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Servo-level, Sensor-based Navigation Using Harmonic Potential Fields

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Abstract— **This paper suggests an integrated navigation control system for time critical missions. The navigation control is derived from a harmonic potential field. It is designed to enable a mobile agent to proceed to a target point in an unknown environment without the need for a dedicated exploration and map-building stage. The agent, en route to the target, collects and processes only the necessary and sufficient sensory data needed to successfully execute the mission. Sensing, processing and all related activities needed to generate mobility are carried-out in real-time at the servo-level. The structure of the navigation control is described in details. Experimental results are provided as a proof of principle.**

I. INTRODUCTION

The interest in first responder robotics is continuing to grow. The aim is to provide a human operator with situational awareness in a timely manner. This mode of operation imposes strict & challenging requirements on a robot. In a first responder situation a mobile robot should be able to move to a designated area in an unstructured and unknown environment. This should be accomplished under zero *a priori* knowledge without engaging in time-consuming activities reserved to exploration and mapping only. The robot is expected to dedicate all its effort to reaching the assigned target zone using necessary and sufficient information its sensors pick-up while attempting to reach the target. Other requirements complicate the constructions of such robots. For example, a robot of this type must have an agile and robust behavior that is communication-aware. The robot is also expected to operate in a hazardous situation where the probability of damage, even loss, is high. Therefore, a first responder robot is expected to be affordable. The above requirements represents a challenge, to say the least, to existing paradigm for autonomous mobility generation.

Mobility is a composite activity that emerges from the interaction of basis activity modules (Figure-1). One of these modules is concerned with the acquisition of environment data [1]. The data is processed and structured by a representation module [2,3] to create a map. A localization module [4] is then used to make the position of the agent on the map corresponds to its true location in the environment. The guidance module provides the agent with the direction along which it has to proceed in order to reach the target [5,6]. The control module [7,8] converts the reference direction into a control signal that is fed to the agents actuators. Despite the intensive work on each module, many issues relating to how they function are still considered to be an open area of research. There is a growing concern that a modularized view of mobility leads to an overly complicated system with shaky performance. The trend is growing to develop theoretical frameworks that jointly examine the construction of more than one of these modules. Examples of this are: simultaneous localization and mapping [9], direct guidance from sensory (observation) space [10,11], joint guidance and control [12]. To the best of these authors' knowledge, a theoretical framework that jointly tackles all the modules needed for providing an autonomous agent with mobility does not exist. Putting together a complete mobility system seems to be mainly dependant on the experience of the designer [13,14].

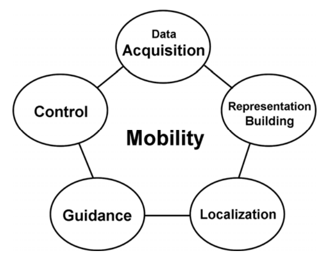

Fig. 1. Components of a mobility structure

The authors strongly believe that modularization is not conducive to achieving an agent mobility that suits a first response situation. They provide in this paper a proof of principle that an integrated view of mobility can meet these requirements. The integration of the mobility elements is carried-out using harmonic potential fields (HPFs) [15,16]. A harmonic potential can efficiently host the representation, guidance and control modules to cerate an integrated navigation action that is implementable at the servo-level. The navigation control can accept virtually unprocessed data even from difficult to use sensors such as ultrasonic sensors [17]. This data may be used to incrementally construct a representation of a safe space [18] for the agent to move through en-route to the target. The representation appears as a pattern imprinted on the gradient field of the HPF. This field is used to guide the robot. Guidance is computed by solving the Laplace boundary value problem, while what the

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control action attempt to do is to align the velocity of the robot with the gradient field. HPFs have an interesting property that makes the amount of processing commensurate with the sensory update hence achieving an output-sensitive navigation action. That is: if HPFs are disturbed by introducing local constraints, the effect of the newly introduced constrains is localized to the vicinity of the changes.

This paper suggests components and workflow for constructing an HPF-based navigation control at the servo-level. The navigation control is able to move the robot along an obstacle-free path in a fully unknown environment without engaging in an exploration and mapping stage. The controller is implemented and tested on the X80 robotics platform. It is restricted to using only the front ultrasonic sensor of the robot. Naïve dead-reckoning is used for localization where the pause of the robot is obtained by directly integrating its linear and angular speeds. All processing is one on-board a host computer. Sensory and control signals are exchanged in real-time with the X80 using a wireless communication link.

The paper is organized as follows: section 2 contains the problem statement. Section 3 describes the mobility components while in section 4 these components are interconnected to yield the navigation control. Experimental results are in section 5 and conclusions in section 6.

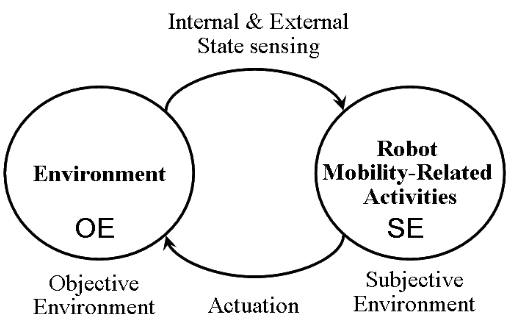

Fig. 2. Objective (OE) and subjective (SE) environments of the robot

II. PROBLEM STATEMENT

Instead of using objective geometric maps (OE) to represent the environment, the paper suggests the use of the easily constructible subjective safety maps (SE) (Figure-2). All what is required is that: if the robot is at a safe place in the environment, then its place on the safety map is marked safe. The horizon of the safety map is a square domain (Γ) of width D. A Cartesian coordinate system (x,y) is assumed so that the origin is at the center of the domain. Initially, the robot is assumed to lie at the origin (x=0,y=0) of the SE coordinates with orientation along the x-axis (θ=0).

The motion equations of a velocity controlled differential drive robot (Figure-3) are:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos(\theta)&0\\ \sin(\theta)&0\\ 0&1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix},\quad \begin{bmatrix}v\\ \omega\end{bmatrix}=\begin{bmatrix}\frac{r}{2}&\frac{r}{2}\\ \frac{r}{W}&\frac{-r}{W}\end{bmatrix}\begin{bmatrix}\omega_R\\ \omega_L\end{bmatrix},\quad \begin{bmatrix}\omega_R\\ \omega_L\end{bmatrix}=\begin{bmatrix}\frac{1}{r}&\frac{W}{2r}\\ \frac{1}{r}&\frac{-W}{2r}\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}. \qquad (1)$$

where v is the tangential velocity of the robot, ω is its angular speed, ω_R, ω_L are the angular speeds of the right and left wheels, r is the radius of the wheels and W is the separation between the two wheels.

The main lobe of the ultrasonic sensor is aligned along the principal axis of the robot. The sensor produces the continuous output S(t). Ideally, S(t) provides a measurement of the distance between the sensor and the closest obstacle that lies along the principal axis of the robot. A zero value of S(t) is an indicator that either no obstacle exist along the principal axis or the obstacle is out of sensor range. In both cases, if S(t)=0 it is assumed that no obstacle exist.

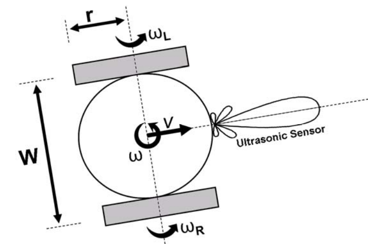

Fig. 3. Velocity controlled differential drive robot

Using only the wheels speeds ($\Omega=\begin{bmatrix}\omega_R & \omega_L\end{bmatrix}^T$), the target location in SE ($X_T=\begin{bmatrix}x_T & y_T\end{bmatrix}^T$) and readings from the sensor (S(t)), a control velocity signal ($\Omega c=\begin{bmatrix}\omega c_R & \omega c_L\end{bmatrix}^T$) is synthesized

$$\Omega c = F(\Omega, X_T, S(t))$$

such that $\lim_{t\to\infty} X \to X_T \quad \& \quad R(t)\cap O \equiv \phi \quad \forall t.$ (2)

where X_T is the target position, R(t) and O are the regions occupied by the robot and obstacles respectively in OE.

III. THE NAVIGATION CONTROL COMPONENTS

In this section, the modules used to construct a mobility system with the above capabilities are described.

It is widely believed that the signal from an ultrasonic sensor needs to undergo extensive processing in order to be usable for robot navigation. This paper demonstrates that the raw output from only one ultrasonic sensor aligned along the principal axis of the robot is highly likely to provide enough information to navigate an autonomous mobile robot in a challenging unknown environment.

A. Safety map construction:

The context in which the robot is operating is recorded at a resolution Δ using an NxN matrix DSE(i,j) (Δ=D/N). If DSE(i,j) is marked by 1, the location indexed by i & j is considered unsafe. If it is marked by 0, the location is considered possibly safe. DSE is constructed as follows (S=0, i.e. no obstacles detected is treated as a special case): first the matrix is initialized

$$\begin{aligned}&DSE(1,i)=DSE(N,i)=DSE(i,1)=DSE(i,N)=1 \quad i=1,..N,\\ &DSE(i,j)=0 \quad i=2,..N-1,\ j=2,..N-1\end{aligned} \qquad (3)$$

At a certain instant in time, given a robot's pose (x,y,θ), DSE is populated as follows:

$$Io = \left[\frac{x + (S+R)\cdot\cos(\theta+\delta)}{\Delta}\right], \quad Jo = \left[\frac{y + (S+R)\cdot\sin(\theta+\delta)}{\Delta}\right] \quad (4)$$

$$DSE(Io+m, Jo+n)=1 \quad \begin{array}{l} n=-Im,..Im,\ m=Im,..Im \\ N>Io+m>1,\ N>Jo+n>1 \end{array}$$

where R is the distance from the center of the robot to where the sensor is located, Im is a nonnegative integer used as a safety margin surrounding the sensed obstacle, [X] is the rounding integer function of the real number X, 1>>δ>0 and m,n are positive integers used to specify a safety zone around the point (Io,Jo). It ought to be noticed that sensing errors caused by spurious reflections do not endanger robot's safety. These errors will cause a safe location to be marked unsafe. Since the sensor is aligned along the direction of motion, it is not possible for the robot to move into an unsafe location. Therefore the sensor-map pair does provide a safe and dynamic representation for the robot. The worst case sensing artifacts and localization error could lead to is partial loss of safe and usable space.

B. Localization

Advanced deadreckoning techniques [19], even precise optical deadreckoning [20], do exist. However, here, basic deadreckoning is used by directly computing the robot's pose from its wheels' speeds:

$$\begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{W} & \frac{-r}{W} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix},$$

$$\nu x = \nu\cdot\cos(\theta),\ \nu y = \nu\cdot\sin(\theta) \quad (5)$$

$$x(t+\Delta T) = x(t) + \Delta T\cdot\nu\cdot x(t),$$

$$y(t+\Delta T) = y(t) + \Delta T\cdot\nu y(t)$$

$$\theta(t+\Delta T) = \theta(t) + \Delta T\cdot\omega(t) \quad x(0)=0,\ y(0)=0,\ \theta(0)=0$$

C. Guidance:

There is a large number of techniques one may choose from to guide a mobile robot. The harmonic potential approach to planning seems to fit well the task described in this paper. The approach amasses a lot of critical properties needed for successful integration in a mobility system [21]. The approach is provably-correct, it can operate in a model-based or sensor-based modes [22], it can process vague information [16], it can enforce, in a provably-correct manner, a variety of constraints on motion [15] and it yields analytic trajectories guaranteeing the construction of a provably-correct control. A basic setting of the harmonic approach is:

Solve $\quad \nabla^2 V(x,y) \equiv 0 \qquad x,y \in \Pi$

Subject to:

$$V(x_T, y_T) = 0, \quad V(x,y) = 1 \quad x,y \in \partial\Pi \quad (6)$$

Motion is safely guided to the target using the gradient dynamical system:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = -\begin{bmatrix} \partial V/\partial x \\ \partial V/\partial y \end{bmatrix} = \begin{bmatrix} Gx \\ Gy \end{bmatrix} \quad (7)$$

where Π is the workspace of the robot and $\partial\Pi$ is the boundary of Π.

The HPF approach can localize the disturbance caused by introducing a new environment component. Let V be a harmonic function constructed from the set $\partial\Pi$. Also, let Vp be a harmonic potential constructed from the set $\partial\Pi \cup P$, where P is a newly introduced point obstacle. If B is a spherical region with center P and radius ε, then one can show that an ε maybe found such that

$$|V - Vp| < \delta \quad \forall x, y \notin B. \quad (8)$$

where δ is an arbitrarily small positive number. The following example illustrates this property. Figure-4 shows the guidance field from a harmonic potential and the same field perturbed by a localized, newly introduced obstacle. A deviation measure (Figure-5) between the two fields is computed and normalized to unity. As can be seen, the deviation between the two guidance fields quickly diminishes with motion away from the disturbance. This means that far from the disturbance one does not need to recomputed the guidance field.

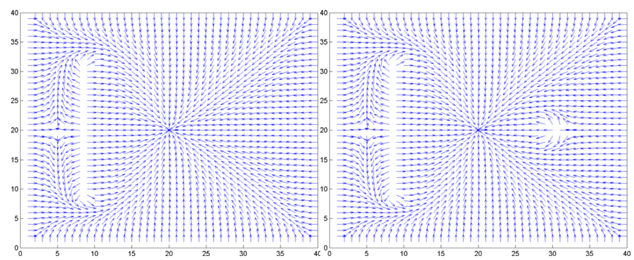

Fig. 4. a guidance field & its slightly perturbed counterpart.

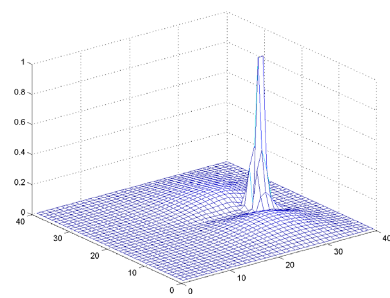

Fig. 5. deviation between the fields in Figure-5.

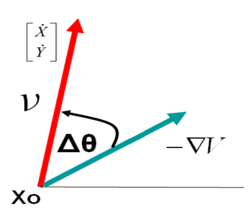

Fig. 6. Controller aligns guidance signal with robot's velocity

D. Control

Generating the navigation control signal is based on the work in [23]. The aim of the control signal is to synchronize the velocity of the robot with the guidance velocity from the negative gradient of the harmonic potential (Figure-6). The result is a provably-correct navigation control signal that can be made to inherit all the properties of the guidance signal. A hardware friendly counterpart of the navigation control in [23] may be constructed as follows: first the sine and cosine of the angle between the velocity of the robot and the guidance vector ($\Delta\theta$) along with the straight line distance to the target (dst)

$$\eta_d = \cos(\Delta\theta) = \frac{Gx \cdot vx + Gy \cdot vy}{\sqrt{vx^2 + vy^2}},$$
$$\eta_c = \sin(\Delta\theta) = \frac{Gx \cdot vy - Gy \cdot vx}{\sqrt{vx^2 + vy^2}} \qquad (9)$$
$$dst = \sqrt{(x - x_T)^2 + (y - y_T)^2}$$

The desired tangential (vc) and angular (ωc) speeds of the robot are computed as:

$$vc = vd \cdot \begin{cases} 1 & dst > Rc \\ \dfrac{dst}{Rc} & dst \le Rc \end{cases},$$
$$\omega c = \omega d \cdot \begin{cases} \eta_c & \eta_d > 0 \\ +1 & \eta_c > 0 \text{ and } \eta_d < 0 \\ -1 & \eta_c < 0 \text{ and } \eta_d < 0 \end{cases} \qquad (10)$$

where vd and ωd are the maximum tangential and angular speeds the robot should assume.

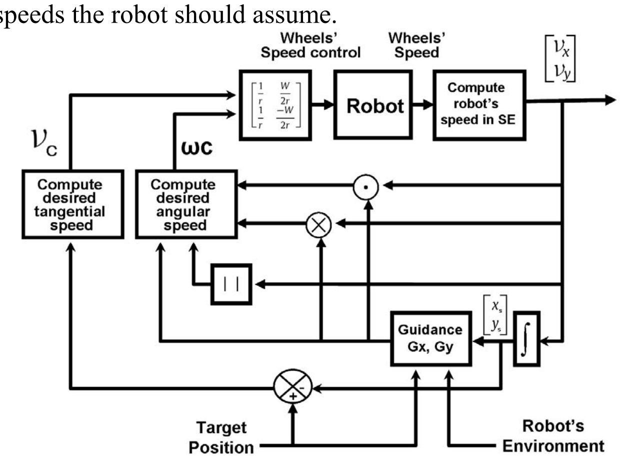

Fig. 7. The suggested, hardware-friendly controller

The velocity control signals that are to be applied to the robot's wheels are computed using equation-11. The overall structure of the navigation controller is shown in Figure-7.

$$\begin{bmatrix} \omega C_R \\ \omega C_L \end{bmatrix} = \frac{1}{r} \cdot \begin{bmatrix} 1 & \dfrac{W}{2} \\ 1 & -\dfrac{W}{2} \end{bmatrix} \begin{bmatrix} vc \\ \omega c \end{bmatrix} \qquad (11)$$

IV. THE NAVIGATION CONTROLLER

In this section the modules discussed above are integrated to yield the mobility structure. The structure is described using the flowchart in Figure-8. As mentioned before, the structure is self-contained where the only piece of information it needs from the external user is where the target is located. It provides the robot will full autonomous capabilities with no restrictive assumptions on the robot's space.

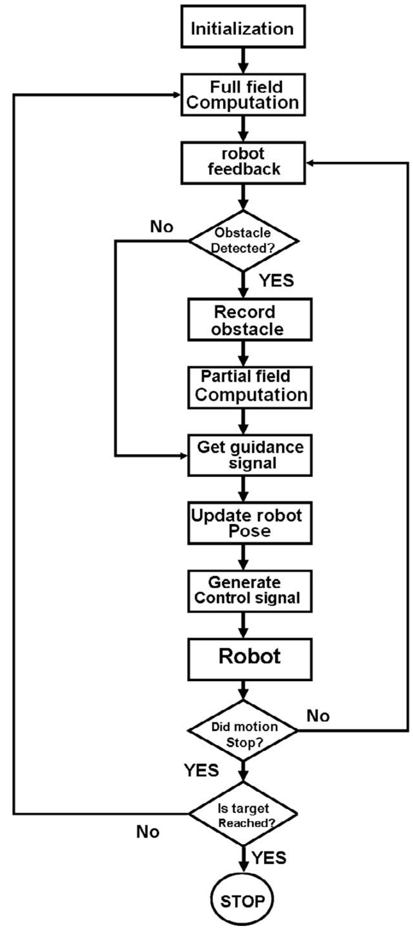

Fig. 8. The ultrasonic navigation control structure

First, the structure needs to be initialized by specifying the subjective coordinates which data is recorded with respect to. The perimeter (D) of SE is also supplied along with the target in SE which has to be the image of the target in OE. The guidance field is then globally computed given the initial information available. The wheels' speeds of the robot and the ultrasonic sensor measurements are recorded. If no obstacles are within the sensor range of the robot (S=0), the pre-computed guidance information is used. If an obstacle facing the robot is detected, it is mapped into the subjective environment of the robot. The guidance field is then locally recomputed around the added environment component and the modified guidance information is obtained. Using the robot's wheels' speeds the pose of the robot in SE is updated. This information is combined with the guidance signal to compute the control signals (wheels' speeds control signals). The control signals are applied to the robot and the speeds of the robot's wheels are monitored. The procedure for constructing the navigation control is provably-correct. In other words, if there is a path to the target, the robot will not stop until the target is reached. If the robot stops short of reaching its target, then the cause of the problem has to be the partial field computation stage. Since motion did halt, computing the guidance field in real-time is no longer important. Therefore, the full field computation stage is invoked to correct this problem.

V. EXPERIMENTAL RESULTS

An inexpensive platform (X80 UGV, Figure-9) is used to experimentally validate the structure. The servo navigation signal is sent to the robot via a wireless link that was not designed for real-time operation. Only sensor-based experiments are reported. The obstacles of the environment are constructed from cable drums. The structure of cable drums creates considerable scattering of the ultrasonic signal.

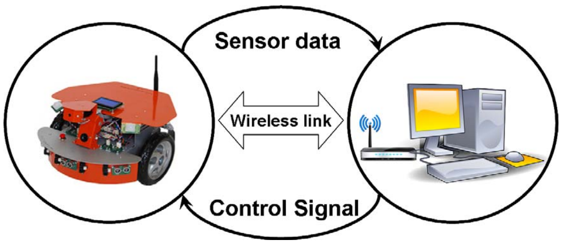

Fig. 9. The X80 differential drive mobile robot platform

The ability of the navigation controller to move the X80 to a target point under zero initial information about the environment using only one front ultrasonic sensor was tested for many obstacles' configurations. The runs produced satisfactory results (Figure-10)

Fig. 10. Trials of X80 moving to a target in an unknown environment

In the following, a detailed example is provided for the proposed control structure. Photos of the environment along with snapshots of the path taken by the robot are shown in Figure-11. The control structure managed to drive the robot to the target zone using only the raw measurements recorded by the front ultrasonic sensor.

Figure-12 shows both the final guidance field along with the trajectory computed by the localization module. Figure-13 shows a sketch of the actual environment (drawn to scale), the trajectory of the robot superimposed on the safety map (SE) constructed from the raw ultrasonic sensor data along with the actual map of the environment (OE). As can be seen, the recorded subjective map significantly differs from the objective map of the environment. It is interesting to notice that the path corresponds nicely to the geometry of the OE, even keeps a good safety margin from the obstacle. Although motion is being generated using impoverished and somewhat unreliable sensing with zero *a priori* information, the differential and integral properties of the robot's trajectory are reasonably good.

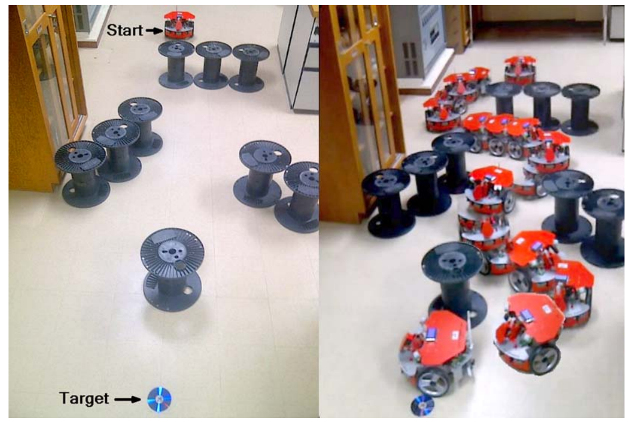

Fig. 11. Environment along with the path generated to the target using only the front ultrasonic sensor.

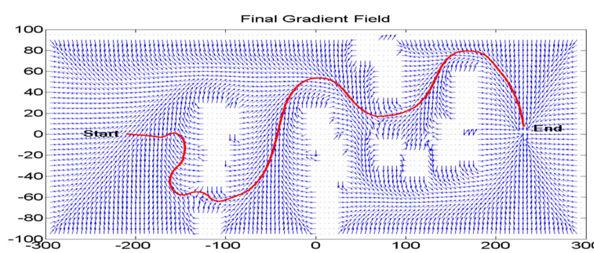

Fig. 12. Final subjective map built from accumulating the sensory data and the corresponding guidance map, trajectory superimposed.

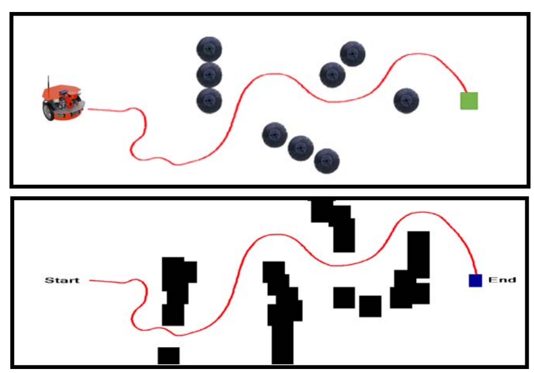

Fig. 13. Robot's trajectory corresponds well to the objective map of the environment

The signal from the sensor is shown in Figure-14. Although the signal is highly unstructured, rapidly fluctuating and has severe discontinuities, the control signals (Figure-15) are continuous and well-behaved. The orientation of the robot as a function of time is shown in Figure-16. Notice the smoothness of the angular profile and the low curvature.

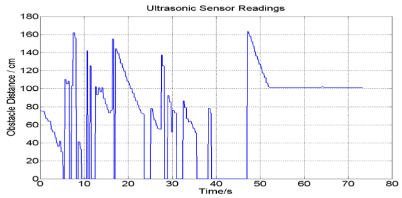

Fig. 14. Signal from the X80 ultrasonic sensor

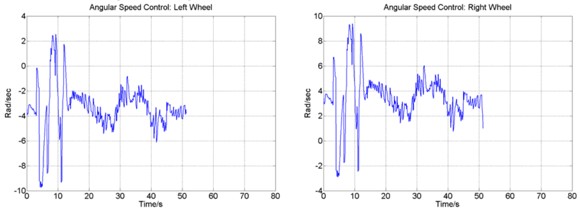

Fig. 15. The X80 wheels control signals

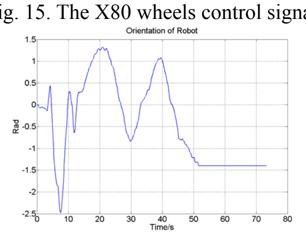

Fig. 16. Orientation of the X80 body

VI. CONCLUSIONS

This paper provides a proof of principle that the potential field approach is not only suitable for motion planning at the servo-level of a robot, it also have the ability to provide provide a provably-correct, ultrasonic sensor-based servo-level navigation control signal. The suggested structure, despite the simplicity of the processing used, can successfully tackle challenging environments both in terms of geometry and sensory signal distortion. The paper also demonstrates the centrality of the guidance module (motion planner) to the overall mobility structure. The work in this paper provides a strong reason to re-examine the belief that accurate, geometrical mapping of the environment is a perquisite to satisfactorily navigate an unstructured environment. Experimental results show that proceeding towards the target under zero *a priori* information while collecting only the data needed to guarantee safety can produce trajectories with good differential, state and integral characteristics.

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