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A Modified, Hybrid, PDE-ODE Controller with Integrated Directional and Region Avoidance Constraints

by

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Abstract

This paper extends the capabilities of the Evolutionary, Hybrid, PDE-ODE controller (EHPC) that is suggested in [4] for navigating an agent in an unknown, multidimensional, stationary environment. This is accomplished by modifying the Hybrid, PDE-ODE controller (HPC) used in constructing the EHPC so that it can incorporate directional requirements among the set of constraints it is enforcing. Theoretical developments along with simulation results are provided.

1. Introduction

Despite the diversity of motion planning methods [1] all existing techniques, to the best of the authors' knowledge, are unified in considering isotropic workspaces where, at any point in the workspace, the agent is permitted to arbitrarily direct the motion of its state, motion actuators permitting. Practical workspaces, on the other hand, face a serious traffic management task that is usually handled by dividing the available space into structured domains each assigned a set of a priori rules for directing traffic. In most cases such rules extend beyond region avoidance constraints to that of restricting the direction along which motion is allowed to proceed. In a typical environment it is customary to find regions where traffic is prohibited, regions where traffic flow is regulated (e.g., ENTER & EXIT signs, or any other traffic signal), and others where traffic is free. It is highly unlikely to find a modern road or building where the above does not apply. From an AI point of view, the incorporation of directional constraints along with obstacle avoidance in directing the actions of an agent while making no assumptions about the geometry or topology of the environment is a formidable planning challenge which, to the best of the authors' knowledge, has not been addressed in the motion planning literature. It fundamentally differs from planning under nonholonomic constraints in which an agent may not be able to project motion along certain directions in the workspace due to the in ability of its actuators to direct motion along these directions. In other words, in the nonholonomic case, the constraints in the control space, which are limiting the efficacy of the motion actuators, are the one responsible for this behavior. On the other hand, directional constraints that are imposed in the workspace cannot be violated even if the agent's actuators permit it to do so.

While there are many planning approaches from which one may choose a candidate to modify in order to incorporate directional constraints, the authors believe that the Harmonic Potential Field approach to motion planning is an ideal candidate for such a choice [2,3]. The Harmonic Potential Field approach is an expression of the more general Hybrid, Partial Differential Equation-Ordinary Differential Equation (PDE-ODE) paradigm to motion planning [4,5]. A Hybrid, PDE-ODE controller (HPC) function to convert the data that is available to the agent about its environment into information that is encoded in the structure of the differential control action group which the agent is using to steer itself. Therefore, implicit in the ability of the agent to project successful actions, is the availability of a necessary and sufficient level of data for the HPC to grind into actions. Unfortunately, in a realistic situation, no guarantees of such a sort are provided. This is a serious weakness which HPCs suffer from that negatively impact on their ability to successfully steer the utilizing agent to its target state. This weakness, however, may be remedied by grounding the agent in its physical environment using Evolutionary, Hybrid, PDE-ODE controllers (EHPCs) [4,5]. EHPCs are situated, embodied, intelligent, and emergent mechanisms for behavior generation. They require no a priori knowledge of their multidimensional environment to guarantee that an agent with an arbitrary shape will converge to its target from the first attempt (First Attempt Completeness (FAC) characterizes the state where the agent has no information what so ever about its environment). Moreover, in this class of planners, the range of the sensors has no influence on convergence where even local sensing such as tactile sensing is enough to guarantee convergence in a multidimensional environment. The range of the sensors controls only the rate of convergence. In this paper, the capabilities of EHPCs are upgraded to enable them to plan in nonlinear, anisotropic workspaces. This is accomplished by modifying the second component of an EHPC, the Hybrid, PDE-ODE controller, so that it can incorporate directional requirements among the set of constraints it is enforcing. The modification is carried out through the use of Nonlinear, Anisotropic, Harmonic Potential Fields which are capable of incorporating directional constraints along with obstacle avoidance in a motion planning process.

2. The Hybrid, PDE-ODE Controller

The motion generated by a HPC that is expressed in a harmonic potential manifold setting [4] is described by the following gradient dynamical system (Figure-1) :

$$\begin{aligned} \dot{X} &= -\nabla V(X,\Gamma,X_t)\ , \qquad X\in\boldsymbol{\Omega}. \\ \textit{such that} \ & \lim_{t\to\infty} X(t)\to X_t,\ \ X(t)\cap O\equiv\phi\ . \end{aligned} \tag{1}$$

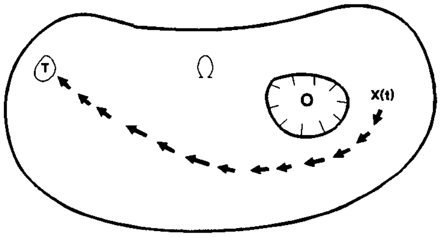

Figure-1: Environment with region avoidance constraints only.

On the other hand, the modified planner generates a safe, constraint-satisfying trajectory to the target using the gradient dynamical system:

$$\begin{aligned} \dot{X} &= -\nabla V(X,\Psi(X),\Gamma,X_t)\ , \qquad X\in\boldsymbol{\Omega}. \\ \textit{such that} \ & \lim_{t\to\infty} X(t)\to X_t,\ X(t)\cap O\equiv\phi\ ,\ \dot{X}^t\Psi(X)>0 \quad X\in\boldsymbol{\Omega}'. \end{aligned} \tag{2}$$

where V is a scalar potential field (Figure-2). The Potential field is conditioned for navigation using the BVP: solve

$$\begin{aligned} & \nabla^2 V(X)=0 && X\in\boldsymbol{\Omega}-\boldsymbol{\Omega}', \\ \textit{and}\quad & \nabla\cdot\Sigma(X)\nabla V(X)=0 && X\in\boldsymbol{\Omega}', \\ \textit{subject to}\quad & V(X_t)=0,\ V(\Gamma)=1\ . \end{aligned} \tag{3}$$

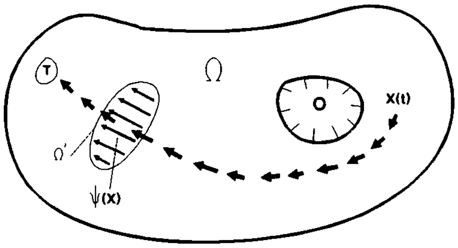

Figure-2: Environment with both region avoidance and directional constraints.

where X and $\dot{X}$ are the N-D position and velocity vectors respectively, Γ is the boundary of the forbidden regions (O, Γ=∂O), X_t is the target state, Ω is the workspace of the robot, Ω' is a subset of Ω ($\Omega' \subset \Omega$),

$$\Sigma(X) = \begin{bmatrix} \sigma_{x_1}(\Psi(X)) & 0 & \ldots & 0 \\ 0 & \sigma_{x_2}(\Psi(X)) & \ldots & 0 \\ \cdot & & & \\ 0 & 0 & \ldots & \sigma_{x_N}(\Psi(X)) \end{bmatrix}, \qquad (4)$$

$\Psi(X)$ is a unit vector field that is defined in Ω' to mark the direction along which motion is to proceed, and $\sigma_{xi}(\Psi(X))$ is defined as:

$$\sigma_{x_i}(\Psi(X)) = \begin{bmatrix} \sigma_f & \Psi^t(X)\nabla V(X) > 0 \\ \sigma_b & \Psi^t(X)\nabla V(X) \le 0 \end{bmatrix}, \qquad (5)$$

σ_f and σ_b are positive constants ($\sigma_f \gg \sigma_b$) depicting the permissibility of the associated part of the workspace.

3. Results

The capabilities of the planner are demonstrated using the simple example shown in figures 3,4 of a road with two unidirectional lanes. At each lane the agent is required to restrict its direction of motion in accordance with the large arrows shown to mark the admissible directions. Also, the agent can only switch lanes at either the beginning or end of the road and is forbidden from doing so along the solid line separating the two lanes. First, an isotropic, harmonic potential field planner is used for steering motion from the initial starting point to the target point. As can be seen from figure-3a the planner totally disregarded the constraints on direction and proceeded to the target along the shortest path (a straight line). Figure-3b shows the corresponding gradient navigation vector field that the agent is using to steer itself. In Figure-4a a nonlinear, anisotropic, harmonic planner is used for steering motion. As can be seen, the planner enforced the directional constraints, avoided the obstacles, and drove the agent to its target. Figure-4b shows the corresponding gradient field .

4. Conclusions

In this paper a novel and complete motion planner (HPC) that is capable of integrating directional constraints along with obstacle avoidance constraints in the navigation process is suggested. The proposed planner is an important addition to the motion planning literature enabling the utilizing agent to enforce the important directional constraints which realistic workspaces often present. It also demonstrates the effectiveness of the harmonic potential field approach to planning, not only as an effective motion planner, but also as a prototype for generating other planning techniques that realistically address the needs of agents operating in real world environments.

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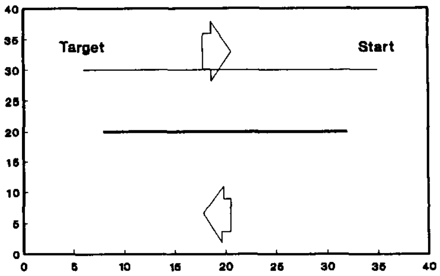

Figure-3a: Harmonic Potential Field Planner.

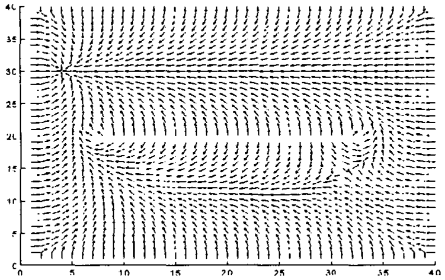

Figure-3b: Corresponding Gradient Field.

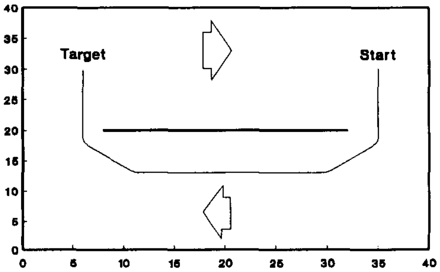

Figure-4a: Modified Harmonic Planner.

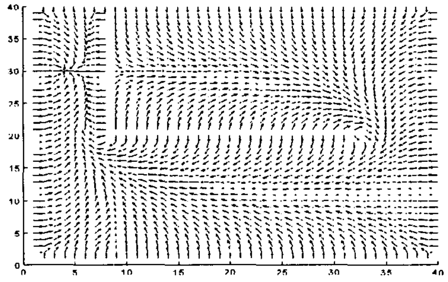

Figure-4b: Corresponding Gradient Field