

Harmonic Potential-Based Communication-Aware Navigation of Mobile Agents in Cluttered Spaces

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Abstract— In this paper we propose a Communication-Aware Navigation (CAN) controller to safely guide an agent to a target position in a cluttered environment while maintaining a reliable Wireless Communication Link (WCL). The CAN controller is based on the Harmonic Potential Field (HPF) approach to motion planning. The navigator can generate a communication-aware navigation control signal that is able to safely steer an agent with non-trivial dynamics to a target point along a direct trajectory that avoids obstacles and Dead Communication Zones (DCZ). The approach is developed and basic proofs are provided. The capabilities of the method are demonstrated using simulation results.

I. INTRODUCTION

There is a recent surge in interest in the use of autonomous agent(s) in areas like search and rescue, reconnaissance, surveillance and exploration (oil, minerals, land-mines). The main task is for the agent(s) to extend the informational access of humans, especially, in regions which become inaccessible to humans (like collapsed buildings). The agent(s) is expected to carry a multitude of sensors. It has to relay real-time data to the central command to help assess the situation at hand and effectively strategize the activities to counter the situation.

Enabling autonomous agents to function in an environment of complex and varying nature while satisfying multiple constraints is a multidisciplinary research problem. The navigation control used by the system has to integrate the following aspects [1]:

- 1) **Guidance:** Guidance lies at the heart of autonomous task accomplishment. Guidance is a multifaceted intersection of the ability of the agent to acquire information, reason and decide on future course of actions.
- 2) **Communications:** A WCL enables real-time transmission of data and reception of tactical commands from the command center.
- 3) **Control:** Control instructs actuators to realize the guidance signal.
- 4) **Energy:** Typically, the trajectory length, its smoothness, quality of control signal and wireless communication directly affect the energy consumption. For mission longevity, it is desired that energy efficiency is incorporated in the navigation control as the agents are battery powered.

This work was not supported by any organization
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Communication-aware path planning techniques are currently being the focus of investigation. Researchers have looked into jointly optimizing sensing and communication performance, [2], [3], [4], [5]. Moving in a formation among obstacles with Line Of Sight (LOS) constraint was presented in [6]. [7] introduced an indoor search while maintaining a WCL with an external Base-Station (BS). All these techniques use simplistic representations of the Wireless Communication Channel (WCC). They assume the signal quality to be dependent on distance or line of sight communication. [8] proposes restricting motion to preserve communication connectivity. [9] use mobile relaying agents to strengthen a weak communication link. Another approach is to define the communication link in binary terms [10], where communication is possible within a certain region but not outside it. In practical applications, especially, in highly populated areas, or indoors, using these simplified models, ignores the physical phenomena that occur from electromagnetic radiation [11]. Moreover, they are incapable of tackling the situation where only one agent is present in the environment. Researchers have started to incorporate realistic models and develop strategies that modulate the spatial position of the agent (Receiver (RX)) so as to improve its Received Signal Strength (RSS).

Lindhé et. al. [12] developed a probabilistic framework to find locations with desired Signal to Noise Ratio (SNR) with a certain probability. They accomplish that by randomly deviating from the reference trajectory. They also developed different kinds of stop-go strategies to improve the amount of data transmitted [13] where the agent traverses the reference trajectory without deviation.

[14] introduced a communication-aware target sensing problem, where a group of autonomous agents locally sense the presence of targets and relay the sensed data to a BS via a WCL. The WCC was modeled as a random variable described probabilistically using underlying parameters that can be estimated based on the previous observations. control commands for the agent were computed by jointly maximizing the connectivity with the BS and probability of detection at the BS. The agents were considered as point, massless and holonomic robots, thus their dynamics were ignored.

[15] developed a modified version of the navigation function to obtain path plans for tracking a mobile target while avoiding obstacles and remain connected to a BS. The DCZ were modeled as obstacles. The navigation function approach restricts the workspace, the agents and obstacles to a sphere.[16] proposed motion strategies for point robots based on reducing the error in estimation of the SNR or Mean

Squared Error (MSE) of WCC assessment. [17] proposes a Mixed Integer Linear Program (MILP) to obtain an optimal policy for periodic sensing of multiple point of interests in an obstacle free environment while minimizing communication and motion energies at the agent.

Yuan et. al. in [18] examined the problem of maximizing the amount of information sent to a BS while moving on a fixed trajectory within a fixed time. [19] proposes a periodic trajectory to sense multiple points of interests and then transmitting the sensed data to a BS at a point most suited for wireless communication.

Most of the work found in the literature focuses on a specific aspect of CAN while assuming simplistic models for the other aspects. This can result in the actual performance significantly (sometime destructively) deviating from the desired one. Our focus is on developing a framework that is able to fuse together, in a provably-correct and energy efficient manner, the three critical aspects, wireless communication, path planning and control of an agent. At the heart of our work is the integration of wireless communication constraints with HPFs [20] to obtain realizable goal seeking guidance fields. The problem is introduced in Section II, followed by the description of the Gamma-Harmonic Potential Field (GHPF) based path-planning algorithm in Section III. We provide claims and proofs in Section IV followed by the results in Section V and conclusions in Section VI.

II. PROBLEM DESCRIPTION

We consider the class of agents described by the dynamic equation as follows [21]

$$\begin{aligned}\dot{\mathbf{p}} &= G(\boldsymbol{\lambda}) \\ \dot{\boldsymbol{\lambda}} &= F(\boldsymbol{\lambda}, \mathbf{u})\end{aligned}\quad (1)$$

where \mathbf{p} ($\mathbf{p} \in \mathcal{W}$) belongs to a n-dimensional workspace \mathcal{W} ($\mathcal{W} \in \mathbb{R}^n$). The Hard Obstacle or Hazardour Region (HO) (regions in the environment inaccessible to the agent) in the workspace are denoted by \mathcal{O} ,

$$\mathcal{O} \subset \mathcal{W} = \bigcup_{i=1}^{N_O} o_i$$

N_O is the number of HO in the environment and $o_i, i = 1, \dots, N_O$ is the i^{th} HO of any arbitrary geometry. They can be convex or non-convex and disjoint or overlapping. Let $\mathcal{W}_a = \mathcal{W} - \mathcal{O}$ be the admissible workspace. The trajectory of the agent is denoted by $\mathbf{p}(t)$, ($\mathbf{p}(0) = \mathbf{p}_S$).

In Model-Based Communication-Aware Navigation (MB-CAN), the agent is assumed to have complete knowledge of the wireless RSS and HO in its environment. Thus, the agent is able to build a soft representation of the RSS (by computing the SNR map) and the hard representation of the HO for the complete environment. It is assumed that the WCC is narrow-band and the environment affecting the wireless signal is stationary and there is no external interference at the agent's RX. The SNR at a point \mathbf{p} in the workspace is then denoted by γ ($\gamma : \mathbf{p} \rightarrow \mathbb{R}_+$) and is

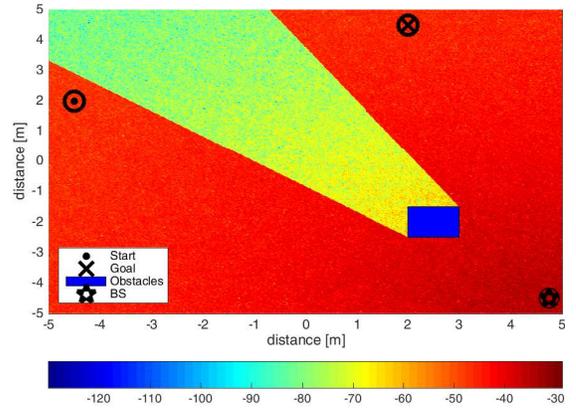


Fig. 1: SNR [dB] map with the HO superimposed

defined as

$$\gamma(\mathbf{p}) = \frac{\text{received signal power at } \mathbf{p}}{\text{receiver noise power}} \quad (2)$$

Figure 1 shows a SNR map and HO representation fused together. The agent is required to navigate towards a known goal position \mathbf{p}_G ($\mathbf{p}_G \in \mathcal{W}_a$) from its starting point \mathbf{p}_S ($\mathbf{p}_S \in \mathcal{W}_a$) while avoiding the HO and the so called DCZ.

DEFINITION 1: DCZ, \mathcal{O}_{DCZ} , are regions where the SNR goes below a threshold such that communication performance becomes prohibitively low. .

$$\mathcal{O}_{DCZ} = \{\mathbf{p} \in \mathcal{W}_a | \gamma(\mathbf{p}) \leq \epsilon, \epsilon \rightarrow 0\}, \epsilon \in \mathbb{R} \quad (3)$$

where,

$$\mathcal{O}_{DCZ} \subset \mathcal{W}_a = \bigcup_{i=1}^{N_{DCZ}} o_{dcz_i}$$

where N_{O_c} is the number of DCZ in the workspace.

A. PROBLEM FORMULATION

The problem can be formulated as follows.

Find (4)

$$\mathbf{u}(\mathbf{p}, \mathbf{p}_G, \mathbf{p}_S, \gamma, \mathcal{O})$$

For agents described by eq. 1

Such that,

$$\lim_{t \rightarrow \infty} \mathbf{p}(t) \rightarrow \mathbf{p}_G$$

$$\mathbf{p}(t) \cap \mathcal{O}_{DCZ} \equiv \emptyset, \forall t$$

$$\mathbf{p}(t) \cap \mathcal{O} = \emptyset, \forall t$$

$$\mathbf{p}(t) \in \mathcal{T} | \Upsilon(\mathbf{p}(t)) \leq \Upsilon(\mathbf{x}_i(t)), \mathbf{x}_i(t) \in \mathcal{T}, \forall i = 1, \dots, \infty$$

Where Υ is a performance functional that should be minimized and \mathcal{T} is the set of all possible trajectories the agent can take to the target.

III. COMMUNICATION-AWARE HPF BASED PATH-PLANNING

In this work we modify the planner in [22] to generate a guidance signal that can control an agent with single integrator dynamics, ($\dot{\mathbf{p}} = u$), in accordance with the

requirements in eq. 4. In [23], it is shown that the guidance signal for the single integrator dynamics can be converted into control signal for the dynamical system described by eq. 1 with a dynamic trajectory exactly equal to the kinematic trajectory of the single integrator agent. Let $\mathcal{O} \in \mathbb{F}_2$ be a scalar binary field representing the HO in the workspace. \mathcal{O} is 1 for regions where obstacles are present and 0 otherwise. Let $\Gamma_{\mathcal{W}}$ represent the boundary of the workspace and $\Gamma_{\mathcal{O}}$ be the boundary of all the HO in the workspace. Finally, let $\phi(\mathbf{p})$ be a scalar Potential Field (PF) at each point in the workspace.

Generating the communication-aware path begins by solving the Laplace Equation as shown below

$$\begin{aligned} \text{solve} \quad & \nabla(\gamma(\mathbf{p})\nabla\phi(\mathbf{p})) = 0, \quad \mathbf{p} \in \mathcal{W} \quad (5) \\ \text{s. t.} \quad & \phi(\mathbf{p}_S) = 1, \quad \phi(\mathbf{p}_G) = 0 \\ & \frac{\partial\phi}{\partial\mathbf{n}} = 0, \quad \text{at } \mathbf{p} \in \Gamma_{\mathcal{W}}, \mathbf{p} \in \Gamma_{\mathcal{O}} \end{aligned}$$

Then a sphere \mathcal{S} is defined around \mathbf{p}_S

$$\mathcal{S} = \{\mathbf{p} \mid |\mathbf{p} - \mathbf{p}_S| = \delta\}, 0 < \delta \ll 1 \quad (6)$$

Letting,

$$\rho(\mathbf{p}) = \gamma(\mathbf{p}) |\nabla\phi(\mathbf{p})| \quad (7)$$

A point \mathbf{p}'_S belonging to \mathcal{S} is selected such that

$$\rho(\mathbf{p}'_S) > \rho(\mathbf{p}), \forall \mathbf{p} \in \mathcal{S} \quad (8)$$

Finally, the path is generated using the following gradient dynamical system.

$$\dot{\mathbf{p}} = -\nabla\phi(\mathbf{p}), \quad \mathbf{p}(0) = \mathbf{p}'_S \quad (9)$$

eq. 6 and eq. 8 are needed because the starting point \mathbf{p}_S is a Dirichlet boundary condition ($\phi(\mathbf{p}_S)=1$) and therefore, lies outside of the domain of ϕ . Thus, we choose a point \mathbf{p}'_S in the vicinity of \mathbf{p}_S such that the flux at that point is maximum using eq. 8.

As demonstrated in [22] an intuition on how the planner works can be developed by expanding eq. 5 as

$$\begin{aligned} \nabla \cdot (\gamma(\mathbf{p})\nabla\phi(\mathbf{p})) &= \gamma\nabla^2\phi + \nabla\gamma \cdot \nabla\phi = 0 \\ \nabla^2\phi &= -\frac{1}{\gamma}((-\nabla\gamma) \cdot (-\nabla\phi)) \quad (10) \end{aligned}$$

Where the Right Hand Side (RHS) is a dot product of, $-\nabla\gamma$ which points in the direction of decreasing SNR and $-\nabla\phi$ which points in the direction the agent's motion will take at that point to reach the goal. If the dot product is +ve then the divergence at that point is positive and would repel the flow (flux of the PF radially away. On the other hand if the divergence is -ve, it would attract the flow. Since the trajectory is obtained by following the gradient (flow) of the PF, the trajectory is expected to be attracted towards a high SNR region and repelled away from low SNR regions. The MBCAN procedure is summarized in algorithm 1.

Also, from [24], it's proven that the trajectories obtained using eq. 9 follow the tangent law of refraction for PFs obtained from eq. 5.

$$\frac{\gamma_1}{\gamma_2} = \frac{\tan\theta_1}{\tan\theta_2} \quad (11)$$

where θ_1 and θ_2 are angles the trajectory makes with the normal to the boundary between the regions of different SNRs. The tangent law increase the components of the trajectory in higher SNR regions, thus, refracting it towards the higher SNR regions. The above procedure is summarized in algorithm 1

Algorithm 1 MBCAN

- 1: Get 2D Workspace \mathcal{W} .
 - 2: Get SNR knowledge $\gamma(\mathbf{p})$ for \mathcal{W} .
 - 3: Get HO knowledge $\mathcal{O}(\mathbf{p})$ for \mathcal{W} .
 - 4: Get start position \mathbf{p}_S of agent.
 - 5: Get goal position \mathbf{p}_G .
 - 6: GHPPF($\mathcal{W}, \gamma, \mathcal{O}, \mathbf{p}_S, \mathbf{p}_G$). ▷ Calling function GHPPF
 - 7: Find \mathbf{p}'_S using eq. 8
 - 8: Solve $\dot{\mathbf{p}} = \int_0^\infty -\nabla\phi dt$, $\mathbf{p}(0) = \mathbf{p}'_S$ for $\mathbf{p}(t)$.
 - 9: **function** GHPPF($\mathcal{W}, \gamma, \mathcal{O}, \mathbf{p}_S, \mathbf{p}_G$)
 - 10: Set $\frac{\partial\phi}{\partial\mathbf{n}} = 0$ at $\Gamma_{\mathcal{W}}$.
 - 11: Set $\frac{\partial\phi}{\partial\mathbf{n}} = 0$ at $\Gamma_{\mathcal{O}}$.
 - 12: Set $\phi(\mathbf{p}_S) = 1$.
 - 13: Set $\phi(\mathbf{p}_G) = 0$.
 - 14: Solve $\nabla \cdot (\gamma\nabla\phi) = 0$ for ϕ .
 - 15: **return** ϕ .
 - 16: **end function**
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IV. ANALYSIS

Let the communication 'effort' be defined as the following product.

$$\gamma |\nabla\phi|^2 \quad (12)$$

Since, γ is based on the received signal, the path-planner can only affect the velocity of the agent based on the value of γ . Note that the Boundary Value Problem (BVP) eq. 5 minimizes the Dirichlet Integral [25]

$$\int_{\mathcal{W}_\phi} \gamma |\nabla\phi|^2 d\mathcal{W}_\phi \quad (13)$$

Since the agent can choose from infinitely many trajectories, $\mathbf{q}_i, i = 1, 2, \dots, \infty$, as shown in figure 2 eq. 13 can be

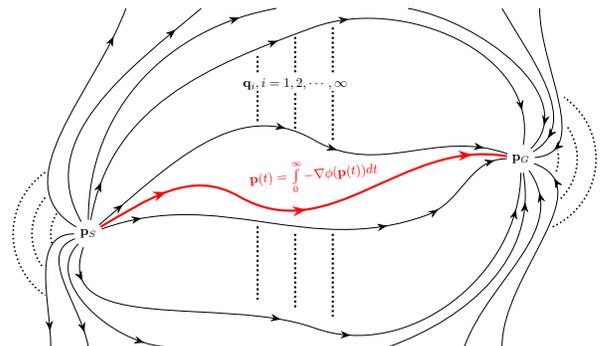


Fig. 2: Infinitely possible trajectories

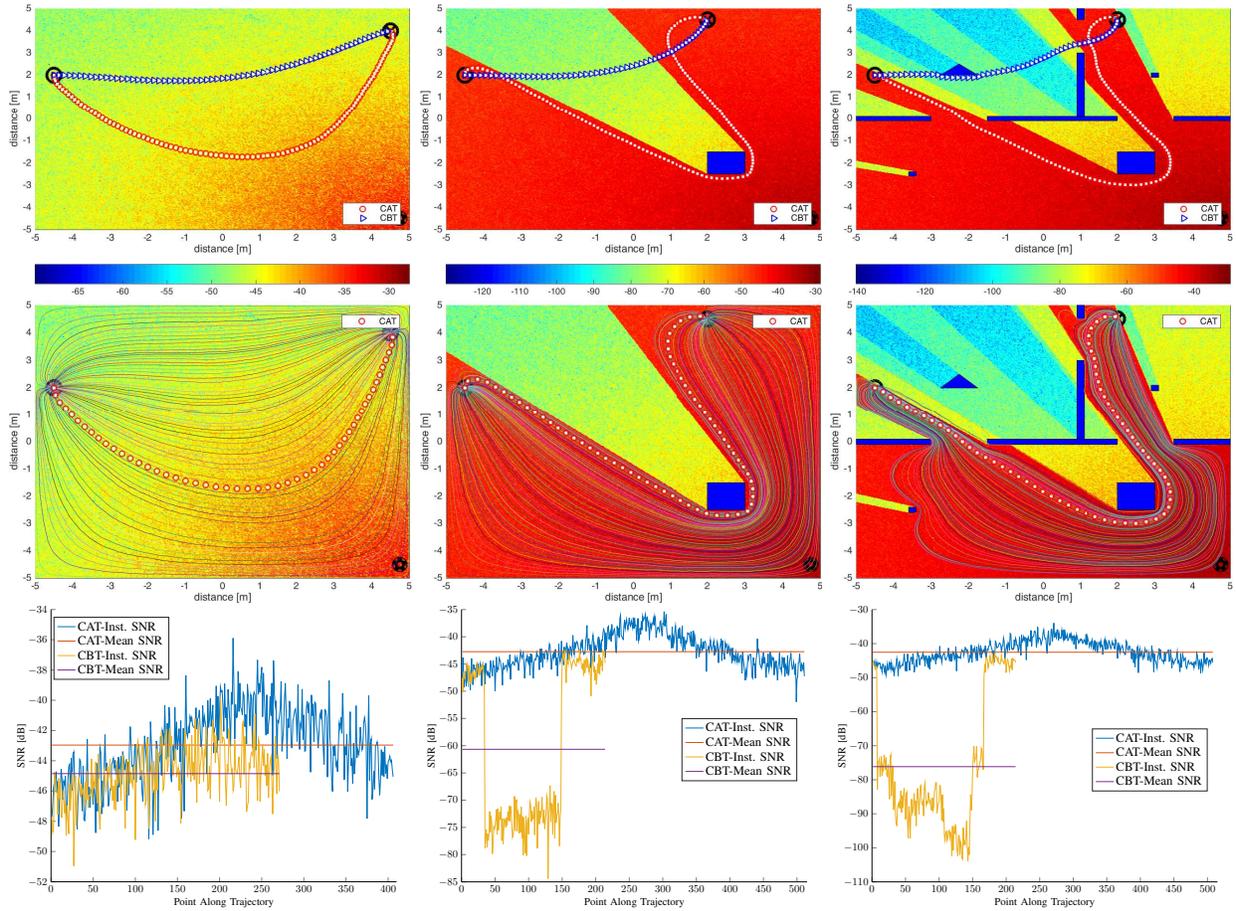


Fig. 3: SNR-aware and -blind trajectories in different environment

written as

$$\sum_{i=1}^{\infty} \int_{\mathbf{p}_S}^{\mathbf{p}_G} \gamma |\nabla \phi|^2 dq_i \quad (14)$$

Which is a sum of line integrals that accumulate eq. 12 over each trajectory \mathbf{q}_i . Any increase in path length of a trajectory will increase the line integral value. More over locally at each point on the trajectory if the SNR is high, eq. 12 can only be minimized if $|\nabla \phi|$ is minimized. Thus, minimization of eq. 13 will assign the shorted possible trajectories to the region and minimize $|\nabla \phi|$ in regions with high γ .

In the following, we provide our propositions and their proofs.

PROPOSITION 1: The trajectory generated by the MB-CAN procedure will always avoid the HO

Proof: The proof follows from the fact that Neumann boundary conditions at the HO boundary $\Gamma_{\mathcal{O}}$ excludes the HO from the domain in which eq. 5 is solved for ϕ .

$$\mathcal{W}_{\phi} = \mathcal{W} - \mathcal{O} \quad (15)$$

where \mathcal{W}_{ϕ} is the domain (a connected open subset) in which ϕ exists. The Neumann boundary conditions force the motion to be tangent to the HO boundary $\Gamma_{\mathcal{O}}$. Thus the trajectory cannot cross the HO boundary and will always remain in \mathcal{W}_{ϕ} . ■

PROPOSITION 2: If $\mathbf{p}_S, \mathbf{p}_G \notin \mathcal{O}_{DCZ}$ and no external force is acting on the agent, then the trajectory obtained by MBCAN procedure will always avoid the DCZ.

$$\mathbf{p} \in \mathcal{W}_C \subseteq \mathcal{W} \quad (16)$$

where,

$$\mathcal{W}_C = \mathcal{W}_{\phi} - \mathcal{O}_{DCZ} \quad (17)$$

Proof: Since $\gamma(\mathbf{p}) \rightarrow 0$ at the boundary of \mathcal{O}_{DCZ} , therefore, from eq. 10 we get

$$((-\nabla \gamma(\mathbf{p})) \cdot (-\nabla \phi(\mathbf{p}))) \rightarrow 0, \mathbf{p} \in \Gamma_{\mathcal{O}_C}$$

Where $-\nabla \gamma(\mathbf{p})$ point towards the DCZ as the SNR decreases in that direction. Thus the motion of the agent $-\nabla \phi$ will be forced perpendicular to $-\nabla \gamma(\mathbf{p})$ directed along the tangent to the boundary of the DCZ $\Gamma_{\mathcal{O}_C}$. Therefore \mathcal{O}_{DCZ} will be avoided. ■

PROPOSITION 3: The trajectory obtained using MB-CAN procedure will minimize the functional, Υ , defined as

$$\Upsilon(\mathbf{q}_i(t)) = \int_0^{\infty} \frac{1}{\gamma(\mathbf{q}_i(t))} dt \quad (18)$$

The derivation of eq. 18 and the proof of proposition 3 is not presented here due to limited space. It will be presented

in future extensions of this work. However, we provide simulation results that support proposition 3. The functional in eq. 18 accumulates the reciprocal of the SNR along the path. Thus minimization of the functional results in trajectories that have higher SNR along the path.

V. SIMULATION & RESULTS

For the simulation, a 2-D rectangular workspace was assumed with a single Transmitter (TX). A single TX and RX antenna was assumed. The RSS was obtained by using the WINNER II channel models [26] for an indoor environment (frequency 2.5GHz). Zero mean Additive White Gaussian Noise (AWGN) unit variance σ^2 was assumed at the RX

Figure 3 shows three different scenarios with SNR (in dB) represented along with HO in blue color. The first contains no HO or DCZ, the second contains a HO of simple geometry and a DCZ due to shadowing of the RSS caused by the obstacle and finally, the third scenario consisting of a complex environment with both convex and non-convex HOs. In all the scenarios, the communication-aware trajectories are compared with trajectories computed using HPF independent of the SNR. We can see in the first scenario that the communication-aware trajectory refracts towards the high SNR region, while the communication-unaware trajectory goes directly to the target point. We also see that all trajectories computed using the MBCAN procedure avoid the DCZ while reaching the goal in the second and third scenario. We also see that the instantaneous SNR along the communication-aware trajectories is significantly improved as compared to the SNR-blind trajectories especially in environments with DCZ. We can also observe improvements in the mean SNR in all the scenarios. The mean SNR of the communication-aware trajectory is around 2 dB greater than that of the normal trajectory. In the second scenario the gain in mean SNR is around 20 dB and around 40 dB in the third scenario. Thus, we can see that by just avoiding the DCZ a significant increase in communication performance can be attained. Finally, it was observed that the trajectories are able to avoid any HO in the environment. Moreover, the trajectories kept a safe distance from the HO and do not graze the obstacles.

Figure 4 shows the functional (eq. 18) computed for each of the trajectory computed in scenario 1 in figure 3. We can see that the maximum value of the product in eq. 8 on a sphere (circle in 2D) around the starting position coincides with minimum value of the functional. Figure 5 shows the kinematic trajectory to the target along with the dynamic trajectory of a Differential Drive Robots (DDR), whose system equation is given by

$$\begin{aligned} \dot{x} &= v \cos \theta \\ \dot{y} &= v \sin \theta \\ \dot{\theta} &= u \\ \dot{v} &= a \end{aligned} \quad (19)$$

It can be seen that the kinematic and dynamic paths are almost identical. The DDR is controlled by its angular

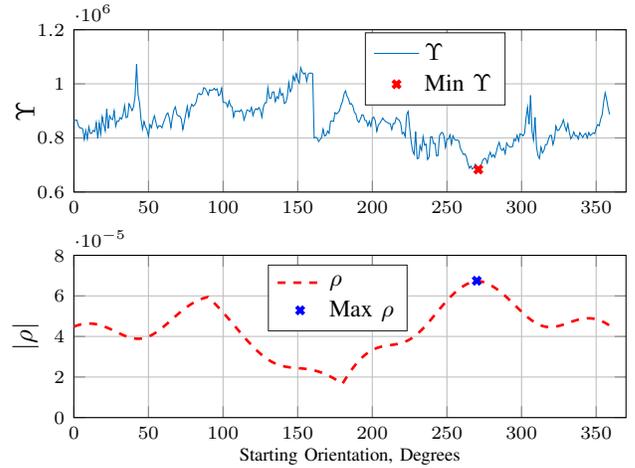


Fig. 4: Starting values of ρ (eq. 7) and the functional (eq. 18) for different starting positions in scenario 1

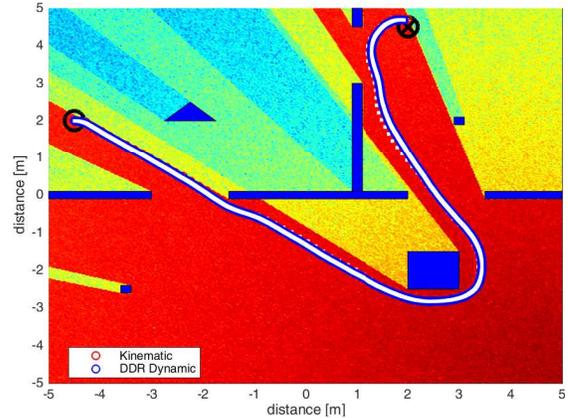


Fig. 5: Dynamic communication-aware trajectory of DDR

velocity u and tangential acceleration a . Figure 6 shows the navigation control signals, obtained using [23], responsible for generating the dynamic path.

VI. CONCLUSIONS & FUTURE WORKS

This paper suggests a provably-correct, communication aware navigation control system. The system can safely steer an agent with nontrivial dynamics to a target zone while maintaining good wireless communication link with a base station. The navigation control can accomplish the above using the exact raw signal strength profile of the wireless channel without any simplifying assumptions. Although the paper tackles the model-based case, the method can be easily adjusted to generate the navigation control signal based on on-line sensory-based measurement of the signal strength and obstacle fields. We are currently working on developing the theoretical framework for this case along with the necessary proofs. The results for the sensor-based case are quite promising. Figure 7 shows results for trajectories obtained by updating the SNR knowledge locally with some apriori knowledge and in the case when no initial knowledge of SNR is available.

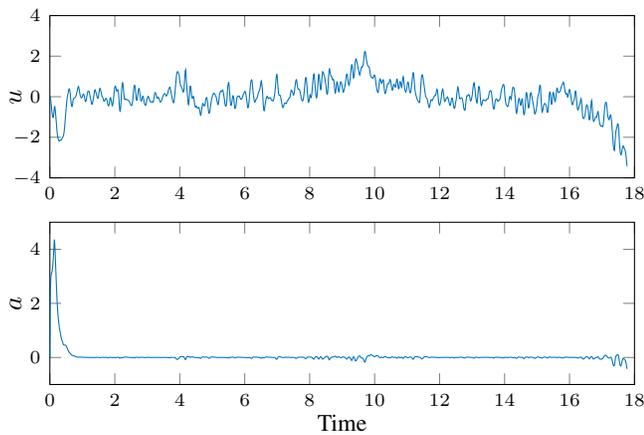


Fig. 6: Control signals that generated the dynamic trajectory

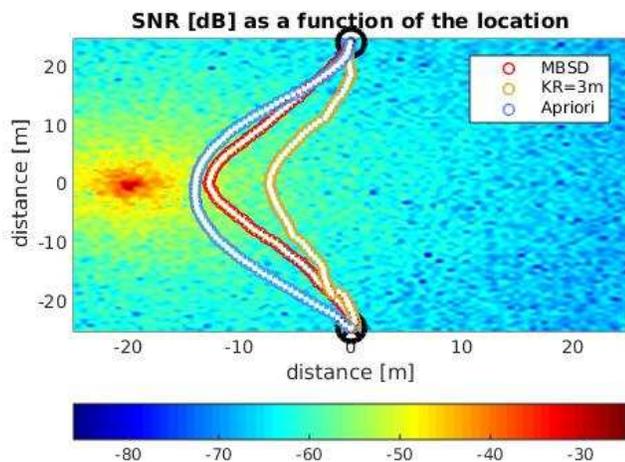


Fig. 7: Trajectories obtained for sensor-based strategies

VII. ACKNOWLEDGMENTS

The authors acknowledge the assistance of King Fahd University of Petroleum and Minerals and the high performance computing resources provided by KFUPM that contributed to the research results reported within this paper.

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