Proceedings of the 2000 IEEE
International Conference on Robotics & Automation
San Francisco, CA • April 2000

Motion Planning in the Presence of Directional and Obstacle Avoidance Constraints Using Nonlinear, Anisotropic, Harmonic Potential Fields

by
Ahmad A. Masoud, Samer A. Masoud *
*Mechanical Engineering Department, Jordan University of Science & Technology, Irbid, Jordan.
e-mail: masoud@just.edu.jo

Abstract

Recently, the authors suggested a new class of Intelligent Motion Controllers that are called Evolutionary, Hybrid, PDE-ODE Controllers (EHPCs) [33]. A controller of such a class is designed for the special task of guiding an agent in a fully unknown environment to a target set along an obstacle-free trajectory. In a short compaion paper, the authors breifly described an extension that would allow an EHPC to jointly condition a motion trajectory with both directional and region avoidance constraints [34]. In this paper, an indepth investigation of the proposed extension is provided. Also, mathematical proofs of both convergence, and the ability to enforce directional and region avoidance constraints are supplied.

1.0 Introduction

Utility and meaning in the behavior of an agent are highly contingent on the agent's ability to semantically embed its actions in the context of its environment. Such an ability is cloned into the agent using a class of intelligent motion controllers that are called motion planners. Despite the diversity of motion planning methods [1-3] all existing techniques, to the best of the authors' knowledge, are unified in considering isotropic workspaces (a workspace is an admissible subset of statespace) where, at any point in the workspace, the agent is permitted to arbitrarily direct the motion of its state, motion actuators permitting. Practical workspaces, on the other hand, face a serious traffic management task that is usually handled by dividing the available space into structured domains each assigned a set of a priori rules for directing traffic. In most cases such rules extend beyond region avoidance constraints to that of restricting the direction along which motion is allowed to proceed. In a typical environment it is customary to find regions where traffic is prohibited, regions where traffic flow is regulated (e.g., ENTER & EXIT signs, etc.), and others where traffic is free. It is highly unlikely to find a modern road or building where the above does not apply. From an AI point of view, the incorporation of directional constraints along with obstacle avoidance in directing the actions of an agent while making no assumptions about the geometry or topology of the environment is a formidable planning challenge which, to the best of the authors' knowledge, has not been addressed in the motion planning literature. It fundamentally differs from planning under nonholonomic constraints [4] in which an agent may not be able to project motion along certain directions in the workspace due to the inability of its actuators to direct motion along these directions (i.e., the constraints in the control space, which are limiting the efficacy of the motion actuators, are the ones responsible for this behavior). On the other hand, directional constraints that are imposed in the admissible region of state space (i.e., workspace) cannot be violated even if the agent's actuators permit it to do so.

While there are many planning approaches from which one may choose a candidate to modify in order to incorporate directional constraints, the authors believe that the Harmonic Potential Field approach to motion planning is an ideal candidate for such a choice. Although a paradigm to describe motion using Harmonic Potential Fields has been available for more than three decades [5-7] it was not till 1987 when Sato [8] formally used the approach as a tool for motion planning. Unfortunately, the work was written in Japanese and had very little exposure (an English version of the work may be found in [20]). Shortly after, the approach was formally introduced to the robotics and intelligent control literature through the independent work of Connolly et. al. [13], Prassler [9] & Tarassenko et. al. [15] who expressed the approach using an electric network analogy, Lei [10] & Plumer [16] who used a neural network setting, and Keymeulen et. al. [14,12] & Akishita et. al. [11] who utilized a fluid dynamic metaphor in their development of the approach. Other work may be found in [16-31].

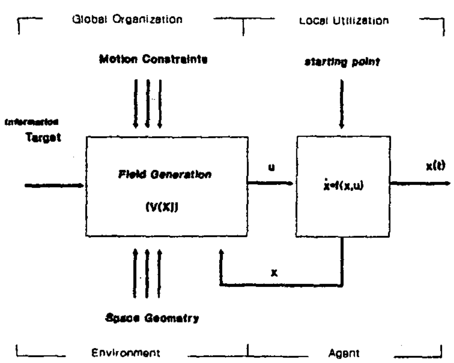


Figure-1: A Hybrid PDE-ODE Control Structure.

The Harmonic Potential Field approach is an expression of the more general Hybrid, Partial Differential Equation-Ordinary Differential Equation (PDE-ODE) paradigm to motion planning (Figure-1) [32-34]. A Hybrid, PDE-ODE controller (HPC) functions to convert the data that is available to the agent about its environment into information that is encoded in the structure of the differential control action group which the agent is using to steer itself. In this class of controllers the conversion mechanism is constructed in conformity with the Artificial Life (AL) approach to behavior generation [35]. To achieve this mode of operation, first the lucidity of the control action is established by inducing the control action group on a potential surface (a manifold) using a vector partial differential operator. The behavior of each member of the group (differential control action) is constrained with respect to the other members in its immediate neighborhood using a proper partial differential operator (G-type of behavior). The group control action (P-type) evolves in space and time as a result of the interpretation of the G-type in the context of the environment. This is achieved by using boundary conditions to factor the influence of the environment in the behavior generation process. Figure-2 shows an evolving control action group in an HPC. In essence, HPCs function to convert available data about the environment into information which the agent uses to steer its actions. Therefore, implicit in the ability of the agent to successfully reach its target, is the availability of a necessary and sufficient level of data for the HPC to grind into actions. Unfortunately, in a realistic situation, no guarantees of such a sort are provided.

This is a serious weakness which HPCs suffer from that negatively impact on their ability to steer the utilizing agent to its target state. This weakness, however, may be remedied by grounding the agent in its physical environment using Evolutionary, Hybrid, PDE-ODE controllers (EHPCs) [32-34].

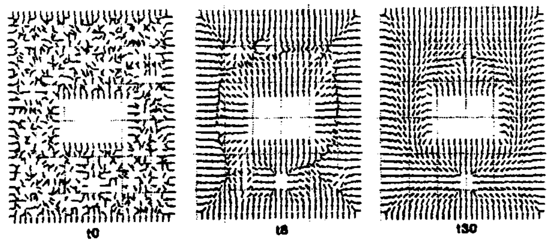


Figure-2: Evolution of the control field in an HPC.

An Evolutionary, Hybrid, PDE-ODE (Figure-3) controller consists of two parts:
1- a discrete time-continuous time system to couple the discrete-in-nature data acquisition process to the continuous-in-nature action release process,
2- a Hybrid, PDE-ODE controller to convert the acquired data into in-formation that is encoded in the structure of the differential control action group.

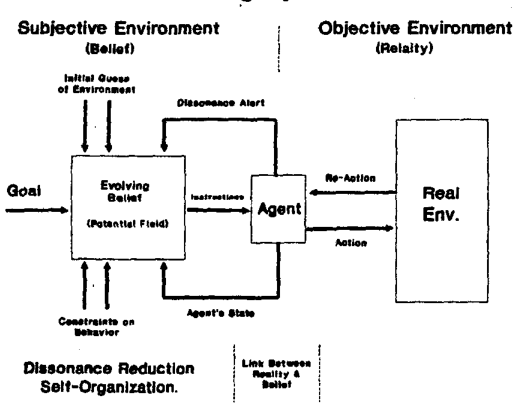


Figure-3: An Evolutionary, Hybrid, PDE-ODE Controller.

EHPCs are situated, embodied, intelligent, and emergent mechanisms for behavior generation [36]. They require no a priori knowledge of their multidimensional environment to guarantee that an agent with an arbitrary unknown shape will converge to its target from the first attempt (First Attempt Completeness (FAC) characterizes the state where the agent has no information about its environment). Moreover, in this class of planners, the range of the sensors has no influence on convergence where even local sensing such as tactile sensing is enough to guarantee convergence in a multidimensional environment. The range of the sensors controls only the rate of convergence. Figure-4 shows three attempts of a point agent to reach its target at the center of the maze. Despite the total lack of a priori knowledge about the maze and the use of proximity sensing, the agent manages to reach its target every attempt, each time enhancing its performance till it converged along an optimal path to the target. In this paper, the capabilities of EHPCs are upgraded to enable them to plan in nonlinear, anisotropic workspaces supporting directional constraints along with region avoidance constraints. This is accomplished by modifying the second component of an EHPC, the Hybrid, PDE-ODE Controller, so that it can incorporate directional requirements among the set of constraints it is enforcing.

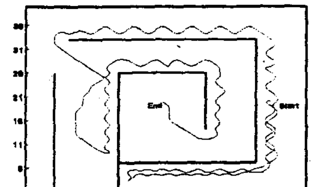


Figure 4a: First Attempt

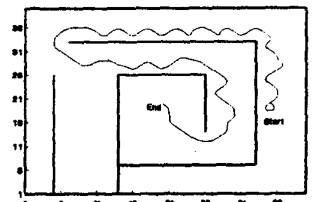


Figure 4b: Second Attempt

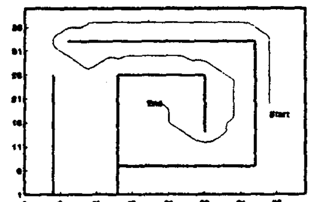


Figure 4c: Third Attempt

This paper is organized as follows: section 2 contains the problem formulation describing the behavior of the modified Hybrid, PDE-ODE controller. Section 3 describes the physical metaphor used in deriving the modified HPC. Section 4 contains the mathematical description of the modified HPC. Section 5 contains a proof of the validity of the suggested HPC. Results, and conclusions are placed in section 6 and section 7 respectively.

2.0 Problem Formulation

In its most general form, a Hybrid PDE-ODE controller is required to synthesize a control u for a dynamical system that is described by the nonlinear state space equation (Figure-5):

$$\dot{X} = f(X,u) ,$$

such that

$$\lim_{t\to\infty} X(t) \to X_T , \quad \& \qquad (1)$$
$$X(t) \cap O \equiv \phi \quad \forall t ,$$

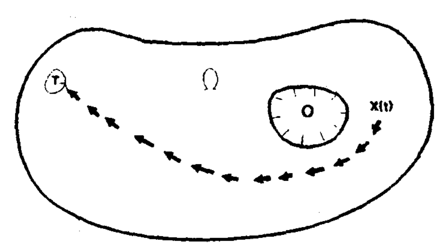


Figure-5, Domain of available HPCs

where X is a N-dimensional vector describing the agent's state in its natural coordinates, u is a M-dimensional control vector, f: $R^N \times R^M \to R^N$, X_T is the target state, and O is the set of forbidden regions in state space which the agent should always avoid, Γ is the boundary of O (Γ=∂O). While ongoing work is focusing on developing HPCs that can tackle both the dynamics and kinematics of an agent [37-39], here the focus is only on kinematics, i.e., the equation of motion is

$$\dot{X} = u . \qquad (2)$$

Also the HPC used here is expressed using a harmonic potential field (V) in the Dirichlet setting. Other ways for expressing an HPC may be found in [25,32]. For this case the HPC is required to synthesize the control signal

$$u = -\nabla V(X,\Gamma,X_T) \ , \tag{3}$$

so that for a system described by (2), the conditions in (1) are satisfied. V is constructed by solving the Boundary Value Problem (BVP)

$$\begin{aligned} &\nabla^2 V(X) \equiv 0 \ , && X \in \Omega \ , \\ &V(X_T)=0, \quad V(X)|_{X\in\Gamma} = C \ , \end{aligned} \tag{4}$$

where Ω is the workspace of the agent (Ω=R^N - O), ∇^2 is the Laplacian operator, and ∇ is the gradient operator.

The directional constraints which the modified HPC is required to enforce are defined on Ω' which is a subset of Ω ($\Omega' \subset \Omega$). They assume the form of the vector field $\Psi(X)$, $X \in \Omega'$, and Ψ: $R^N \to R^N$ The compliance of the agent with these constraints is detected using the inner product

$$\dot{X}^t \Psi(X) \ , \tag{5}$$

such that if $\dot{X}^t\Psi(X) > 0$, the constraints are enforced

$\dot{X}^t\Psi(X) \leq 0$, the constraints are violated.

The modified HPC is required to synthesize the control signal (Figure-6)

$$u = -\nabla V(X,\Psi(X),\Gamma,X_T) \qquad X \in \Omega \ , \tag{6}$$

such that for a system described by (2)

$$\begin{aligned} &\lim_{t\to\infty} X(t) \to X_T \ , \\ &X(t) \cap O \equiv \phi \qquad \forall\ t, \quad and \\ &\dot{X}^t\Psi(X) > 0 \qquad X \in \Omega' \ ,\forall t \ . \end{aligned} \tag{7}$$

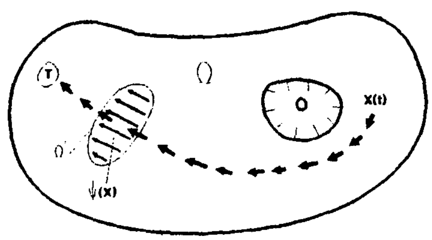


Figure-6, Domain of modified HPC

For convenience, in the remainder of the paper, V(X) is used to refer to $V(X,\Psi(X),\Gamma,X_T)$.

3.0 A Physical Metaphor

A proper analogy between a well-understood natural process and a problem at hand may serve as a feasible alternative to the arduous task of mathematically deriving a provably-correct solution to the problem. The Harmonic field approach lends itself to this mode of problem solving. It is well-known that a path generated by the gradient dynamical system from a Harmonic potential field in the Dirichlet setting is analogous to the path marked by the electric current moving in a resistive grid (Figure-7) with the potential set at a positive constant value at the nodes marking the boundary of the forbidden regions and to zero at the node which is located at the target point [9,15,26,28].

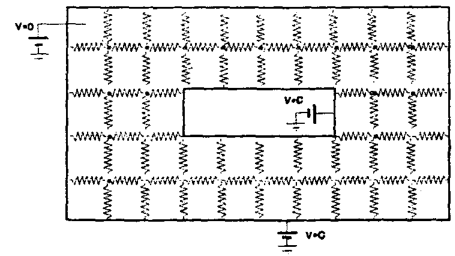


Figure-7, resistive grid equivalence of a harmonic planner

The correctness of such an analogy may be easily deduced by discritizing the 2-D Laplacian operator

$$\nabla^2 V(x,y) = \frac{\partial^2 V(x,y)}{\partial x^2} + \frac{\partial^2 V(x,y)}{\partial y^2} \tag{8}$$

in order to construct the difference equation

$$V(i,j) = \frac{1}{4}[V(i,j+1)+V(i,j-1)+V(i+1,j)+V(i-1,j)] \tag{9}$$

which may be interpreted as an element with four equal resistors (R=1) connected to a node with a V(i,j) voltage (Figure-8)

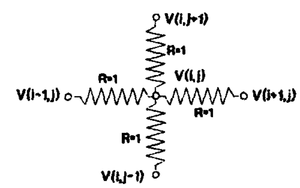


Figure-8, resistive element of harmonic field

As can be seen, in the harmonic approach, a resistive element is the one manipulating the electric current. A resistor is a linear, bilateral electric component with characteristics that remain unchanged regardless of the direction in which the current passes through the element (Figure-9).

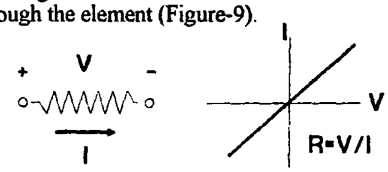


Figure-9, characteristics of a resistor

To add the needed directional sensitivity, an element that is sensitive to the direction in which the current passes through needs to be used along with the resistor for building a grid that would manipulate the flow of the electric current in the desired manner. The new element is a diode [40]. Ideally, a diode is a voltage controlled switch that can be in either one of two states (Figure-10): a. forward biased state, in which its resistance is zero, b. backward biased, in which its resistance is infinite.

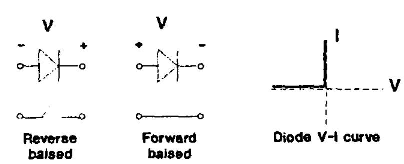


Figure-10, characteristics of a diode as a switch

A more realistic model of a diode is that of a resistive element whose resistance (R_d) varies depending on the direction in which the current flow (Figure-11):

$$R_d = \begin{bmatrix} R_f & \textit{if forward baised} \\ R_b & \textit{if backward baised} \end{bmatrix}, \qquad R_b >> R_f. \quad (10)$$

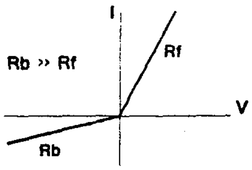


Figure-11, characteristics of a realistic diode

A diode and a resistor are sufficient elements for building a grid that would control the flow of the electric current in a manner that is analogous to the behavior of the suggested planner. In the regions of the worksapce marked as free traffic zones (Ω-Ω') a resistive element only is used for building the motion control grid (Figure-12). In regions where the direction of traffic is constrained (Ω'), a diode element is used in the construction of the grid so that it is placed in a forward biased mode along the admissible direction of motion along which traffic is allowed to proceed. The voltage of the nodes marking the boundary of the forbidden regions is set to a constant, positive voltage, and the voltage of the node marking the target is set to zero.

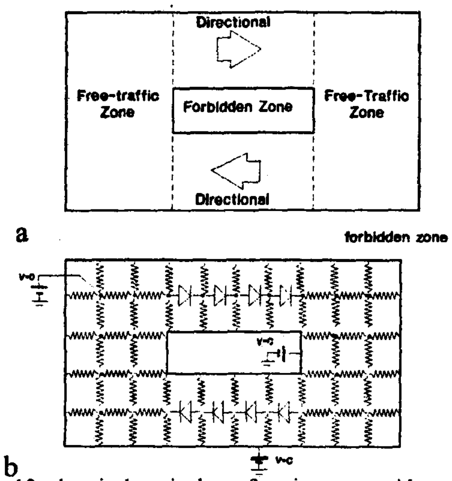


Figure-12: electrical equivalent of environment with directional constraints.

4.0 Nonlinear, Anisotropic Field Synthesis

Based on the above metaphor, the modified BVP is derived.

4.1 The Modified Differential Operator:

The Laplacian operator is constructed by forcing the divergence ($\nabla\cdot$) of the gradient of the potential field, which is analogous to the current in a resistive grid, to zero inside Ω,

$$\nabla\cdot\nabla V(X) = \nabla^2 V(X) \equiv 0 \qquad X\in\Omega\,. \quad (11)$$

This condition guarantees the continuity of the current, which in turn guarantees the continuity of motion, inside Ω in order to steer motion to the global minimum of V(X) which is situated at X_T. As can be seen, by choosing the Laplacian operator as the governing relation of the differential behavior of the electric current (i.e., motion), no preferable direction for motion to proceed along can be encoded in the behavior of the agent (i.e the workspace is linear and isotropic). To modify the governing differential operator so that along with guaranteeing the continuity of motion inside Ω, favorable directions of motion inside Ω' may also be enforced, the metaphor in section 3.0 is used. At a point $X\in\Omega'$ a diode is assumed to be present and oriented in a manner such that the favorable direction of motion, which is marked by the vector $\Psi(X)$, coincides with the direction in which the diode is in a forward biased mode. This means that the current experiences low resistance R_f , or equivalently high conductance σ_f , along that direction. On the other hand, the current experiences high resistance R_b , or equivalently low conductance σ_b , along the opposite direction. Therefore, the electric current inside Ω' may be expressed as:

$$-\Sigma(X)\,\nabla V(X)\ , \quad (12)$$

where

$$\Sigma(X) = \begin{bmatrix} \sigma_{x1}(X) & 0 & \ldots & 0 \\ 0 & \sigma_{x2}(X) & \ldots & 0 \\ \cdot & & & \\ 0 & 0 & \ldots & \sigma_{xN}(X) \end{bmatrix}$$

and

$$\sigma_{xi}(X) = \begin{cases} \sigma_f & -\nabla V(X)^t\Psi(X) > 0 \\ \sigma_b & -\nabla V(X)^t\Psi(X) \le 0 \end{cases},$$

$\sigma_f >> \sigma_b$. After sensitizing the electric current to a favorable direction of motion, the motion continuity constraints are applied by forcing its divergence to be identically zero inside Ω'

$$\nabla\cdot\,[\Sigma(X)\,\nabla V(X)] \equiv 0 \qquad X\in\Omega'\,. \quad (13)$$

4.2 The Modified BVP

A BVP which may be used to generate a potential field for constructing the control signal in equation (6) for both region avoidance and the directional constraints is: solve

$$\nabla^2 V(X) \equiv 0 \qquad \Omega\text{-}\Omega' \quad (14)$$

and

$$\nabla\cdot\,[\Sigma(X)\,\nabla V(X)] \equiv 0 \qquad X\in\Omega'\,.$$

subject to $V(X_T) = 0\ ,\quad V(X)|_{X\in\Gamma} = C\ ,$

5.0 Performance Verification

A mathematical proof is provided to verify the capabilities of the suggested planner. It is based on Lyapuonov method [41,42] for stability of nonlinear dynamical systems.

5.1 Lyapunov Stability Theorem:

In order for the system $\dot X = f(X)$ (15)

to be globally asymptotically stable (i.e., $\lim_{t\to\infty} X(t) \to X_T$) it is sufficient that there exists a scalar function V(X) with continuous first partial derivatives with respect to X, so that:

a. $V(X)=0 \qquad X=X_T$ (16)
b. $V(X)>0 \qquad X\neq X_T$ (i.e., V(X) is positive definite),

and for $\dot V(X) = \frac{dV(X)}{dt}$

c. $\dot V = 0 \qquad X=X_T$.

d. $\dot V < 0 \qquad X\neq X_T$ (i.e., V(X) is negative definite).

A V(X) that satisfies a & b is called a Lyapunov Function Candidate (LFC). If V(X) satisfies all of the above conditions, it is called a Lyapunov Function (LF). Usually, another condition for V(X) to be a LF is for $V(X)\to\infty$ with $\|X\|\to\infty$. However, since we are dealing with finite domains, this condition is not applicable.

First, (13) is reexpressed as follows:

$$\sum_{i=1}^{N} \frac{\partial}{\partial x_i} \sigma_{xi}(X) \frac{\partial V(X)}{\partial xi} \tag{17}$$

$$= \sum_{i=1}^{N} [\ \sigma_{xi}(X) \frac{\partial^2 V(X)}{\partial xi^2} + \frac{\partial V(X)}{\partial xi} \frac{\partial \sigma_{xi}(X)}{\partial xi}\ ]$$

since $\sigma_{xi}(X) = \sigma_b + (\sigma_f - \sigma_b)\ U(\nabla V(X)\ '\Psi(X)\ )$,

the above expression becomes:

$$= \sum_{i=1}^{N} [\sigma_{xi}(X)\frac{\partial^2 V(X)}{\partial xi^2} + (\sigma_f - \sigma_b)\delta(\nabla V(X)\ '\Psi(X))\frac{\partial V(X)}{\partial xi}\frac{\partial \nabla V(X)\ '\Psi(X)}{\partial xi} \tag{18}$$

where U( ) is the unit step function, and δ( ) is the dirac-delta function. As can be seen the second term of the operator is either zero or infinite. Since the BVP in (14) is constructed based on an analogy with a natural process, its solution exists and is unique. In other words, the value of the differential operator should be zero everywhere in Ω'. Therefore, the governing differential relation may be written as

$$\sum_{i=1}^{N} \sigma_{xi}(X) \frac{\partial^2 V(X)}{\partial xi^2} \equiv 0 \qquad X \in \Omega' \tag{19}$$

Proposition-1: A potential field V(X) which is generated by the BVP in (14) is a Lyapunov Function Candidate.

Proof: Since by construction $V(X_T)=0$, showing that V(X) is a LFC requires only to show that $V(X)>0\ \forall\ X \neq X_T$. Since V(X) satisfies the second order elliptic partial differential equation

$$\sum_{i=1}^{N} k_i(X) \frac{\partial^2 V(X)}{\partial xi^2} \equiv 0 \qquad X \in \Omega,\ k_i > 0\ , \tag{20}$$

and $V(X) = C\ \ \forall\ X \in \Gamma$, where

$$k_i(X) = \begin{cases} \sigma_f & X \in \Omega - \Omega' \\ \sigma_{xi}(X) & X \in \Omega' \end{cases},$$

if at any point $X \in \Omega$ $V(X) \leq 0$, $\exists$ a local minima such that $\partial^2 V(X)/\partial xi^2 > 0$. Since all $k_i(X)$'s are positive, equation (20) will be violated. Therefore, V(X) should be greater than zero for all $X \in \Omega$. In other words, V(X) is a LFC. In a similar way one can also prove that $V(X) < C\ \forall\ X \in \Omega$.

Proposition-2: If $X(0) \in \Omega$, the motion steered by the gradient dynamical system in (6) will always remain inside Ω (i.e. $X(t) \cap O \equiv \phi\ \ \forall t$).

Proof: Consider the part of Ω near an obstacle O (Figure-13). Let n(x) be a vector normal to the surface of the obstacle Γ, $x \in \Gamma$.

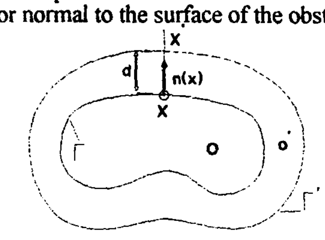


Figure-13, region near obstacle

Let O' be a region created by infinitesimally expanding the forbidden region O such that $O \subset O'$, and Γ' be the boundary of that region ($\Gamma' = \partial O'$). The radial derivative of V(x) along n(x) may be computed as

$$\frac{\partial V(x)}{\partial n(x)} = \frac{V(x') - V(x)}{(x' - x)\ 'n(x)}\ , \qquad x' \in \Gamma'\ , \tag{21}$$

where x' is computed as the minimum distance between x and Γ'. Since the value for the potential everywhere in Ω is less than C, and x' lies inside Ω the radial derivative of the potential along n(x) is negative

$$\frac{\partial V(x)}{\partial n(x)} < 0\ . \tag{22}$$

Let us assume that X(t) is initially located at x', and d is the distance between X(t) and x

$$d = n(x)\ '(X(t) - x)\ . \tag{23}$$

Note that since x' is initially inside Ω, d is initially positive. Let L be a measure of that distance

$$L = d^2\ . \tag{24}$$

Noting that n(x) is not a function of time, and $\partial V/\partial n = n' \nabla V$, the rate of change of L with respect to time may be computed as:

$$\begin{aligned} \frac{dL}{dt} &= 2d\dot{d} = 2\ d\ n'(x)\ \dot{X}(t) \\ &= -2d\ n'(x)\nabla V(x) = -2d\frac{\partial V(x)}{\partial n(x)} > 0\ . \end{aligned} \tag{25}$$

Therefore d is increasing with time and X(t) is being steered away from Γ. This makes it impossible for X(t) to intersect O, or in other words

$$X(t) \cap O \equiv \phi \qquad \forall t\ . \tag{26}$$

Proposition-3: The motion generated by the control signal in (6) will globally, asymptotically converge to X_T,

$$\lim_{t \to \infty} \to X_T \qquad \forall\ X(0) \in \Omega\ . \tag{27}$$

Proof: To prove the above it is sufficient to show that the time derivative of V(X) is negative definite (i.e. V(X) is a Lyapunov Function). The time derivative of V may be computed as:

$$\dot{V} = \nabla V(X)\ '\ \dot{X}(t) = -\nabla V(X)\ '\nabla V(X) = -\|\ \nabla V(X)\ \|\ . \tag{28}$$

∇V(X) does not vanish any where in Ω with the exception of some isolated points and a subset of the interface between the nonlinear, anisotropic region and the rest of the workspace (Γ'). Since the equilibrium in each of these cases has an unstable nature that does not threaten convergence to the only stable equilibrium point which is situated at X_T, we have

$$\begin{aligned} &\dot{V}(X) = 0 \qquad X = X_T\ . \\ \&\ &\dot{V}(X) > 0 \qquad \forall X \in \Omega\ , \end{aligned} \tag{29}$$

Therefore dV/dt is negative definite and V is a valid Lyapunov Function. Therefore, X(t) will globally asymptotically converge to X_T.

Proposition-4: For any point $X \in \Omega'$

$$\dot{X}'\Psi(X) > 0 \qquad X \in \Omega'\ . \tag{30}$$

Proof: Let xo be a point inside Ω' (Figure-14). Let x1 be another point in that region constructed by extending xo a small distance (Δr) along the direction of the unit vector Ψ(xo), i.e. x1= xo + Ψ(xo)Δr .

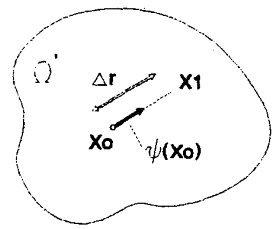


Figure-14, points inside Ω'

By integrating the partial differential relation in (19) along the Ψ(xo) direction, one can approximate the potential at x1 as:

$$V(x1) \approx V(xo) + \frac{k}{\sigma_{xi}(xo)} \|\nabla V(xo)^t \Psi(xo)\| \Delta r ,$$
$$= V(xo) + \frac{k}{\sigma_{xi}(xo)} \|\frac{\partial V(xo)}{\partial \Psi(xo)}\| \Delta r , \qquad (31)$$

where k is a finite positive constant. Let us assume that the directional constraints are violated, i.e.,

$$\dot{X}^t \Psi(xo) \leq 0 . \qquad (32)$$

Note that $\dot{X} = -\nabla V(X)$ .There are two possibilities:

1- $\dot{X}^t \Psi(xo) = 0$: Here, the second term of equation 18,

$$\sum_{i=1}^{N} (\sigma_f - \sigma_b)\ \delta(\nabla V(X)^t \Psi(X))\ \frac{\partial V(X)}{\partial xi}\ \frac{\partial \nabla V(X)^t \Psi(X)}{\partial xi} \qquad (33)$$

will become infinite, making the relation in 19 impossible to satisfy. This possibility is already ruled out by the fact that the solution to the suggested BVP exists and is unique.

2- $\dot{X}^t \Psi(xo) < 0$: for this case, $\sigma_{xi}(xo) = \sigma_b$. When constructing the potential field σ_b is chosen to be a very small positive number ($\sigma_b << 1$). This results in a high value for V(x1) that will exceed the value of the potential at Γ, i.e., V(x1)>C. This possibility was ruled out since it leads to the violation of equation 20.

As can be seen the only remaining possibility is that $\dot{X}^t \Psi(xo) > 0$, $xo \in \Omega'$. In other words, the directional constraints must be satisfied.

6.0 Simulation Results

6.1: Results:

The behavior which the modified HPC form is capable of projecting subsumes that of a Linear Harmonic HPC. While there are salient similarities in the behavior from both HPC forms, there are, never-the-less, profound differences that are not amenable to analysis or interpretation using the framework of linear potential fields in general, and harmonic potential fields in particular. The following simulation results are intended to highlight some of these differences along with some basic capabilities of the planner. An environment similar to the one in figure-12a is chosen to test the new method. It consists of three different types of domains: 1- forbidden regions, 2- constrained traffic regions, and 3- free traffic regions. As can be seen from figure-15.a, the planner disregarded the directional constraints and drove motion along the shortest path to the target. The corresponding navigation field is shown in figure-15b.

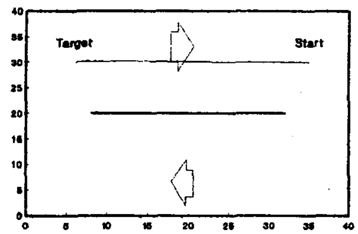


Figure-15a: Harmonic Planner.

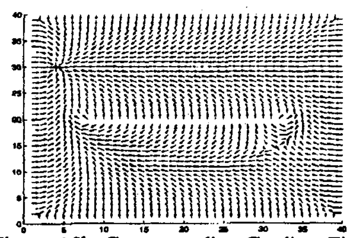


Figure-15b: Corresponding Gradient Field.

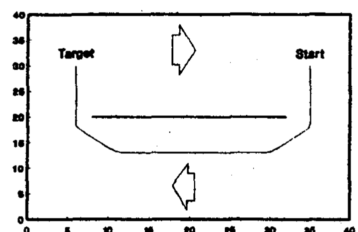


Figure-16a: Modified Harmonic Planner.

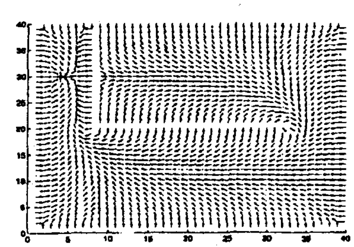


Figure-16b: Corresponding Gradient Field.

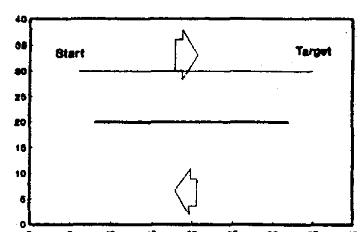


Figure-17a: The Modified Harmonic Planner.

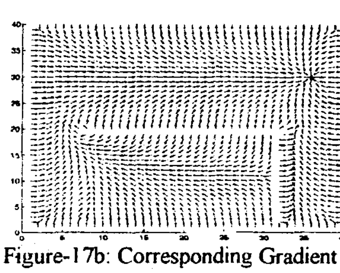


Figure-17b: Corresponding Gradient Field.

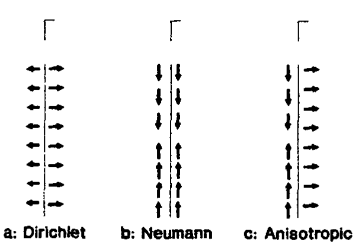
The path generated by the modified planner is shown in figure-16a, and the corresponding gradient field is shown in figure-16b. As can be seen, the gradient field from the modified potential successfully steered motion to the target avoiding the forbidden regions, and enforcing the directional constraints. In figure-17a, the target and starting point are interchanged. As can be seen, the modified planner drove motion along a straight line to the target as if it were being steered by a linear, harmonic planner. The steering gradient field is shown in figure-17b.

6.2 Discussion:

Although the gradient field of the modified potential may appear similar to that from a linear harmonic potential, the fact is that the gradient of the modified potential possess unique structural properties that are significantly different from those of a gradient field generated from an underlying harmonic potential. Consider for example the vertical straight line pattern that appears at the left corner of the upper corridor in the modified field (figure-16b). It may appear as if the field from the modified potential is obtained by adding a boundary condition and solving an augmented linear harmonic BVP. In the following the fallacy of this assumption is proven. There are two basic settings in which boundary conditions can be applied to a harmonic BVP:

1- homogeneous, Dirichlet boundary conditions in which the value of the potential at the boundary is kept constant, i.e. $V(X)=C \; X \in \Gamma$.
2- homogenous Neumann boundary conditions in which the radial derivative of the potential is set to zero $\partial V(X)/\partial n = 0 \; X \in \Gamma$.

Since in the first case the voltage is kept constant along the boundary, the gradient along the tangent to the boundary is zero (i.e., $\partial V(x)/\partial t = 0 \; X \in \Gamma$). In other words, for this case, the gradient field can only be projected normal to Γ (Figure-18a). For the homogeneous Neumann case, the choice of the boundary condition forces the radial component of the gradient field along Γ to zero. Therefore, in the Neuman setting the gradient field has to be tangent to the boundary (Figure-18b). It is not hard to see that the two settings are mutually exclusive in the sense that the presence of one field pattern at any side of the boundary immediately excludes the presence of the other field pattern on the other side. Now let us examine the structure of the gradient field around both sides of the line pattern in figure-16b. At one side of the line the gradient field is normal to Γ, at the other side the gradient field is tangent to Γ (figure-18c). As can be seen, attempting to attribute the appearance of the above straight line pattern in the gradient field of the modified harmonic field to an added boundary condition will immediately lead to a logical contradiction.



a: Dirichlet b: Neumann c: Anisotropic

Figure -18, field configurations around boundary

6.3 More Results:

In figure 19 the planner is presented with a more complex environment. Figure-19a shows the laid trajectory and figure-19b shows the corresponding gradient navigation field. It can be seen that nonlinear anisotropic harmonic gradient navigation fields suffer from the same vanishing field problem as their linear counterparts. In [25] the authors introduced Biharmonic fields as an alternative that does not suffer from this problem. The authors hope to be able to extend the biharmonic approach to accommodate nonlinear, anisotropic spaces.

7.0 Conclusions

In this paper a novel and complete motion planner (HPC) that is capable of integrating both directional and region avoidance constraints in the planning process is suggested. The proposed planner is an important addition to the motion planning literature enabling the utilizing agent to enforce the important directional constraints which realistic worspaces often present. The new method also demonstrates the effectiveness of the harmonic potential field approach not only as an effective planning method, but also as a prototype for generating other planning techniques that realistically addresses the needs of agents operating in real world environments. As was discussed in the paper, the gradient field of the modified potential possesses unique characteristics that cannot be analyzed using existing tools. Since understanding the characteristics of the gradient field is the key to understanding the behavior of the modified planner, future work will focus on developing the necessary tools for carrying out this task.

References

[1]J. Latombe,"Robot Motion Planning", Kluwer Academic Publishers, Boston, Dordrecht, London, 1991.
[2]J. Schwartz, M. Sharir," A Survey of Motion Planning and Related Geometric Algorithms", Artificial Intelligence Journal, Vol. 37, 1988, pp. 157-169.
[3]Y. Hwang, N. Ahuja, "Gross Motion Planning", ACM Computing Surveys, Vol. 24, No.3, Sept., 1992, pp.219-91
[4]H. Kolmanovsky, N. McClamroch,"Developments in Nonholonomic Control Problems", IEEE Control Systems, December, 1995, Vol. 15, No. 6, pp. 20-36.
[5]P. Doyle, J. Snell, "Random Walks and Electric Networks", The Carus Mathematical Monographs: number twenty-two, Published & Distributed by the Mathematical Association of America, 1984.
[6]R. Hersh, R. Griego, "Brownian Motion and Potential Theory", Scientific American, March 1969, Vol. 220, No. 3, pp. 66-74.
[7]C. Nash-Williams, "Random Walk and Electric Currents in Networks", Proceedings of the Cambridge Philosophical Society (Math. & Phys. Sciences), Vol. 55, 1959, pp. 181-194.
[8]K. Sato,"Collision Avoidance in Multi-dimensional Space Using Laplace Potential", Proc. 15th Conf. Rob. Soc. Jpn. pp.155- 156, 1987.
[9]E. Prassler,"Electrical Networks and a Connectionist Approach to Path-finding", Connectionism in Prespective, R. Pfeifer, Z. Schreter, F. Fogelman, L. Steels (Eds.), Elsevier Science Publishers, North-Holland, 1989, pp. 421-428.
[10]G. Lei,"A Neuron Model With Fluid Properties for Solving Labyrinthian Puzzle", Biological Cybernetics, Vol. 64, 1990, pp. 61-67.
[11]S. Akishita, S. Kawamura, K. Hayashi,"New Navigation Function Utilizing Hydrodynamic Potential for Mobile Robot. IEEE Int. Workshop on Intelligent Motion Control, Istanbul, Turkey, August 20-22, 1990, pp. 413-417.
[12]J. Decuyper, D. Keymeulen,"A Reactive Robot navigation System Based on a Fluid Dynamics Metaphor", H. Schwefel, R. Hartmanis (Eds.), Parallel Problem Solving From Nature, 1st Workshop, PPSN1, Dortmund, Oct. 1-3, 1990, pp. 356-362.
[13]C. Connolly, R. Weiss, J. Burns,"Path Planning using Laplace Equation", 1990 IEEE Int. Conf. On Rob. And Aut., May 13-18, 1990, Cincinnati, Ohio, pp. 2102-2106.
[14]D. Keymeulen, J. Decuyper," A Reactive Robot Navigation System Based on a Fluid Dynamics Metaphor", AI MEMO # 90-5, A. I. Lab., Vrije Universiteit Brussel, 1990.
[15]L. Tarassenko, A. Blake,"Analogue Computation of Collision- free Paths", 1991 IEEE Int. Conf. On Rob. And Aut,

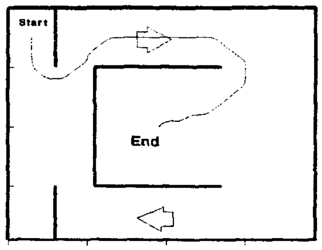


Figure-19a: A more complex environemnt.

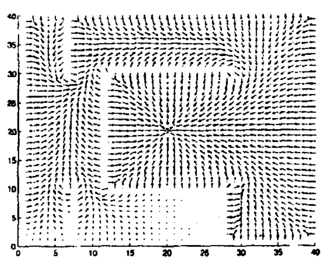


Figure-19b: Corresponding gradient field.

Sacramento, California, April 1991, pp. 540-545.

[16]E. Plumer,"Cascading a Systolic Array and a Feedforward Neural Network for Navigation and Obstacle Avoidance Using Potential Fields", NASA Contractor Report 177575, Prepared for Ames Research Center Contract NGT-50642, Feb. 1991.

[17]Z. Li, T. Bu, "Robot Path Planning and Navigation Using Fluid Model", Second Int. Conf. On Aut. , Rob, and Comp. Vision, 16-18, Sept., 1992, pp. RO-10.3.1-RO-10.3.5.

[18]D. Megherbi, W. Wolovich,"Real-Time Velocity Feedback Obstacle Avoidance Via Complex Variables and Conformal Mapping", 1992 IEEE Int. Conf. On Rob. And Aut., Nice, France, May 1992, pp. 206-213.

[19] J. Kim, P. Khosla,"Real-time Obstacle Avoidance Using Harmonic Potential Functions", IEEE Trans. On Rob. And Aut., Vol. 8, No. 3, June 1992, pp. 338-349.

[20]K. Sato, "Deadlock-free Motion Planning Using the Laplace Potential Field", Advanced Robotics, Vol. 7, No. 5, pp. 449-461, 1993.

[21]S. Akishita, S. Kawamura, T. Hisanobu,"Velocity Potential Approach to Path Planning for Avoiding Moving Obstacles", Advance Robotics, Vol. 7, No. 5, 1993, pp. 463-478.

[22]C. Connolly, R. Grupen,"The Application of Harmonic Functions to Robotics", Journal of Robotics Systems Vol. 10 No. 7, 1993, pp. 931-946.

[23]J. Guldner, V. Utkin,"Sliding Mode Control for an Obstacle Avoidance Strategy Basen on a Harmonic Potential Field", Proceeding of the 32nd Conference on Decision and Control, San Antonio, Texas, December 15-17, 1993, pp. 24-429.

[24]D. Keymeulen, J. Decuyper,"The Fluid Dynamics Applied to Mobile Robot Motion: the Stream Field Method", 1994 IEEE International Conference on Robotics and Automation, May 8-13, San Diego, California, pp.378-85.

[25]A. Masoud, S. Masoud,"Robot Navigation Using a Pressure Generated Mechanical Stress Field 'The Biharmonic Potential Approach", 1994 IEEE Int. Conf. On Rob. And Aut., May 8-13, 1994, San Diego, 124-129.

[26]M. Stan, W. Burleson, C. Connolly, R. Grupen,"Analog VLSI for Path Planning", Journal of VLSI Signal Processing, 8, 1994, pp. 61-73.

[27]J. Guldner, V. Utkin,"Sliding Mode Control for Gradient Tracking and Navigation Using Artificial Potential Fields", IEEE Transactions on Robotics and Automation, Vol. 11, No.2, April, 1995, pp. 247-254.

[28]K. Althofer, D. Fraser, G. Bugmann,"Rapid Path Planning for Robotic Manipulators Using an Emulated Resistive Grid", Electronic Letters, Vol. 31, No. 22, Oct. 1995, pp.1960-61.

[29]A. Masoud,"Techniques in Potential-Based Path Planning", Ph.D. Thesis, Electrical Engineering Department, Queen's University, Kingston, Ontario, February, 1995.

[30]H. Jacob, S. Feder, J. Slotine,"Real-Time Path Planning Using Harmonic Potentials In Dynamic Environment", Proceedings of the 1997 IEEE International Conference on Robotics and Automation, Albuquerque, New Mexico, April 1997, pp. 874-881.

[31]C. Connolly,"Harmonic Functions and Collision Probabilities", The International Journal of Robotics Research, Vol. 16, No. 4, August 1997, pp. 497-507.

[32] A. Masoud, S. Masoud, "Evolutionary Action Maps for Navigating a Robot in an Unknown, Multidimensional, Stationary, Environment, Part II: Implementation and Results", Proceedings of the 1997 IEEE International Conferenceon Robotcs and Automation, Albuquerque, New Mexico, April 1997, pp. 2090-2096.

[33]Ahmad A. Masoud, Samer A. Masoud,"A Self-Organizing, Hybrid,PDE-ODE Structure for Motion Control in Informationally-deprived Situations", The 37th IEEE Conference on Decision and Control, Tampa Florida, Dec. 16-18, 1998, pp. 2535-40.

[34]Ahmad A. Masoud, Samer A. Masoud,"A Modified, Hybrid, PDE-ODE Controller with Integrated Directional and Region Avoidance Constraints", 37th IEEE Conference on Decision and Control, Tampa Florida, Dec. 16-18, 1998, pp. 2021-22.

[35]C. Langton,"Artificial Life", Artificial Life SFI Studies in The science of Complexity, Ed. C. Langton, Addison-Wesley, 1988, pp. 1-47.

[36]R. Brooks,"Intelligence Without Reason", MIT AI Lab., Memo No. 1293, April 1991.

[37]A. Masoud,"Robot Navigation Using the Vector Potential Approach", 1993 IEEE Int. Conf. On Rob. And Auto., Atlanta Georgia, May 2-7, 1993, 1:805-811.

[38]A. Masoud ,"Constraining the Motion of a Robot Manipulator Using the Vector Potential Approach", 1993 IEEE Regional Conference on Aerospace Control Systems, Westlake Village, CA, May 25-27, 1993, pp. 493-497.

[39]A. Masoud, S. Masoud "Constrained Motion Control Using Vector Potential Fields," to appear in the IEEE Transactions on Systems, Man, and Cybernetics: Part A.

[40]B. Streetman,"Solid State Electronic Devices", Second Edition, Printice-Hall Inc. , Englewood Cliffs, N.J. 1980.

[41]D. Siljak,"Nonlinear Systems, The Parameter Analysis and Design", Jhon Wiley & Sons Inc., New York, London Sydney, Toronto, 1969.

[42]N. Krasovskii, J. Brenner,"Stability of Motion, Applications of Lyapunov's Second Method to Differential Systems and Equations With Delay", Stanford University Press, Stanford California, 1963.