{\partial t})\frac{\partial V}{\partial n}. \quad 28$$

We are going to assume that motion starts outside Γ ($X_n>0$). This means that if we can prove that a measure of the length of X_n is always increasing in Γ^δ (Liapunov theory of instability is used), we can prove that the path will never intersect {O}. Let us define Va as a measure of the length of X_n:

$$Va = \|X_n\|^2 = X_n^2. \quad 29$$

Its derivative with respect to time ($\dot{V}a$) is

$$\dot{V}a = 2\cdot X_n\cdot\dot{X}_n = -2\cdot X_n\cdot(1+\frac{1}{\|\nabla V\|^2}\frac{\partial V}{\partial t})\frac{\partial V}{\partial n} \quad 30$$

since we are constraining $V(X,t)$ on Γ to a constant,

$$\frac{\partial V}{\partial t} = 0 \qquad X \in \Gamma.$$

Since $V(\mathbf{x},t)$ is the solution of the wave equation and it is differentiable in both time and space, for a sufficiently small δ we can make the following approximation:

$$\frac{\partial V}{\partial t} \approx 0 \qquad \mathbf{x} \in \Gamma^\delta \quad 31$$

Since at any frozen instant of time V is harmonic with its global maxima at Γ, we have

$$\frac{\partial V}{\partial n} < 0 \qquad \mathbf{x} \in \Gamma^\delta. \quad 32$$

Therefore, $\dot{V}a$ can be approximated in Γ^δ as

$$\dot{V}a = -2\cdot X_n\cdot\frac{\partial V}{\partial n}. \quad 33$$

Since for $\mathbf{x} \in \Gamma^\delta$, $X_n > 0$, we have

$$\dot{V}a > 0 \qquad \mathbf{x} \in \Gamma^\delta. \quad 34$$

This means that Va is always increasing with time. Therefore, the magnitude of X_n is always increasing, and $\mathbf{x}$ is being steered away from Γ.

3.5 Avoidance of obstacles (the Neumann case)

Since $\partial V/\partial n$ is set to zero at all time, the time derivative of the liapunov function in equation 30 is equal to zero:

$$\dot{V}a = -2\cdot X_n\cdot(1+\frac{1}{\|\nabla V\|^2}\frac{\partial V}{\partial t})\frac{\partial V}{\partial n} = 0. \quad 35$$

As a result the path will never penetrate the forbidden zone and collision with the obstacles will be avoided.

4. RESULTS

In this section, the tracking and obstacle avoidance capabilities of the proposed method are tested for different patterns of motion. Its tracking capabilities are also compared to that of the quasi-stationary Laplace method and the diffusion equation strategy. The three approaches were compared for stationary, linearly moving, and slowly moving targets. It is observed that all techniques exhibit equivalent capabilities in terms of converging to the target and avoiding the obstacles. Although the performance of the different techniques in terms of their tracking abilities is comparable for the above cases, the disparity greatly widens when a fast moving, rapidly fluctuating target is considered. In figure-5a,b,c the linear motion along the x-axis is supplemented with high sinusoidal oscillations along the y-axis. As can be seen, these fluctuations confused the quasi-stationary Laplace scheme leaving it undecided whether to proceed right or left of the corridor exit. The total failure of the quasi-stationary Laplace strategy is a consequence of its total disregard to the temporal dependence of events and total reliance on spatial information in planning a path to the target. The absence of the dimension of time from the strategy of the interceptor opens a "hatch" for the target through which this neglected dimension is exploited not only to evade capture by the interceptor, but even to totally paralyze him. Here the target is aware that the interceptor is following him along the minimum distance path at each instant with no regard to where the target is moving to next or how fast it is going there. Therefore, once the target observes a direction of motion for the interceptor, it quickly repositions itself (an activity related to time) so that the distance to the target is shorter if the interceptor moves along the opposite direction to the previous direction of motion. By continuously repeating this relatively simple maneuver, the target is able to bring the interceptor to a total stand-still along the x-axis and to trap him in a limit cycle along the y-axis (Figure-6). Definitely, more sophisticated maneuvers and methods of entrapments do exist for an intelligent target to use against an interceptor that is utilizing the quasi-stationary Laplace strategy. Although in the diffusion strategy only spatial information is used to lay a path to the target, temporal information is indirectly utilized by creating some degree of correlation between the successive spatial layers of the potential. Despite the ability of the interceptor to proceed in the general direction of the target, it fails to keep up with its rapid movements. As for the wave strategy, the interceptor was able to closely follow the target.

5. CONCLUSIONS

In this paper, a method of verifiable capabilities is suggested to safely intercept a target that is moving amidst known stationary obstacles. The solution is obtained using a suggested canonical form for potential-based navigation techniques along with the isotropic radiator metaphor. The suggested planner is complete in the sense that if there is a way to intercept the target, the planner will find it. The planner also has a causal implementation (i.e. it does not require a priori knowledge of the path of the target in order for it to successfully lay a path for interception). There are many questions that are yet to be answered about the behavior of the proposed technique. For example, the ability of the method to always succeed in intercepting the target regardless of the intelligence of the maneuver that is used by it needs to be carefully examined?! What is the effect of delay on the ability of the technique to capture the target? What is the rate of convergence (i.e. how fast can the method capture the target), and what is the effect of giving the pursuer and the target finite masses instead of assuming that they have a zero mass? To begin answering these questions, a thorough experimentation has to be done first. This requires a robust, efficient, interactive implementation of the method which is the focus of ongoing research.

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Figure-5a: Laplace (oscillating).

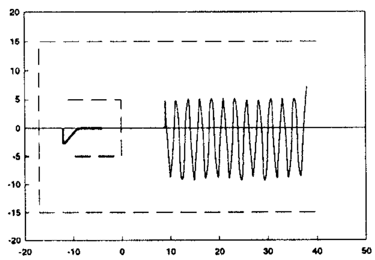


Figure-5b: Diffusion (oscillating).

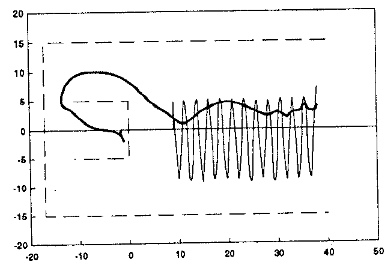
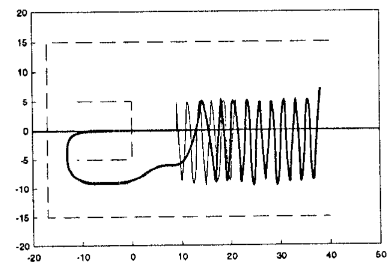


Figure-5c: Wave (oscillating)



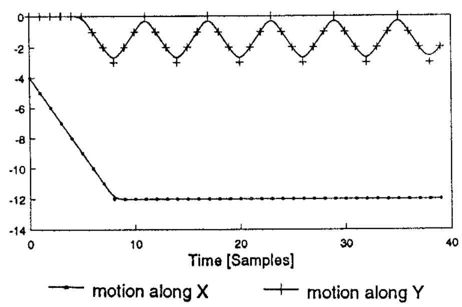


Figure-6: Quasistationary Laplace strategy confused by an oscillating target.

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