A NEW METHOD FOR DESIGNING M-D LINEAR-PHASE FIR DIGITAL FILTERS USING THE WKS SAMPLING THEORY

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ABSTRACT

In this paper a new method is proposed for designing M-D linear-phase FIR digital filters with arbitrary magnitude specifications. The method is based on the WKS sampling theorem (a more general form of Shannon sampling theorem). The main feature of the method is its ability to dissociate, to a great extent, complexity from the order and dimensionality of the filter. Some design examples are provided; the results are compared to those of the optimum Minmax approximation.

I. INTRODUCTION

All of the currently available optimization-based techniques for FIR filter design face serious problems when extended to the M-D case [1]. Design complexity grows very fast with the increase in the filter order and/or dimensionality. Techniques were proposed to alleviate this problem [2]. However, this gain was achieved at the expense of either sacrificing freedom in choosing the desired characteristics or degrading the approximation quality. The main reason for this increase in design complexity is that all the proposed optimization techniques operate directly on the filter coefficients. As a result design complexity becomes strongly dependent on the order and dimensionality of the filter. To weaken this dependence, the proposed scheme manipulates the filter response by controlling a set of variables other than the filter coefficients. The interrelations between the controlling variables and the coefficients is achieved through a set of well-defined rules.

Here, a method based on the Whittaker, Kotelnikov,Shannon (WKS) Sampling Theorem [3] is proposed for designing FIR digital filters. As will be seen later if the method is appropriately configured design complexity can be isolated to a large extent from the order and dimensioanlity of the filter. Although complexity is greatly reduced, approximation quality remains acceptable compared to that of the optimum Minmax approximation.

The paper is organized as follow. In section II the WKS approach is developed. In section III the the control parameters are stated, section IV gives some design examples, with conclusions given in section V.

II. THE WKS APPROACH

It is well known that Shannon's sampling theorem [4] is used to construct bandlimited functions from their samples, provided that the samples are taken sufficiently close. The construction is carried out by introducing at the specified frequency samples pulse-like interpolating function (Drichlet kernels) scaled to the value of the specifications at these samples. Since FIR filters are characterized by a time-limited pulse response, and a real magnitude frequency response, the Shannon sampling theorem can be adapted (through the use of duality between the time domain and the frequency domain) to design linear-phase FIR digital filters. Indeed, the frequency sampling method [5] already proposed for FIR filter design bears great resemblance to Shannon's theorem. Due to the nature of the Drichlet kernel and the fact that the majority of characteristics are not time-limited, Shannon's theorem is not expected to yield very close approximation. Still, the approximation quality can be improved without introducing any inconvenient modifications to the basic body of the sampling theorem.

A new version of the sampling theorem, called the WKS theorem is used in the design. In essence, the WKS theorem is the same as Shannon's theorem with the exception that the Drichlet kernel is replaced with other kinds of kernels (Figure 1) with more desirable convergence properties. According to the WKS theorem a frequency response $H(\omega)$ can be reconstructed from the samples of the desired characteristics ($H_d(\omega)$) by using the following interpolating formula :

$$H(\omega) = \lim_{N\to\infty} \sum_{|n|\le N} H_d(\omega_n)\cdot\Phi(\omega - \omega_n) \qquad (1)$$

where $\Phi(\omega)$ is the interpolating kernel. It should be noticed that the time domain representation of $\Phi(\omega)$ is the well-known window function $W(n)$. A list of kernels and the corresponding window function can be found in [6]. One important property of the WKS theorem is the ease and the straightforwardness by which it can be extended to the M-D case. For this case the response is constructed the same way as in the 1-D case with the only exception that the building blocks are multidimensional kernels constructed by cross multiplying M 1-D kernels. The interpolating formula used to construct an M-D response is :

$$H(\omega_1,.,\omega_M)=\sum_{k_1}..\sum_{k_M} H_d(\omega_{1_{k1}},.,\omega_{M_{kM}})\prod_{i=1}^{M}\Phi_i(\omega_i-\omega_{i_{ki}}) \qquad (2)$$

<u>The Pulse Response</u>

In this section a realizable pulse response based on (2) is derived. Emphasis is placed on the 3-D even symmetric case :

Applying the M-D Inverse Discrete Fourier Transform to get the pulse response, we have :

$$h(m_1,.,m_M) = F^{-1}(H(\omega_1,.,\omega_M))$$

$$= F^{-1}\Big(\sum_{k1}..\sum_{kM} H_d(\omega_{1_{k1}},.,\omega_{M_{kM}})\cdot\prod_{i=1}^{M}\Phi_i(\omega_i - \omega_{i_{ki}})\Big)$$

$$= \sum_{k1}..\sum_{kM} H_d(\omega_{1_{k1}},.,\omega_{M_{kM}})\cdot\prod_{i=1}^{M} F^{-1}(\Phi_i(\omega_i - \omega_{i_{ki}}))$$

$$= \prod_{i=1} W_i(m_i)\Big(\sum_{k1}..\sum_{kM} H_d(\omega_{1_{k1}},.,\omega_{M_{kM}})\cdot e^{j\sum_{i=1}^{M}\omega_{i_{ki}}\cdot m_i}\Big) \qquad (3)$$

<u>The 3-D Case</u>

For the 3-D case (3) reduces to :

$$h(n,m,q)=W(n)W(m)W(q)\sum_i\sum_j\sum_k H_d(\omega_{1_i},\omega_{2_j},\omega_{3_k})\cdot (\cos(\omega_{1_i}n+\omega_{2_j}m+\omega_{3_k}q)+j\sin(\omega_{1_i}n+\omega_{2_j}m+\omega_{3_k}q))$$

To obtain a real symmetric frequency response, the pulse response is constrained to :

$$h(n,m,q) = h(-n,m,q) = h(n,-m,q) = h(n,m,-q)$$

The resulting pulse response is :

$$h(n,m,q)=W(n)W(m)W(q)\sum_i\sum_j\sum_k H_d(\omega_{1_i},\omega_{2_j},\omega_{3_k})\cdot \cos(\omega_{1_i}n)\cos(\omega_{2_j}m)\cos(\omega_{3_k}q)$$

Obviously, the 2-D response is a special case of the above.

<u>The Real Building Blocks (RBB)</u>

If the pulse response is constrained to be real, the building block constructing the magnitude frequency response is no longer a simple kernel. Instead, it becomes a set of kernels situated at the corners of a hypercube. For the M-D even symmetric case the RBB consists of 2^M kernels; with a time domain representation expressed as :

$$\prod_{i=1}^{M} W_i(m_i)\cdot\cos(\omega_{i_i}\cdot m_i)$$

and corresponding frequency domain representation :

$$\sum_{i1=0}^{1} . \sum_{iM=0}^{1}\Phi(\omega_1 + (-1)^{i1}\omega_{1_j},.,\omega_M + (-1)^{iM}\omega_{M_j})$$

where

$$\Phi(\omega_1,.,\omega_M) = \prod_{i=1}^{M}\Phi_i(\omega_i)$$

These blocks have to be scaled by a factor of 2^{-r}, where r is the number of ω_{i_j}'s equal to zero.

III. Control Parameters

In order to use (3) in the design, the following must first be specified :

1. The dimensionality of the filter, 2. The desired characteristics, 3. Maximum deviation in both stopband (δ_s) and passband (δ_p), and 4. Either the transition width (TW) or the filter order.

The parameters to be manipulated :
1. The type of the kernel and the parameters controlling it (if any), 2. The number of RBB used to construct the response, 3. The location of the kernels in the M-D frequency space (Ω).

Kernels can be allocated in any order as long as the symmetry that produces real pulse response is preserved. In order to control the degree of coupling between the optimization variables and the order and dimensionality of the filter a structured scheme for allocating the kernel (a grid) has to be adopted. In the following two coordinate systems are provided to allocate the kernels. The coordinates are given for the 3-D case.

a. Rectangular coordinates :
The location of a kernel in the $\omega_1\omega_2\omega_3$ space (Ω_{ijk}) can be written as :

$$\Omega_{ijk} = \omega_i\vec{\omega}_1 + \omega_j\vec{\omega}_2 + \omega_k\vec{\omega}_3$$

where $\vec{\omega}_i$ is a unit vector in the i'th direction.

b. Spherical coordinates.
The location can be written as :

$$\Omega_{ijk}=r_i(\sin(\theta_j)\cos(\gamma_k)\vec{\omega}_1 + \sin(\theta_j)\sin(\gamma_k)\ \vec{\omega}_2 + \cos(\theta_j)\vec{\omega}_3\ )$$

the 2-D case is easily obtained by placing $\theta_j=\pi/2$. Other coordinates can, also, be used to allocate the kernels, see [7].

IV. EXAMPLES

Circular 2-D L.P.F.

The following ideal characteristics are to be approximated :

$$H_d(\omega_1,\omega_2) = \begin{cases} 1 & 0 < \sqrt{\omega_1^2 + \omega_2^2} < .4\pi \\ 0 & \text{else where} \end{cases}$$

The following 2-D polar grid is used to allocated the kernels:

$$\Omega_{ij}= \Delta r \cdot (\cos(\frac{2\pi}{R_i}j_i)\vec{\omega}_1 + \sin(\frac{2\pi}{R_i}j_i)\vec{\omega}_2)$$

$$i = 1,.,2 \qquad j_i= 0,.,R_i-1$$

where

$$R_i = INT(2\pi/\cos^{-1}(2i^2-1)/2i^2)$$

The separation between the concentric circles is taken equal to that between successive kernels on each circle. The filter has an order of 11x11. The Chebychev kernel (α=2) is used in the realization :

$$\Phi(\omega) = \cos(N\cdot\cos^{-1}(B\cdot\cos(\frac{\omega}{2})))$$

$$B = \cosh(\frac{1}{N}\cdot\cosh^{-1}(C))\ ,\ C = 10^{\alpha}$$

The results obtained using the proposed method are shown in Figure 2 for $\Delta r=.2\pi$. The Transition Width (TW) is equal to $.213\pi$, and the maximum error (δ) is .0406. The result of the optimum Minmax approximation [1] is shown in Figure 3, TW=$.2\pi$, and δ=.0569 .

Rectangular 3-D L.P.F.

The following characteristics are approximated using the proposed method :

$$H_d(\omega_1,\omega_2,\omega_3) = \begin{cases} 1 & 0 \le |\omega_1|,|\omega_2|,|\omega_3| \le .4\pi \\ 0 & \text{else where} \end{cases}$$

a rectangular grid with uniform separations ($\Delta\omega$) is used, Chebychev kernels (α=2) are used as a building block. Results are shown in Figure 4. The filter order is 11x11x11, and $\Delta\omega=.285\pi$.

V. CONCLUSIONS

A fast, simple, and efficient method based on the WKS sampling theorem is proposed for the design of M-D arbitrary magnitude, linear-phase, FIR digital filters. The structure of the method allows the use of apriori information in the design. Such a feature makes it possible to control the degree of interrelation between complexity, order, and dimensionality of the filter to the degree of complete isolation. As can be seen in Example 1 a reasonable choice of the kernels location (with no use of optimization) can produce acceptable results (compared to the Minmax approximation). It must be mentioned that more work is still required in order to further investigate and fully exploit the proposed method.

REFERENCES

[1] D. Harris, R. Mersereau," A Comparison of Algorithms for Minmax Design of Two-Dimensional

Linear-phase FIR Digital Filters, IEEE Trans. on ASSP, Vol.25,No.6, PP 492-500, 1977.

[2] J. McClellan," The Design of Two-Dimensional Digital Filters by Transformations", Proc. 7'th Ann. Princeton Conf. on Infor. Scie., and sys., 1973, P.P. 247-251.

[3] H. P. Kramer, " A Generalized Sampling Theorem", J. Math. Phys., Vol.38, PP 68-72, 1959.

[4] A. Jerri," The Shannon Sampling Theorem-Its Various Extensions and Applications: A Tutorial", Proc. IEEE, Vol.65, No.11, PP 1565-96, Nov. 1977.

[5] L. Rabiner, R. Schafer, " An approach to the Approximation Problem for Nonrecursive Digital Filters", IEEE Trans. A.U., Vol.18, PP. 83-106, June 1970.

[6] F. Harris, " On the Use of Windows for Harmonic Analysis with Discrete Fourier Transform", Proc. IEEE, Vol.66, No.1, PP 51-83, Jan. 1978.

[7] P. Moon, D. Spencer," Field Theory Handbook", Spinger Verlag, Berlin, Gottingen, Heildlberg, 1961.

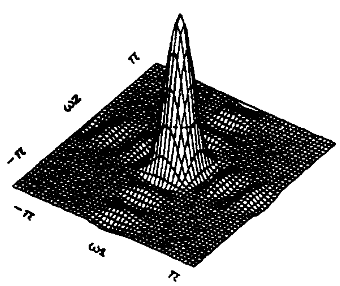

Figure 1 : A 2-D Chebychev kernel

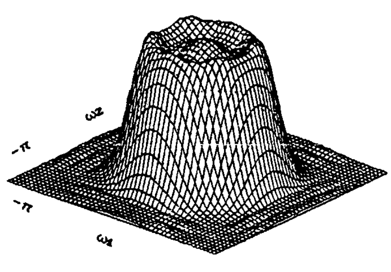

Figure 2 : A Circular L.P.F designed using the proposed method.

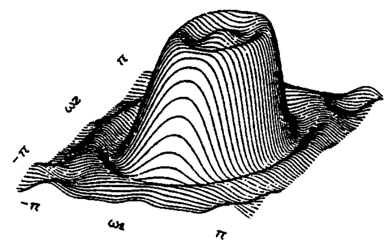

Figure 3 : A Circular L.P.F designed using Minmax approximation.

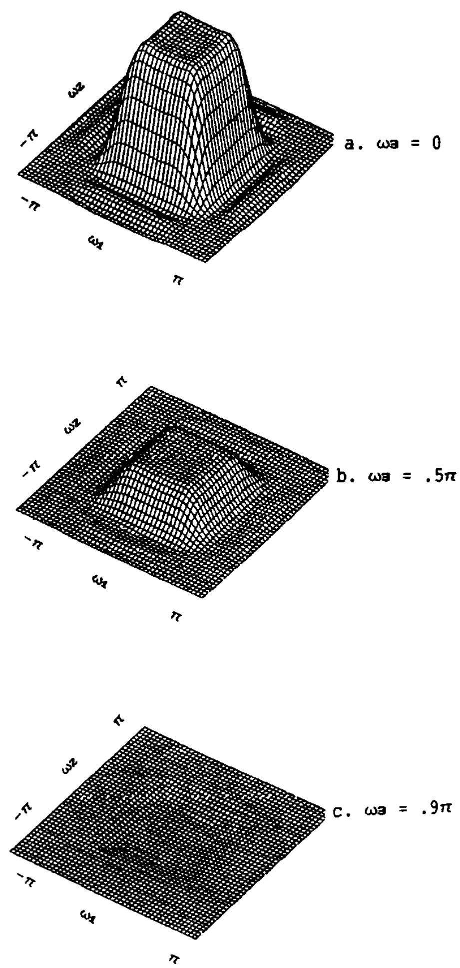

Figure 4 : A 3-D rectangular L.P.F designed using the proposed method.