

A Self-Organizing, Hybrid, PDE-ODE Structure for Motion Control in Informationally-deprived Situations

Ahmad A. Masoud

Unit 12A, 71 Brock St., Kingston, Ontario, Canada K7L 1R8, E-mail: masoud@host.king.igs.net

Samer A. Masoud

Mechanical Engineering Department, Jordan University of Science and Technology, Irbid, Jordan

Abstract

One of the most challenging problems in robotics is the design of navigators. Navigators are motion controllers that are used to generate a constrained solution trajectory linking a starting point to a target zone for an agent of arbitrary, unknown shape that is operating in an unknown environment. In other words, this class of controllers is required to carry out the unusual task of solving an ill-posed problem. This paper discusses the construction of a hybrid control structure that would enable an agent to successfully deal with the informationally-deprived situation that is described above. The control structure has an evolutionary nature that allows for Autonomous Structural Adaptation of the control relying only on a sequentially acquired sensory input as a source of information. It also has the ability to integrate any available a priori information in its database, regardless of its fragmentation or sparsity, in the planning process to accelerate convergence. The structure is based on a bottom-up, Artificial Life approach to behavior synthesis that allows the integration of experience along with synergy as drivers of the action selection process. Theoretical development of the structure, a realization, and simulation results are provided.

1. Introduction

Utility and meaning in the behavior of an agent are highly contingent on the agent's ability to semantically embed its actions in the context of its environment. Such an ability is cloned into the agent using a class of intelligent motion controllers that are called motion planners. The above task becomes significantly difficult when the agent is to tackle an environment that denies the agent's operator/designer access both informational and physical. Two examples of the above are: underwater sea exploration where due to the severe attenuation high frequency electromagnetic waves experience by water no communication between the exploring agent and the operator is possible. The other involves planetary exploration where long delays in information transmission severely hampers the communication process. An agent operating under such conditions can not rely on guidance from the operator to avoid hazard or to proceed towards the goal. To complicate things more, the agent may not have the benefit of utilizing experience in selecting its actions. In such situations failure to properly act the first time may lead to damaging the agent (a swim or sink situation). These restrictions do not only rule out active-guidance where the operator, on-line, steers the actions of the agent, it also includes indirect guidance that assumes the form of algorithms implanted into the agent to steer its actions in a manner that meets the acceptance of the designer, not necessarily the demands of the environment. It is not hard to see that if an agent is to successfully operate under such an informationally-deprived situation it must be capable of autonomous operation.

Some of the concerns which autonomous operation may give rise to are:

- 1- The compatibility of the representation with the manner by which the agent makes decisions and actuates motion.
- 2- The informational adequacy of the representation (i.e., does the representation encode enough information to generate a successful action).
- 3- If needed, the ability to augment the available information to, at least, the minimum level needed to execute the task.
- 4- The ability to convert the acquired information into successful actions.

An area of research that falls under the above category is planning in an unknown, stationary environment. Here, an agent is required to lay

a safe trajectory to a stationary target state relying only on the local information that is sequentially being acquired by its finite range sensors. It must be able to coherently tie the stream of fragments of sensory input in a manner that permits the generation of a continuous stream of action instructions to the actuators of motion. The structure of such a stream is required to successfully embed the agent in its environment.

This paper examines the construction of a Hybrid Control Structure that would allow an agent to move to a stationary target in a multidimensional space containing stationary, forbidden regions while guaranteeing convergence from the first attempt which is marked by the absence of any information about environment from the database of the agent. Subsequent attempts result in the agent improving its performance. The suggested structure is situated, embodied, intelligent, and emergent [1]. It utilizes both experience and synergetic interaction among a massive number of local differential systems in driving the action selection process [3]. The structure is required to satisfy the four requirements stated at the beginning of this section.

This paper is organized as follows: In section 2 the planning problem is formulated. Section 3 discusses the control structure. Section 4 discusses a realization of the structure. Section 5 presents the simulation results, and conclusions are placed in section 6.

2. Problem Formulation

The type of agents tackled here is described by the differential system:

$$\dot{X} = u \quad (1)$$

where u is the control input ($u \in \mathbb{R}^N$). X and \dot{X} are N -dimensional position and velocity vectors. Unfortunately, the above system implies the unrealistic assumption that the agent can execute any action instruction (u) that the controller supply. However, the above system can still be used with mechanical agents moving at low speeds.

Let O be a set of a priori unknown regions in \mathbb{R}^N which the agent is required to avoid, Γ is the boundary of O ($\Gamma = \partial O$), and Ω is the space in which the agent is permitted to operate ($\Omega = \mathbb{R}^N - O$). Let Γ be the subset of Γ that is a priori known ($\phi \subset \Gamma \subset \Gamma$). Let Q be the state of a Discrete Event System (DES) [4]. At any time Q must assume a value from the binary set $\{0,1\}$. Such a value depends on the event the local sensors register regarding the possible future position of X ($X^+(t+dt)$). There are only two events, either X^+ is in Ω ($X^+ \notin O$) which for this case Q assumes the value 0, or X^+ is in a forbidden region ($X^+ \in O$) where Q is 1. The value of Q is driven from 0 to 1 at time t_i by a combination of the current control which the agent believe, given the amount of information available at that moment, that it can successfully direct its actions, the remaining unknown part of the environment ($\Gamma - \Gamma$), and the location of the target. The opposite transition from 1 to 0 which occurs at t'_i is caused by the modified control which the agent uses for directing its future actions. Although Q experiences discrete jumps in value, the cause of these jumps is continuous. Therefore, the planner must have a hybrid nature [2]. Here action is totally carried out by a continuous process while the discrete phenomenon is manifested only as a pattern that uses the continuous process as a substrate. The agent reacts to the $X^+ \in O$ event at t_i by modifying its control so that a transition of Q from 1 to 0 occurs at t'_i . The agent control at t_i is denoted by the vector field $u = f_i(X, T, Q, f_{i-1})$ ($f_i \in \mathbb{R}^N$), where T is the target region and f_i maps the hybrid situation space $X \times T \times Q \times f_{i-1}$ to the N -dimensional continuous action space u . For successful action the agent is required to synthesize a finite set of successively dependent f_i 's $\{f_i: 1, \dots, L < \infty\}$ so that:

$$\begin{aligned} \dot{X} &= f_i(X, T, Q, f_{i-1}) & X(0) &\in \Omega, \\ \lim_{\substack{i \rightarrow L \\ t \rightarrow \infty}} X(t) &\rightarrow T & f_0 &= f(x, \Gamma', T), \\ & & i &\in [1, \dots, L], \\ & & t &\in [t_0, \dots, \infty), \\ \text{also } X(t) \cap O &\equiv \phi & & \forall t. \end{aligned} \quad (2)$$

3. The Suggested Structure

The control structure functions to convert the goal and the available representation of the environment into a sequence of actions $\{u_0, \dots, u_L\}$. These actions must yield a corresponding sequence of states $\{X_0, \dots, X_L\}$ so that the final state X_L is the goal state of the agent. The action sequence is called a plan which is a member of a field of plans (Action Field) that densely covers state space so that regardless of the starting point, a plan always exists to propel the agent to its goal. Constructing a control structure of the above kind for a general agent described by $\dot{X} = g(X, u)$ begins by densely spreading agent-like micro-agents at every point in the domain of the control (Figure-1).

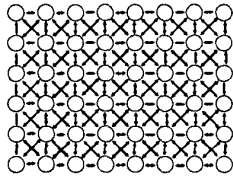


Figure-1: An interacting collective of micro-agents.

The only difference between the “mother-agent” and a micro-agent is that the state of the agent evolves in time and space while the state of a micro-agent is stagnant and immobilized to one a priori known point in the state space

$$\dot{X}_i = g(X_i, u(X_i)), \quad (3)$$

where X_i is the a priori known location of the i 'th micro-agent in state space (i.e., X_i is a constant), u is the action (control input) under the disposal of the micro-agent ($u \in \mathbb{R}^M, M \in \mathbb{N}$), and $g(X_i, u)$ is the change that the i 'th micro-agent induces in the state of the agent so it is driven from X_i to X_j

$$X_j = X_i + dt g(X_i, u(X_i)) \quad (4)$$

and dt is an infinitesimal unit of time. The micro-agent concept is used to construct a control action group for the agent by first covering the state space with a manifold that has locally (point-wise) extractable vector features which homogeneously cover the domain on which the control is defined. The vector features are determined by the vector partial differential operator that is used to operate on the manifold to induce a vector field that may be used to describe the action structure of the micro-agent group, therefore generating the Action Field of the agent. The second step is to provide each micro-agent with the ability to generate a proper differential behavior. Differential behavior is a self-behavior where a micro-agent does not attempt to influence the other micro-agents that it is directly interacting with. Instead, it forms a soft informational coupling with them where it only observes their behavior and uses this information to derive a self-action that governs its and only its behavior in state space (Figure-2). This may be achieved by constraining the vector partial differential operator that is used to emulate the actions of the micro-agents (P) using another partial differential operator (L). This operator encodes how a micro-agent is going to constrain its behavior with respect to the behavior of the other micro-agents it is interacting with. The third step is to induce a proper action structure over the micro-agent group. Unlike centralized approaches where each micro-agent has to search for the “correct” action in order to generate a group structure that unifies all the micro-agents in one goal-oriented unit, in the proposed approach a micro-agent is only required not to exert the “wrong” actions that could result in the failure of the agent to reach the goal.

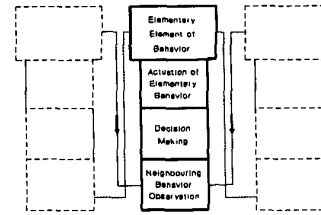


Figure-2: Layers of functions in an interactive micro-agent.

Obviously not selecting the wrong actions is not enough, on its own, for each micro-agent to restrict itself to one and only one admissible action that would constitute a proper building block of the global structure that is needed to turn the micro-agent group into a functioning

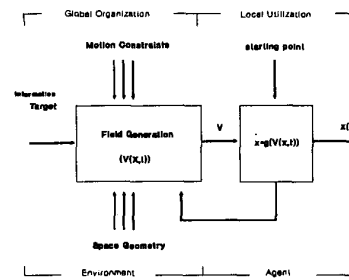


Figure-3: Basic Structure of a Hybrid, PDE-ODE Controller.

unit. The additional effort needed to induce the global structure on the micro-agents is a result of the evolution of the behavior of the micro-agent group in time and space under the guidance of the environment (i.e. morphogenesis, [5]). This guidance is what eventually limits each micro-agent to one and only one action that is also a proper component in a functioning group structure. The environment guidance may be factored into the behavior generation process as state boundary conditions which play the role of self-preserving actions that the agent is a priori equipped with. The behavior of a micro-agent at a location which the agent believes to be harmful is constrained to an a priori known survival action that would drive motion away from it and towards a safe region. The above three steps describe the Hybrid PDE-ODE system shown in Figure-3. Figure-4 demonstrates the ability of the above Hybrid PDE-ODE system to generate, without any assistance from an external agent, the necessary information which is needed to induce the necessary action structure on the micro-agent substrate in order to provide the agent with the necessary action instructions that enable it to reach its target.

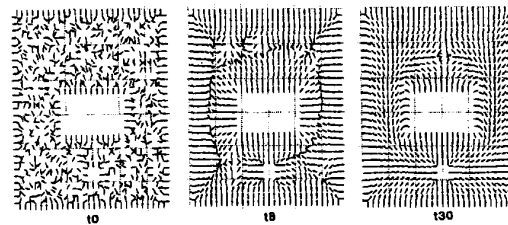


Figure-4: Evolution of Action Structure in a PDE-ODE Controller.

Unfortunately, for a Hybrid, PDE-ODE controller (HPC) to project successful actions, it must have a necessary and sufficient level of data available to grind into actions. In a realistic situation no guarantees of such a sort are provided. This weakness may be remedied by grounding the agent in its physical environment using Evolutionary, Hybrid, PDE-ODE controllers (EHPC). One of the fundamental assumptions which an EHPC structure is based on regards the agent's environment as the inducer of a subjective (self-referential) representation which the agent

can avail itself from. This representation is either incrementally acquired through the sensory input and/or is a priori present in the database of the agent. The PDE-ODE system discussed above transforms the goal, constraints on motion, and initial knowledge of the environment into a continuous sequence of instructions which the agent uses for directing its actions.

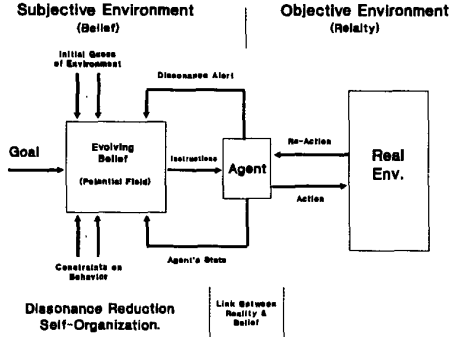


Figure-5: The Suggested, Evolutionary, Hybrid Control Structure.

The HPC receives two feedbacks from the agent, a continuous one and a discrete one. The continuous feedback provides the current location of the agent in state space. This location is measured with respect to the target using a self-referential coordinate system. As for the discrete feedback, it is supplied at random instants in time. Its presence is indicative of a dissonance situation. Dissonance is a generic term that indicates an irregularity in the agent's internal functioning, a mismatch between outcomes and expectations, or the presence of a hazard in the vicinity of the agent. Once a dissonance arises, the agent immediately stops using the Action Field (control) that is based on its current belief whose usefulness has already been falsified by the rise of dissonance. At the same time the dissonance signal sets the HPC in a self-organizing mode to generate a dissonance-free Action Field that is based on the new modified belief. Figure-5 shows the structure of an EHPC. In short, an EHPC consists of two parts:

- 1- a discrete time-continuous time system to couple the discrete-in-nature data acquisition process to the continuous-in-nature action release process,
- 2- a HPC to convert the acquired data into information that is encoded in the structure of the differential, control action group.

4.0 A Realization

This section outlines several tools needed for realizing the above control structure.

4.2.1 Differential Constraints

For the types of agents described in section 2 a scalar manifold (V) is sufficient for emulating the action of a dense group of micro-agents (this is not always the case for a general agent [6-7]). Here two differential surface features may be used for constructing the point vectors needed for describing the actions of the individual micro-agents. Either the slope of V is used to construct the micro-control action:

$$u(X) = -\nabla V(X), \quad (5)$$

or the slope of the curvature of the surface is used:

$$u(X) = -\nabla(\nabla^2 V(X)). \quad (6)$$

To construct a self-behavior (differential behavior) that would enable the micro-agent group to constructively interact, the micro-control at every point in state space is constrained with respect to the other micro-controls that are in its immediate infinitesimal neighborhood. The actions of the micro-agents are related to each other using

$$\nabla \cdot u(X) = 0, \quad (7)$$

where $\nabla \cdot$ is the divergence operator. This choice guarantees the continuity of motion so that no micro-agent may take the "wrong" action of blocking motion before the agent reaches its target. Therefore the governing relation that is used to condition the differential properties of V so that it can emulate a dense, interactive, micro-agent group are the Laplacian or the Bilaplacian operators

$$\nabla^2 V(X) = 0, \quad \nabla^4 V(X) = 0. \quad (8)$$

4.2 State Constraints

As was mentioned earlier, factoring the environment into the action selection process is carried out using behavioral boundary conditions. Whenever the evolving state of the agent is at a location in the environment that affects its internal environment in an unsatisfactory manner, therefore giving rise to a state of dissonance in the agent, the agent responds by constraining the actions of the micro-agents at that location to an a priori known action that is designed to drive it to a safe place in the state space (survival action). This can be indirectly achieved by applying the proper Boundary Conditions (BC) on the manifold at those locations in state space which the agent believes to be harmful to it. In the following two BC's that suits the Laplacian [8], and Bilaplacian [9] differential operators are briefly stated.

4.2.1 The Harmonic PDE-ODE Planner

The generating BVP is:

$$\nabla^2 V(X) = 0, \quad (9)$$

subject to $V(X) = 1 \mid_{X=\Gamma}$, $V(X) = 0 \mid_{X=\Gamma_p}$,

where $\Gamma_p = \{X: |X - T| \leq \rho, \rho > 0\}$. Figure-6 shows an Action Field of a Harmonic planner.

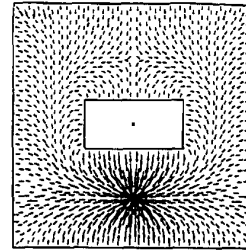


Figure-6: Action Field, Harmonic.

4.2.2 The Biharmonic PDE-ODE Planner

The generating BVP for this case is: jointly solve

$$\nabla^4 V(x,y) = 0, \quad (10)$$

and $\nabla V(x,y)(\nabla V(x,y))^T = \lambda[\nabla \cdot \Delta(x,y)]I + G[J(\Delta(x,y)) + J^T(\Delta(x,y))]$

subject to

$$\Delta(x,y)|_{x,y=1} = 0, \quad \nabla_x \Delta(x,y)|_{x,y=1} = 0,$$

and $\sigma_{xx} = P \cdot n_{px}$, $\sigma_{yy} = P \cdot n_{py}$, $\sigma_{xy} = 0$,

where Δ is a displacement vector, ∇_x is the curl operator, λ , P , and G are positive constants, I is the identity matrix, J is the Jacobian, n_{px} is the x component of the unit vector that is normal to Γ_p , n_{py} is the same but for the y component, and

$$\sigma_{xx} = \frac{\partial^2 V(x,y)}{\partial x^2}, \quad \sigma_{yy} = \frac{\partial^2 V(x,y)}{\partial y^2}, \quad \sigma_{xy} = \frac{\partial^2 V(x,y)}{\partial x \partial y}.$$

Figure-7 shows an Action Field generated by the Biharmonic planner.

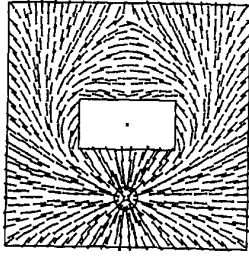


Figure-7: Action Field, Biharmonic PDE-ODE.

4.3 The Over-all Controller

HPCs are reliant on an accurate, a priori known model of the environment in order for them to properly function. In the following, a procedure is suggested to remove this dependence and construct an EHPC.

Let an agent be operating in a space (Ω) that is a subset of an N-D space constituting its environment. Let the hazardous regions in that environment be called O ($O=R^N - \Omega$), Γ be the unknown boundary of that region ($\Gamma=\partial O$, $\Gamma \in R^N$, $M \leq N$), and Γ^0 be a subset of Γ which the agent is initially aware of ($\phi \subset \Gamma^0 \subset \Gamma$). Let the region occupied by the agent at time t be $R(X,t)$ which is a priori known and is initially inside Ω . Let $\gamma(X,t)$ be the boundary of that region ($\gamma(X,t) = \partial R(X,t)$). Let R_s be the sensed region surrounding R and $\gamma_s(X,t) = \partial R_s(X,t)$

$$R_s(X,t) = \{X: |X - \gamma(X,t)| \leq \epsilon, X \in R(X,t)\}, \quad (11)$$

where ϵ is the range of the sensor. In this paper, local sensing is used ($\epsilon \ll 1$). Also let q be the K-D natural coordinates of the agent ($q \in R^K$), and q_s be the starting point, q_e be the final target point, and both $R(X,t_s)$ and $R(X,t_e) \in \Omega$. The following procedure may be used to successfully navigate an agent with arbitrary shape to its target in an unknown environment, therefore satisfying the requirements in equation 2:

- 1- Select one of the HPCs in section 4.2.
- 2- Convert Γ^0 from the workspace coordinates to the natural coordinates of the agent Γ_n^0 (this step is not needed if Γ^0 is set to ϕ).
- 3- set $In=0$.
- 4- Solve the BVP that corresponds to the chosen PDE-ODE planner subject to the proper BC's on Γ_n^0 , q_s , and q_e .
- 5- Apply the proper vector differential operator to the potential field in order to generate the dynamical system

$$\dot{q} = u_i(q, q_s, \Gamma_n^0), \quad (12)$$

$$u_i = \begin{cases} -\nabla V_i(q, q_s, \Gamma_n^0) & \text{for the Laplacian} \\ -\nabla^2 V_i(q, q_s, \Gamma_n^0) & \text{for the BiLaplacian} \end{cases}$$

- 6- As long as $Q=0$, generate a path for the agent in its natural coordinates using:

$$q(t) = q_s + \int_{t_0}^t u_i(q, q_s, \Gamma_n^0) dt. \quad (13)$$

- 7- If $q(t)=q_e$, Halt.
- 8- If at any instant (t_i) Q changes state from 0 - 1 (i.e., the sensors detected a hazardous region that was not previously known to the agent (Γ^0)),

$$\Gamma_s = R_s \cap \Gamma \neq \phi, \text{ and } \Gamma_s \in \Gamma_n^0, \quad (14)$$

- a. Halt motion (i.e. set $q(t)=q(t_i)$),
- b. Add the point in the natural coordinates of the agent to the known-obstacles contours,

$$\Gamma_n^0 = \Gamma_n^0 \cup q(t_i). \quad (15)$$

9- $In=In+1$, go to step 4.

5. Results

Figure-8 shows a point agent attempting to reach a target in a maze. The maze is not known and the agent is restricted to using local proximity sensing. Figure-8.1 shows the agent's trajectory laid during its first attempt to solve the maze. As can be seen, despite the total lack of knowledge about its environment and the local nature of its sensors, the agent manages to reach its target from the first attempt. In the process of reaching its target the agent had to adjust its control 45 times (Figures 8.1.1-8.1.4). Figure-8.2 shows the trajectory of the agent that is generated during its second attempt to reach the target. Using the experience it acquired from the first attempt the agent managed to eliminate the unnecessary detours from its path. It also had to make fewer adjustments to its control (only 12 adjustments to the control field were made during the second attempt). Figures 8.2.1-8.2.4 show the control field of the agent at different stages of evolution during the second attempt. Figure-8.3 shows the trajectory laid during the agent's third attempt at reaching its target. As can be seen, a steady, smooth, optimum path to the target was achieved without the agent having to make any adjustment to the control field it acquired during the second attempt.

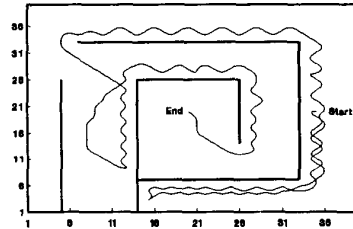


Figure-8.1: Point robot trajectory, First attempt.

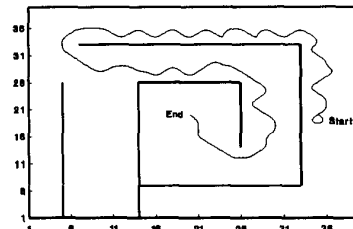


Figure-8.2: Point robot trajectory, Second attempt.

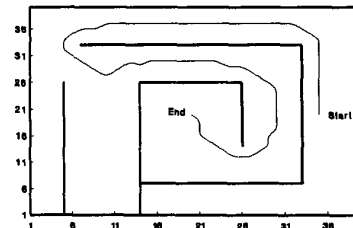


Figure-8.3: Point robot trajectory, Third attempt.

The second example demonstrates the self-referential nature of the control structure. A fixed-base, 2-D manipulator is required to flex in a confined environment (i.e. move from the initial configuration $\Theta_1=60^\circ, \Theta_2=120^\circ$ to the final configuration $\Theta_1=90^\circ, \Theta_2=0^\circ$). Figure-9 show the trajectory of the agent and Figures 9.1-9.4 show the belief of the agent which had to be adjusted 17 times in order for the agent to reach its target.

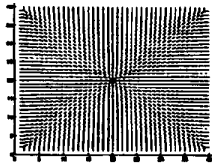


Figure-8.1.1: Action field, first attempt, point robot, t_0 .

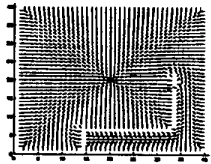


Figure-8.1.2: Action field, first attempt, point robot, t_1 .

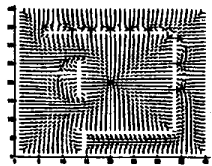


Figure-8.1.3: Action field, first attempt, point robot, t_2 .

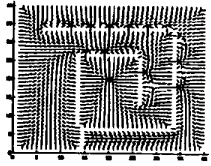


Figure-8.1.4: Actionfield, first attempt, point robot, t_3 .

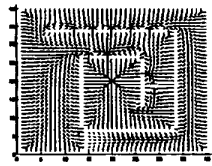


Figure-8.2.1: Action field, second attempt, point robot, t_4 .

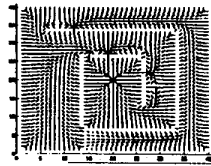


Figure-8.2.2: Action field, second attempt, point robot, t_5 .

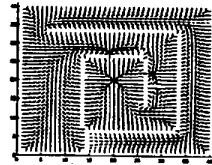


Figure-8.2.3: Action field, second attempt, point robot, t_6 .

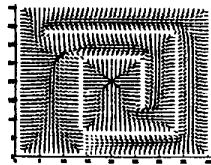


Figure-8.2.4: Action field, second attempt, point robot, t_7 .

In figure-10 the completeness of the planner is demonstrated. Completeness requires the planner to find a solution trajectory if one exist. If no solution exists, the planner should halt operation and provide an indicator that a solution does not exist. Here, the planner provides such an indication by degenerating the action field in all the regions which the target cannot be reached from. In figure-10 the target is placed in a region where it can not be reached from the initial starting point of the agent. Not knowing that the target cannot be reached, the agent proceeded to goal adjusting its belief each time an obstacle is encountered. Once the agent reaches the belief that the target is inaccessible, it halts operation and degenerates the action field in region where the agent cannot be reached from. Here, the agent had to perform 18 adjustments to its belief before reaching the decision that no path to the target exists. Figures 10-(1,2,3,4) show the evolution of the action field.

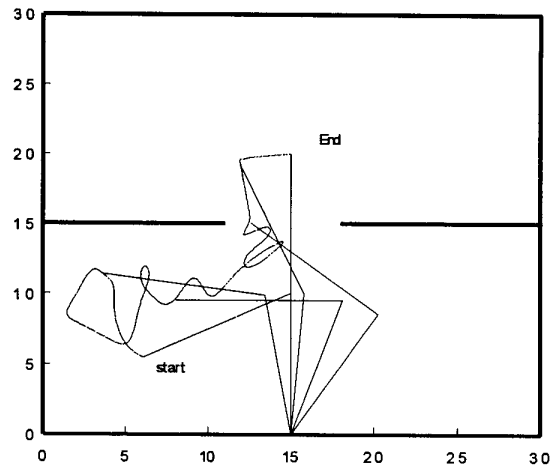


Figure-9: Arm manipulator trajectory, first attempt.

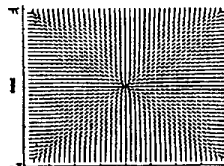


Figure-9.1: Action field, first attempt, arm manipulator, t_0 .

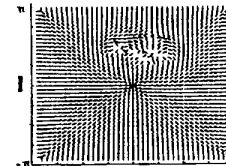


Figure-9.2: Action field, first attempt, arm manipulator, t_1 .

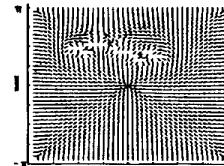


Figure-9.3: Action field, first attempt, arm manipulator, t_2 .

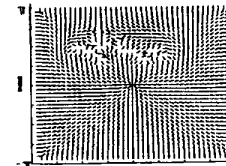


Figure-9.4: Action field, first attempt, arm manipulator, t_3 .

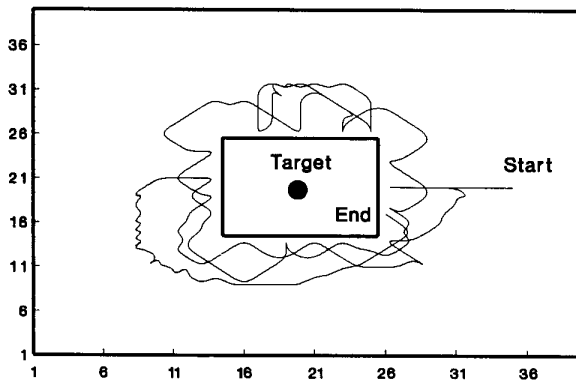


Figure-10: Unattainable goal, point agent.

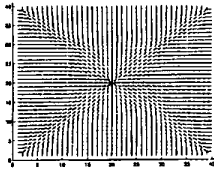


Figure 10-1, t0

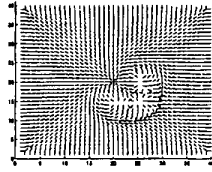


Figure 10-3, t15

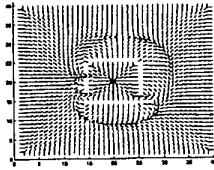


Figure 10-2, t4

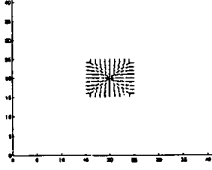


Figure 10-4, t18

6. Conclusions

In this paper an Evolutionary, Hybrid, Control structure is suggested for generating a constrained trajectory for an agent operating in an unknown multidimensional space. Instead of using a Top-Bottom, hierarchical reasoning approach as a driver of action selection, the structure uses a bottom-up synergetic, Artificial Life approach. The suggested structure has several attractive properties, most notably is its ability to guarantee that a trajectory to the target state will be found from the first attempt if one exists (i.e. First Attempt Completeness).

Although the class of agents used in this work is a little simplistic dealing with the kinematic aspects of motion only, ongoing work tackles more realistic agents [7] that incorporate the dynamic aspects in the planning process. The authors are optimistic that the suggested framework for behavior generation is a good first step towards the synthesis of intelligent control structures.

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