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Agile, Steady Response of Inertial, Constrained Holonomic Robots Using Nonlinear, Anisotropic Dampening Forces

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Abstract- In this paper the harmonic potential field (HPF) approach to motion planning is adapted to work with second order mechanical systems. The extension is based on a novel type of dampening forces called: nonlinear, anisotropic, dampening forces (NADFs). It is shown that NADFs are effective aids for planning spatially-constrained kinodynamic trajectories for mechanical systems. Theoretical developments and simulation results are provided in the paper.

I. Introduction

A planner may be defined as an intelligent, purposive, context-sensitive controller that can instruct an agent on how to deploy its motion actuators (i.e generate a control signal) so that a target state may be reached in a constrained manner. The harmonic potential field approach [1-3] is a powerful paradigm for constructing motion planners. In its current form the HPF approach can only deal with the kinematic aspects of trajectory generation. The output from the planner has to be converted into a control signal by a low level controller. Guldner and Utkin suggested an interesting approach based on a sliding mode control for converting the gradient field from an HPF into a control signal [4]. The main drawback of the approach seems to be the high shattering the control signal experiences. Another method to convert the gradient guidance field from a potential surface ($-\nabla V$) into a control signal (u), is to use a viscous dampening force that is linearly proportional to speed:

$$u = -B \cdot \dot{x} - \nabla V(x) \tag{1}$$

This combination will only work provided that the initial speed of the robot is lower than a certain upper bound [5].

In this paper a method is suggested to enable the HPF approach to directly generate the navigation control signal. This is accomplished by augmenting the gradient guidance field from an HPF with a new type of dampening force called: nonlinear anisotropic dampening forces (NADFs). It is shown that an NADF-based control can efficiently suppress inertia-induced artifacts in the dynamical trajectory of the system making it closely follow the kinematic trajectory while maintaining an agile system response. The approach does not require the system dynamics to be fully known. A loose upper bound is sufficient for constructing a well-behaved control signal that can deal with dissipative systems as well as systems being influenced by external forces (e.g. gravity).

This paper is organized as follows: section II provides a brief background of the potential field approach. The NADF approach is presented in section III. Sections IV and V discuss the application of the approach to dissipative systems and systems experiencing external forces respectively. Simulation results are in section VI, and conclusions are placed in section VII.

II. Background

Although the HPF approach was brought to the forefront of motion planning independently and simultaneously by different researchers [6-8], the first work to be published on the subject was that by Sato in 1986 [6]. The HPF approach forces the differential properties of the potential field to satisfy the Laplace equation inside the workspace of a robot (Ω) while constraining the properties of the potential at the boundary of Ω ($\Gamma=\partial\Omega$). The boundary set Γ includes both the boundaries of the forbidden zones (O) and the target point (x_T). A basic setting of the HPF approach is:

$$\nabla^2 V(x) \equiv 0 \qquad x \in \Omega$$

$$\text{subject to:} \quad V = 0|_{X=X_T} \;\&\; V = 1|_{X\in\Gamma} \;. \tag{2}$$

The trajectory to the target (x(t)) is generated using the HPF-based, gradient dynamical system:

$$\dot{x} = -\nabla V(x) \qquad x(0) = x_0 \in \Omega \tag{3}$$

The trajectory is guaranteed to:

$$1\text{-}\ \lim_{t\to\infty} x(t) \to x_T \qquad 2\text{-}\ x(t) \in \Omega \qquad \forall \mathbf{t} \tag{4}$$

whereby a proof of (4) may be found in [3]. Figure-1 shows the negative gradient field of a harmonic potential. Figure-2 shows the trajectory, x(t), generated using (3).

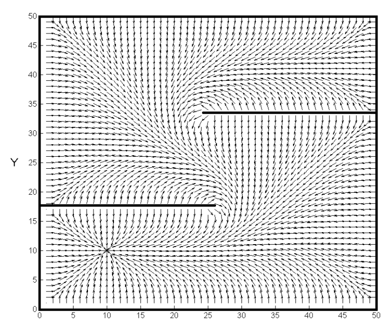


Figure-1: Guidance field of an HPF.

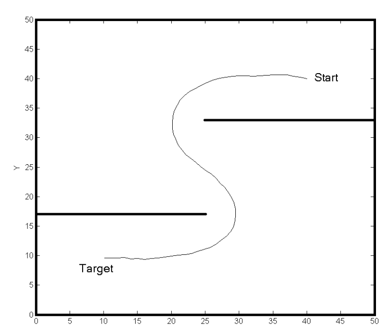


Figure-2: Trajectory generated by the field in figure-2.

The trajectory, x(t), should be fed to a low-level controller in order to generate the control signal, u. In figure-3, the negative gradient of the potential in figure-2 is used to navigate a 1kg point mass. The dynamic equation of the system is:

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} = -\mathbf{B}\cdot\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} - \begin{bmatrix}\partial V(x,y)/\partial x\\ \partial V(x,y)/\partial y\end{bmatrix} \qquad (5)$$

where B=0.1. Despite the fact that the initial speed of the robot is zero, the trajectory violated the avoidance condition and collided with the walls of the room.

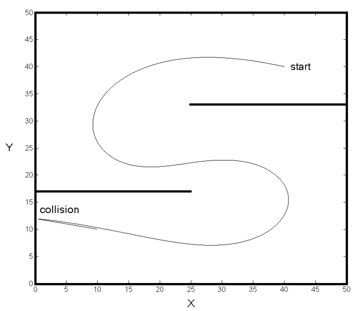


Figure-3: trajectory of a point mass controlled by the field in figure-1.

III. The NADF Approach

The linear velocity component acts as a dampener of motion that may be used to place the inertial force under control by marginalizing its disruptive influence on the trajectory of the robot that the gradient field is attempting to generate. Unfortunately, this approach ignores the dual role the gradient field plays as a control and guidance provider. A dampening component that is proportional to velocity exercises omni-directional attenuation of motion regardless of the direction along which it is heading. This means that the useful component of motion marked by the direction along which the goal component of the gradient of the artificial potential is pointing is treated in the same manner as the unwanted inertia-induced, noise component of the trajectory. The guidance and disruptive components should not be treated equally. Attenuation should be restricted to the inertia-caused disruptive component of motion, while the component in conformity with the guidance of the artificial potential should be left unaffected (figure-4).

A carefully constructed dampening component that treats the gradient of the artificial potential both as an actuator of dynamics and as a guiding signal is:

$$M(x,\dot{x}) = [(n^t\dot{x})n + (\frac{\nabla V(x)^T}{|\nabla V(x)^T|}\cdot\dot{x}\cdot\Phi(\nabla V(x)^T\dot{x}))\frac{\nabla V(x)}{|\nabla V(x)|}] \qquad (6)$$

where **n** is a unit vector orthogonal to ∇V and Φ is the unit step function. This force is given the name: nonlinear, anisotropic, dampening force (NADF).

IV- Dissipative Systems

In this section two propositions are stated and proven for dissipative systems.

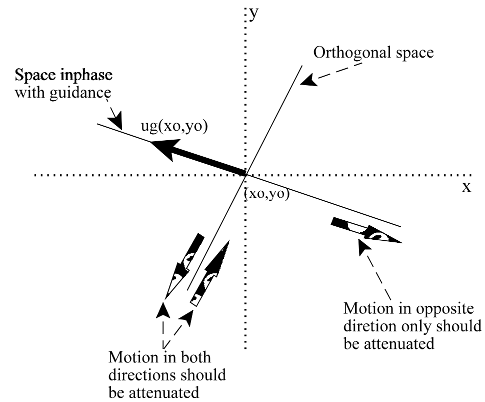
Proposition-1: Let V(x) be a harmonic potential generated using the BVP in (2). The trajectory of the dynamical system:

$$\mathbf{D}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{B}_d\cdot\mathbf{M}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{K}\cdot\nabla\mathbf{V}(\mathbf{x}) = \mathbf{0} \qquad (7)$$

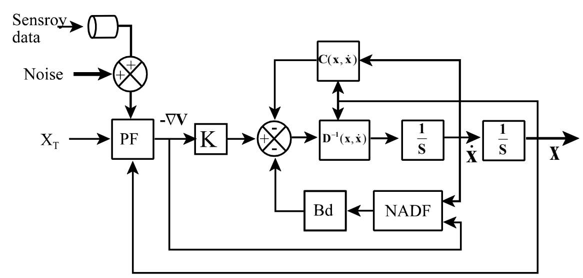
will globally, asymptotically converge to:

$$\lim_{t\to\infty}\mathbf{x}\to\mathbf{x}_T,\quad \lim_{t\to\infty}\dot{\mathbf{x}}\to\mathbf{0} \qquad (8)$$

for any positive constants B_d and K, where $x\in R^N$, $V(x):R^N\to R$, D(x) is an N×N positive definite inertia matrix, $C(x,\dot{x})\dot{x}$ contains the centripetal, Coriolis, and gyroscopic forces.



a- action of the dampening force



b- block diagram

Figure-4: nonlinear, anisotropic, dampening force (NADF).

Proof: Let Ξ be the Liapunov function candidate:

$$\Xi(x,\dot{x}) = K\cdot V(x) + \frac{1}{2}\dot{x}^T D(x)\dot{x} \qquad (9)$$

Note that since V(x) is harmonic, it must assume its maxima on Γ and minima on x_T. In other words, V(x) can only be zero at x_T; otherwise, its value is greater than zero:

$$\Xi(x,\dot{x}) = \begin{cases} 0 & \text{iff}\quad x=0, \dot{x}=0 \\ \text{positive} & \text{otherwise}\end{cases}. \qquad (10)$$

The time derivative of the above function is:

$$\dot{\Xi}(x,\dot{x}) = K\cdot\nabla V(x)^T\dot{x} + \frac{1}{2}\dot{x}^T\dot{D}(x)\dot{x} + \dot{x}^T D(x)\ddot{x}. \qquad (11)$$

Substituting:

$$\ddot{x} = D^{-1}(x)[-C(x,\dot{x})\dot{x} - Bd\cdot M(x,\dot{x}) - K\cdot\nabla V(x)] \qquad (12)$$

along with (7) in the above equation yields:

$$\begin{aligned}\dot{\Xi} = \; & K\cdot\nabla V(x)^T\dot{x} + \frac{1}{2}\dot{x}^T\dot{D}(x)\dot{x} \\ & - K\cdot\nabla V(x)^T\dot{x} - \dot{x}^T C(x,\dot{x})\dot{x} \\ & - B_d\cdot\dot{x}^T(n^T\dot{x})n \\ & - B_d\cdot\dot{x}^T(\frac{\nabla V(x)^T}{|\nabla V(x)|}\cdot\dot{x}\cdot\Phi(-\nabla V(x)^T\dot{x})\frac{\nabla V(x)}{|\nabla V(x)|}\end{aligned} \qquad (13)$$

Using the passivity property:

$$\dot{x}^T(\dot{D}(x) - 2\cdot C(x,\dot{x}))\dot{x} = 0 \qquad (14)$$

and rearranging the terms we get:

$$\dot{\Xi} = -B_d \cdot (n^T\dot{x})^T(n^T\dot{x}) - B_d \cdot \frac{(\nabla V(x)^T \cdot \dot{x})^T}{|\nabla V(x)|} \cdot \frac{(\nabla V(x)^T \cdot \dot{x})}{|\nabla V(x)|} \cdot \Phi(\nabla V(x)^T \dot{x}) \quad (15)$$

as can be seen $\quad \dot{\Xi} \le 0 \quad \forall \quad x, \dot{x}, \quad (16)$

where $\quad \dot{\Xi} = 0 \quad$ for $\quad x, \dot{x} = 0$

according to LaSalle principle [9] any bounded solution of (7) will converge to the minimum invariant set:

$$E \subset \{\dot{x} = 0, x\} \quad . \quad (17)$$

Exactly determining E requires studying the critical points of V(x) where $\nabla V(x)=0$. According to the maximum principle, x_T is the only minimum (stable equilibrium point) V(x) can have. Besides x_T , V(x) has other critical points $\{x_i\}$; however, the hessian at these points is non-singular, i.e. V(x) is Morse [10]. This can be easily shown by expanding V(x) around a critical point x_i ($|x-x_i|<<1$) using Taylor series:

$$\mathbf{V(x) = V(x_i) + \nabla V(x_i)^T(x - x_i) + \frac{1}{2}(x - x_i)^T H(x_i)(x - x_i)} \quad (18)$$

Since x_i is a critical point of V, the linear term vanishes. Rewriting the above equation, we have:

$$\mathbf{V^{'}(x) = V(x) - V(x_i) = +\frac{1}{2}(x - x_i)^T H(x_i)(x - x_i)} \quad (19)$$

Notice that V\` is also a harmonic function. Using eigen value decomposition [11], we have

$$\mathbf{V}^{'} = \frac{1}{2}\xi^T \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \lambda_N \end{bmatrix} \xi \quad = \quad \frac{1}{2}\sum_{i=1}^{N} \lambda_i \xi_i^2 \quad (20)$$

where $\xi=U(x-x_i) = [\xi_1\ \xi_2\ ..\xi_N]^T$, and U is an orthogonal matrix of eigen vectors. Since V\` is harmonic, it cannot be zero on any open subset of $\Omega$; otherwise, it will be zero for all $\Omega$ [12]. Since it is impossible for V\` to be zero for all Ω, non of the λ_i's of the hessian matrix can be zero. In other words, the hessian at x_i cannot be singular. From the above we conclude that E contains only one point which is the point $\mathbf{x = x_T, \dot{x} = 0}$ to which motion will converge.

Proposition-2: Let ρ be the trajectory constructed as the spatial projection of the solution, x(t), of the first order differential system in (3). Also Let ρ_d be the trajectory constructed as the spatial projection of the solution, x(t), of the second order system in (7), figure-5. Then there exist a Bd that can make the maximum deviation between ρ and ρ_d (δ_m) arbitrarily small.

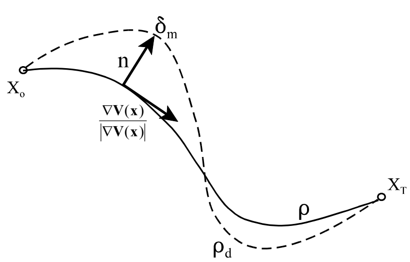


Figure-5: The kinematic and dynamic trajectories.

Proof: The gradient field from an HPF does not only work as a guide of motion to the target; it also may be used to cover Ω with a complete set of boundary-fitted basis coordinates. The radial basis of the system ($\nabla V/|\nabla V|$) marks the useful component of motion. The basis orthogonal to this component span the disruptive component of motion (δ) which NADF is required to attenuate (figure-6).

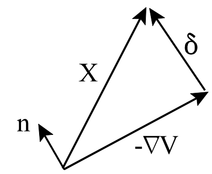


Figure-6: The disruptive component of motion.

The dynamic equation describing the disruptive component is:

$$n^T D(x)\ddot{x} + n^T C(x,\dot{x})\dot{x} + Bd \cdot n^T M(x,\dot{x}) + K \cdot n^T \nabla V(x) = 0 \quad (21)$$

Examining the above equation term by term yields:

1- $n^T \nabla V = 0$, (22)

2- $\mathbf{n}^T[(n^t \dot{x})n + (\frac{\nabla V(x)^T}{|\nabla V(x)^T|} \cdot \dot{\mathbf{x}} \cdot \Phi(\nabla V(x)^T \dot{x}))\frac{\nabla V(x)}{|\nabla V(x)|}] = (n^t \dot{\mathbf{x}})$

3- assuming that an upper bound can be placed on the speed:

$$|\dot{x}| \le \nu_{max} \quad (23)$$

the norm of the matrix C may be bound as:

$$|C(x,\dot{x})| \le c_{max} \quad (24)$$

4- any inertia matrix belonging to a physical system is positive definite, invertible, and have a bounded norm:

$$|D(x)| \le d_{max} \quad (25)$$

where d_{max}, c_{max}, and ν_{max} are finite, positive constants. Based on the above, a dynamic equation that yields an upper bound on δ is:

$$d_{max} \cdot n^T\ddot{x} - c_{max} n^T\dot{x} + B_d \cdot n^T\dot{x} = 0 \quad (26)$$

or

$$\ddot{\delta} + \Delta \cdot \dot{\delta} = 0$$

where $\ddot{\delta} = n^T\ddot{x}$, $\dot{\delta} = n^T\dot{x}$, and $\Delta = \frac{B_d - c_{max}}{d_{max}}$.

To determine the effect of the disruptive time component ($\xi(t)$) that acts normal to ∇V, the impulse response (h(t)) of (26) is obtained:

$$h(t) = \frac{1}{\Delta}(1 - \mathbf{e}^{-\Delta t})\Phi(\mathbf{t}) = \frac{h^{'}(t)}{\Delta} \ . \quad (27)$$

The deviation as a function of time may be computed as:

$$\delta(\mathbf{t}) = \xi(\mathbf{t}) * h(t)$$

where * denotes the convolution operation. Since it was shown in proposition-1 that motion will converge to x_T and all dynamic terms will tend to zero, $\xi(t)$ may be bounded as:

$$\int_0^\infty |\xi(t)| dt \le I \ , \quad (28)$$

therefore: $\quad \delta(\mathbf{t}) = \frac{\mathbf{1}}{\Delta} h^{'}(t) * \xi(\mathbf{t}) \le \frac{I_{max}}{\Delta}$,

where I and I_{max} are positive constants. By properly selecting a value for Δ, the maximum deviation δ_m can be made arbitrarily small. In other words the dynamic trajectory of (7) will closely follow the kinematic trajectory of (3) and the spatial constraints will be preserved.

V. Systems with External Forces

The NADF approach may be adapted for designing constrained motion controller for mechanical systems experiencing external forces (e.g. gravity). The dynamical equation of such systems has the form:

$$D(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) = F \quad (29)$$

where G(x) and F are vectors containing the external forces and the applied control forces respectively. The controller:

$$F = -B_d \cdot M(x,\dot{x}) - K \cdot \nabla V(x). \tag{30}$$

has the ability to make the trajectory of the system in (29) closely follow the kinematic trajectory from an initial starting point (x_o) to the target point x_T. However, due to the presence of the external forces the controller will not be able to hold the state close to the target point and drift will occur (Figure-12). Here an approach for effectively dealing with this type of systems is suggested.

1. Clamping control:

The effect of the clamping control (F_c) is strictly localized to a hyper sphere of radius σ surrounding the target point. If motion is heading towards the target, this control component is inactive. On the other hand, if motion starts heading away from the target, the control becomes active and attempts to drive the trajectory back to the target (Figure-7). A form of a clamping control that behaves in the above manner is:

$$F_C(x,\dot{x}) = (x - x_T)\cdot\Phi(\sigma - |x - x_T|)\cdot\Phi(\dot{x}^T(x - x_T)) \tag{31}$$

The strength of F_c is adjusted using the constant K_c so that the steady state error is kept below a desired level (ϵ). Clamping control maintains stability for any positive K_c.

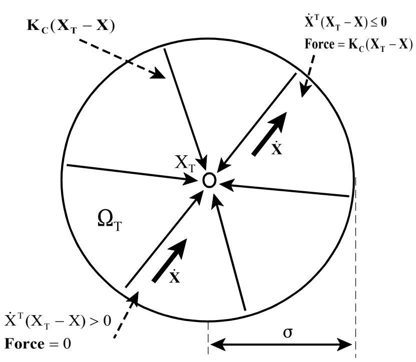


Figure-7: The clamping control.

Proposition-3:

For the mechanical system in (29), a controller of the form:

$$F = -B_d \cdot M(x,\dot{x}) - K\cdot\nabla V(x) - K_C \cdot F_C(\dot{x}, x) \tag{32}$$

can make $\lim_{t\to\infty}|x(t) - x_T| \le \varepsilon < \sigma$ and $\lim_{t\to\infty}\dot{x} = 0$ (33)

provided that:

1- K, B_d, and K_c are all positive,

2- $K_c \ge F_{max}/\epsilon$,

$F_{max} = \max_X |G(x)| \quad x\in\Omega_\sigma$

and $\Omega_\sigma = \{x: |x - x_T| \le \sigma\}$. (34)

3- a high enough value of B_d is selected so that at some instant in time t`

$$|x(t`) - x_T| < \sigma \tag{35}$$

4- K is high enough so that the gradient field is capable of directing the trajectory to Ω_σ

$$|K\cdot\nabla V(X)| > \left|G^T(X)\frac{\nabla V(X)}{|\nabla V(X)|}\right| \quad X\in\Omega\text{-}\Omega_\sigma \tag{36}$$

Proof: Consider a Liapunov function similar to the one in (9) with a gravitational potential energy term (P(X)) added:

$$\Xi(x,\dot{x}) = K\cdot V(x) + \frac{1}{2}\dot{x}^T D(x)\dot{x} + P(x) \tag{37}$$

note that: $G(x) = -\nabla P(x)$ and $P(x) = \int^x G(z)\cdot dz$. (38)

Differentiating (37) with respect to time we get:

$$\dot{\Xi}(x,\dot{x}) = K\cdot\nabla V(x)^T\dot{x} + \frac{1}{2}\dot{x}^T\dot{D}(x)\dot{x} + \dot{x}^T D(x)\ddot{x} + \dot{x}^T G(x) \tag{39}$$

solving for $\ddot{x}$ from equations (29, 30) and substituting the results in (39) we get:

$$\begin{aligned}\dot{\Xi} = & -B_d\cdot(n^T\dot{x})^T(n^T\dot{x}) \\ & -B_d\cdot\frac{(\nabla V(x)^T\cdot\dot{x})^T}{|\nabla V(x)|}\cdot\frac{(\nabla V(x)^T\cdot\dot{x})}{|\nabla V(x)|}\cdot\Phi(\nabla V(x)^T\dot{x}) \\ & -K_C\cdot\dot{x}^T(x - x_T)\cdot\Phi(\dot{x}^T(x - x_T))\cdot F(\sigma - |x - x_T|)\end{aligned} \tag{40}$$

Since K_c and B_d are positive we have:

$$\dot{\Xi} \le 0 \qquad \forall \quad x,\dot{x}, \tag{41}$$

where $\dot{\Xi} = 0$ for $x,\dot{x} = 0$.

Since we are assuming that K and B_d are selected high enough so that the dynamic trajectory will follow the kinematic trajectory and enter Ω_σ, the minimum invariant set to which the trajectory is going to converge may be computed from the equation:

$$G(x) + K\cdot\nabla V(x) + K_C\cdot F_C(x,\dot{x} = 0) = 0 \tag{42}$$

Since Φ(0)=1, and $x\in\Omega_\sigma$ (i.e. $\Phi(\sigma-|x-x_T|)=1$), equation (41) becomes:

$$G(x) + K\cdot\nabla V(x) + K_C\cdot(x - x_T) = 0 \tag{43}$$

As can be seen if condition 2 on K_C is satisfied, the solution of the above equation has to lie in the set $\Omega_\epsilon = \{x: |x-x_T| < \epsilon\}$. This means that the deviation of the end of the dynamic trajectory from the target point should at most be ϵ. Selecting a high gain of the clamping control (K_C) to manage the steady state error will not cause a transients problem. This is due to the fact that this control component is designed to be minimally intrusive affecting the system only when it is needed.

VI. Results

The gradient field in figure-1 is augmented with NADF and used to steer a 1Kg mass from a start point to a target point. A B_d=10, is used. The trajectory of the mass is shown in figure-8, and the mass distance to the target, D(t) is shown in figure-9. As can be seen, the kinodynamic trajectory of the mass is almost identical to that marked by the gradient field (kinematics only) in figure-2 with a settling time (T_S) of about 12 seconds. Figure-10 shows the control signal (X-Y force components).

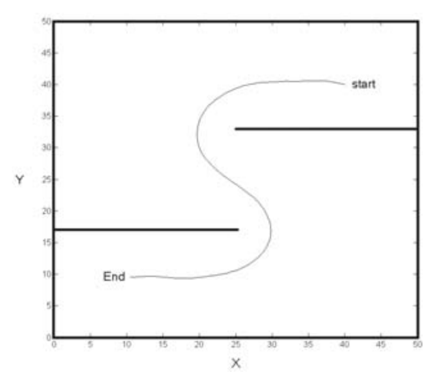


Figure-8: Trajectory, NADF, Bd=10.

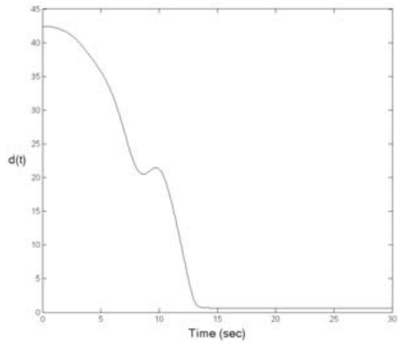


Figure-9: Distance to target versus time, NADF.

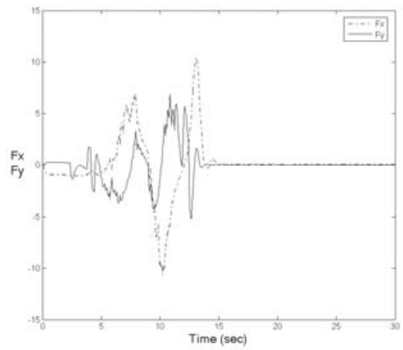


Figure-10: x and y control force components , NADF.

The relation between T_S and the coefficient of NADF (B_d) is a rapidly and strictly decreasing one (figure-11). For a low value of B_d high oscillations will prevent the quick capture of the trajectory in the 5% zone around the target. As the value of B_d increases, NADF, by design, only impedes the component of motion along the coordinate field tangent to the gradient guidance field. This component does not contribute to convergence and it only causes delay in reaching the target. Since NADF attenuates this and only this component of motion leaving the motion along the gradient field unaffected, the delay in reaching the target drops as B_d increases yielding a strictly decreasing profile of the T_S-B_d curve.

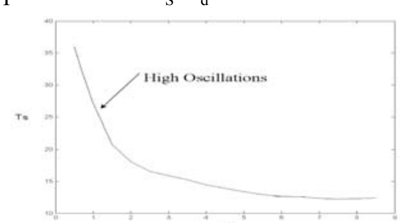


Figure-11: Settling time versus NADF coefficient.

The T_S versus the coefficient of dampening profile is important. It determines the ability to tune the controller so that the specifications are met. In tuning the controller there are two requirements: it is required that the maximum spatial deviation (δ_m) between the kinematic and the dynamic path be as small as possible so that the constraints are upheld. It is also required that the settling time be as small as possible. The first requirement is achieved by making the coefficient of dampening high enough. In the linear viscous dampening case one can only strike a compromise between T_S and $\boldsymbol{\delta}_m$. For the NADF case this compromise is not needed since both T_S and δ_m are strictly decreasing as a function of B_d.

In this example a point mass with constant external forces acting on it having the system equation in (44) is controlled using a gradient field and NADF.

$$\begin{bmatrix}\ddot{\mathbf{x}}\\ \ddot{\mathbf{y}}\end{bmatrix} = \begin{bmatrix}-4\\ -4\end{bmatrix} \tag{44}$$

As can be seen from figure-12, for a sufficiently high B_d the controller will succeed in driving the mass to the target and avoiding the obstacles. However, when the target is reached, drift caused by the external forces occur.

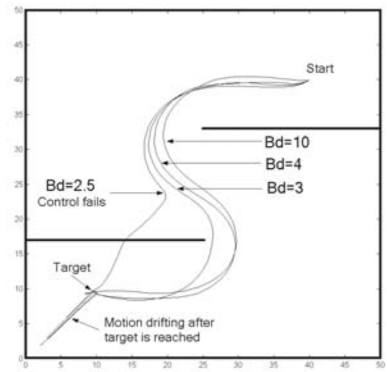


Figure-12: Trajectory, NADF - external force present.

In figure-13 a clamping control similar to the one in (31) is added with K=1, B_d=10, K_C=10. As can be seen, the controller was able to hold the trajectory near the target point relying only on a loose, upper bound estimate of the drift. Despite the high value of K_C , the trajectory settled in an overdamped manner with no oscillations taking place. The Fx, Fy control forces are shown in figure-14.

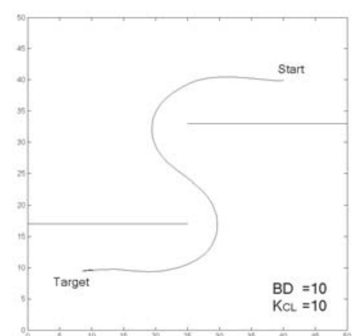


Figure-13: Trajectory, NADF and clamping - external force present.

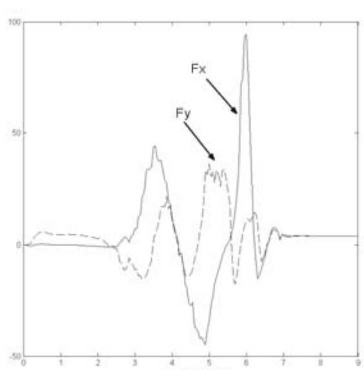


Figure-14: x and y control force components, NADF and clamping - external force present.

The sliding mode (SM) control approach suggested by Guldner and Utkin in [4] for converting a gradient guidance signal in to a control signal has the ability to handle systems with external forces. The parameters of the sliding surface are set so that a settling time of 6 sec similar to the one using NADF and clamping control is obtained. The trajectory is shown in figure-15. The control forces are shown in figures-16. Compared to the NADF approach with clamping the trajectory obtained using the SM approach is a little shaky and experiences some oscillations near the target. However, the biggest difference has to do with the quality and magnitude of the control signals used by both approaches.

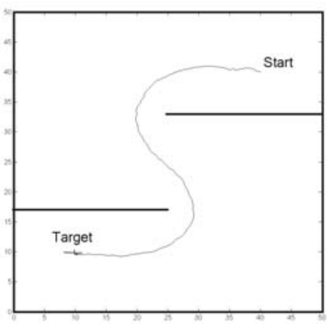


Figure-15: Trajectory - sliding mode control.

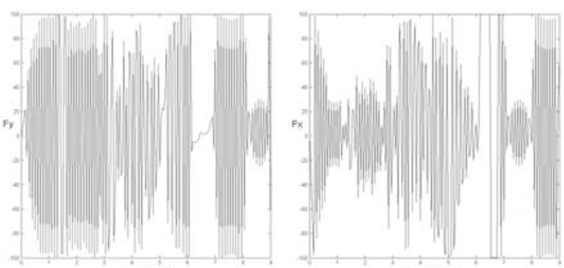


Figure-16: X and Y force control component - sliding mode control.

NADFs may also be applied for the multi-robot case [13,14]. Figure-17 shows the paths for two massless robots that are trying to exchange positions. It also shows that when mass is added (1Kg each), the planner totally fails.

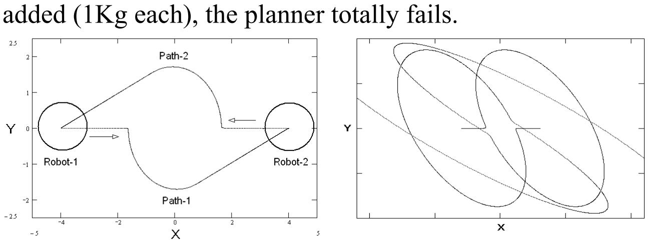


Figure-17: robots exchanging positions, kinematics (Left) Dynamics (right).

In figure-18 linear dampening is added to control the inertial forces (B=1). It took the robot about 13 seconds to reach its target. In figure-19, NADF was used (B_d=10). It took robot-1 only two seconds to reach its target.

VII. Conclusions

In this paper the capabilities of the HPF approach are extended to tackle the kinodynamic planning case. The extension is provably-correct and bypasses many of the problems encountered by previous approaches. It is based on a novel type of nonlinear, passive dampening forces called NADFs. The suggested approach enjoys several attractive properties. It is easy to tune; it can generate a well-behaved control signal; the approach is flexible and may be applied in a variety of situations, it is provably-correct; it is resistant to sensor noise; it does not require exact knowledge of system dynamics, and it can tackle dissipative systems as well as systems under the influence of external forces.

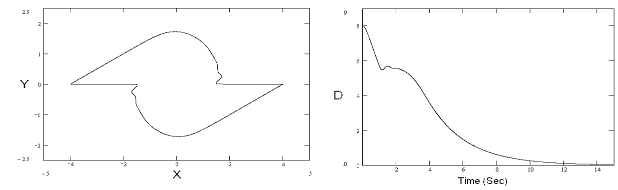


Figure-18: Linear dampening, B=1. Trajectories and robot-1 distance to target.

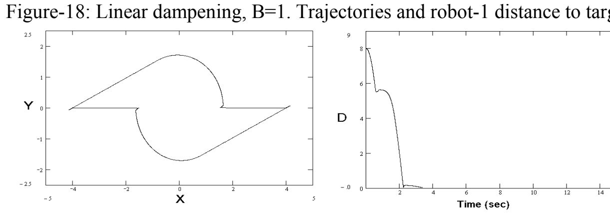


Figure-19: NADF, Bd=10. Trajectories and robot-1 distance to target.

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