Solid Mechanics-Inspired Sensor-based Motion Planner

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Abstract- In this paper a motion planner capable of laying a trajectory for a robot operating in a complex, stationary, unknown environment based on the sensory data it acquires on-line from its finite range sensors is suggested. The planner utilizes concepts from the area of mechanics of solids to generate the navigation field. A new setting for the bi-harmonic potential field approach to planning [1] is suggested. The new setting makes it possible to gradually feed the parts of the environment, as they are discovered on-line by the sensors of the robot, to the bi-harmonic potential-based planner. Theoretical development of the method as well as simulation results are provided.

I. Introduction

A motion planner may be defined as a goal-oriented, context-sensitive, constrained, intelligent controller whose job is to instruct a robot on how to direct its motion so that a goal is achieved. Planning has been the center of attention of researchers in robotics and AI. To address the requirements a planner has to meet in order to have a reasonable chance of success when operating in a realistic environment, many methods, approaches, and ways of thinking were and are still being suggested [2,3,4,11]. One promising approach to planning employs boundary value problems (BVP) to properly combine the data about the environment, the goal, and the constraints on behavior to generate a potential field that is in turn used to induce the vector field steering motion of the robot. To the best of our knowledge, the approach appeared in the late eighties through the work of Sato on harmonic potential fields [5,6]. Harmonic functions have many useful properties[12,13] that make them a good choice for use in motion planning. Most notably, a harmonic potential is also a Morse function and a general form of the navigation function suggested in [14] (see appendix-1).

An extensive survey of this approach and the potential field approach in general may be found in [7] and [8] respectively. The harmonic potential field approach is the most widely known BVP-based planning approach; however, other approaches that use different boundary value problems do exist. One of these approaches is the bi-harmonic potential field approach suggested in [1]. Although the approach is more computationally involved, it offers several advantages over the harmonic approach:

1- the navigation field from a bi-harmonic potential exhibit high numerical stability allowing it to manage environments that are geometrically complex,

2- the curvatures of the generated paths are low and exhibit little fluctuation while maintaining a short path length.

3- unlike many planners who are only capable of point-to-point navigation, the planning action generated by such planners is a region-to-point action. This is an important feature to have if the navigation template is to be used by other robots situated at different places in the environment.

The bi-harmonic method in [1] can only generate the navigation field for fully known environments. In a realistic situation it is unlikely that such a requirement be accommodated. A sensor-based approach that would gradually feed the content of the environment to the planner is more suitable for real-life operation. In this paper a new setting for the bi-harmonic method that is suitable for use as a sensor-based planner is suggested. The new setting jointly takes into consideration the nature of both the BVP generating the field and the database used by the robot to store the environment data.

This paper is organized as follow: section II contains a brief background on planners utilizing boundary value problems for generating navigation fields. In section III the modified bi-harmonic approach is discussed. Simulation results are reported in section IV and conclusions are placed in section V.

II. Background

A properly constructed BVP is the core of a special type of motion planners called: evolutionary, hybrid, PDE-ODE controllers (EHPC). An EHPC (figure-1) consists of two parts:

1- a discrete time-continuous time system to couple the discrete-in-nature data acquisition process to the continuous-in-nature action release process;

2- a hybrid, PDE-ODE controller (HPC) to convert the acquired data into in-formation that is encoded in the structure of the micro-control action group out of which a control action is selected to steer motion.

In general, the control signal provided by an EHPC should simultaneously provide guidance for a robot and manage its dynamics. Ongoing work aims at providing EHPCs with such a capability [7]. The version considered here can only provide a guidance signal in the form:

$$\mathbf{u} = -\nabla V(x, \beta, S(t_n), \Gamma_o(t_n)) \qquad (1)$$

so that for the gradient dynamical system:

$$\dot{x} = -\nabla V(x, \beta, S(t_n), \Gamma_o(t_n)) , \qquad (2)$$

$$\lim_{\substack{n \to Z \\ t \to \infty}} x(t) \in \beta$$

and: $$x \cap O \equiv \phi \qquad \forall t,$$

where V is a potential field, n represents the n'th instant at which the robot's sensors pick up a novel event in the form of new data about the environment (t_n) , Z is a finite, positive integer, ∇ is the gradient operator, and β is the target zone. $S(t_n)$ is a binary variable with ground state 0. It changes state from 0 to 1 at t_n. It is then set back to 0 when the newly discovered information, $\Gamma_o(t_n)$, is added to the evolving database of the robot represented by the set Γ and the control field is adjusted accordingly.

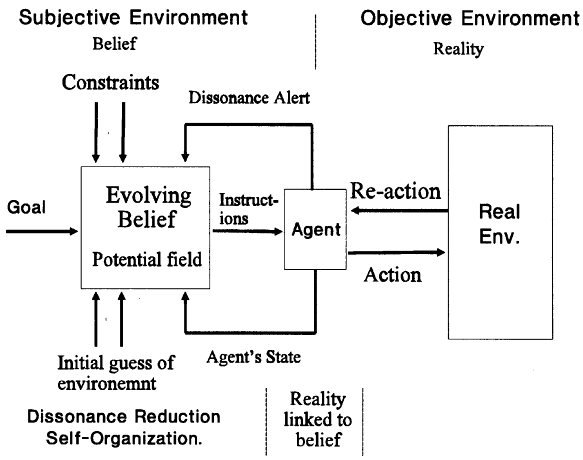

Figure-1: An evolutionary structure for motion planners.

The potential field V is generated using a suitable BVP. Below is one of the BVPs utilized by the harmonic potential field approach

$$\nabla^2 V(x) \equiv 0 \qquad x \in R^N - \Gamma - \beta$$

subject to: $V = 0|_{X=\beta} \;\&\; V = 1|_{X\in\Gamma}$. (3)

The BVP for generating the bi-harmonic navigation field suggested in [1] is:

$$\nabla^4 V(x,y) \equiv 0 \qquad x,y \in \Omega$$

and

$$(\nabla V)(\nabla V)^t = \lambda[\nabla \cdot Q(x,y)] \cdot I + G[J(Q(x,y)) + J^t(Q(x,y))]$$

subject to:

$$\frac{\partial^2 V(x,y)}{\partial x^2}|_\beta = P \cdot n\beta_x, \frac{\partial^2 V(x,y)}{\partial y^2}|_\beta = P \cdot n\beta_y, \frac{\partial^2 V(x,y)}{\partial x \partial y}|_\beta = 0.$$

and

$$Q \equiv 0, \qquad \nabla \times Q \equiv 0 \qquad x, y \in \Gamma \qquad (4)$$

where V is a potential field, Ω is the work space of the robot, Γ is the boundary of the obstacles, I is the identity matrix, J is the jacobian matrix, $n\beta_x$, $n\beta_y$ are the x and y components of the unit vector normal to β where β is the region surrounding the target zone, λ and G are positive constants. $Q = [u\ v]^t$ is the displacement field, where u is its x component and v is its y component. The lines of the minimum principal stress are traversed in order to guide motion to the target along an obstacle-free path. A steering field generated by the bi-harmonic approach is shown in figure-2.

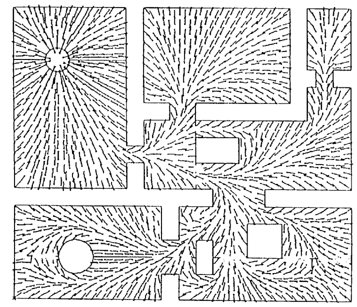

Figure-2: Navigation field from a bi-harmonic potential

III. The planner

In this section the suggested sensor-based planner is described.

A. The building blocks:

Special care has to be taken when the environment of the robot, represented by Γ, is not readily available. One of the issues that needs to be considered is the compatibility of the, most probably, noisy fragments of data about the environment of the robot which are being acquired by the sensors of the robot at discrete instants in time ($\Gamma_o(t_n)$) with the BVP synthesizing the navigation field. The physical nature of a BVP describing mechanics of material fields does not admit all types of geometries. For example, isolated points of the boundary or segments exhibiting high level of discontinuities may cause the solution of the BVP to degenerate. Another important issue is the fact that the discrete-in-time stream of fragmented boundary contours must have a form suitable for storage in a database which the utilizing mobile robot can use to construct an evolving representation of its environment. A third important consideration has to do with the algorithms and software needed to solve for the stress field. As can be seen, the BVP for the bi-harmonic potential has a more involved form than its harmonic counterpart. This, on its own, may make the harmonic potential approach more desirable than the bi-harmonic approach. It is important that whatever method used for feeding back the content of the environment to the planner be compatible with an accessible, off-the-shelf BVP numerical package (e.g. the Matlab PDE toolbox).

These authors believe that the database suggested by Moravec [9] for logging the sensory data acquired online by a mobile robot meets all the above requirements. The database uses circles as the building blocks of representation. Any fragments of data collected by the robot is made to fit into a circle. The radius and center of the circle are then logged in a table format (figuer-3). The smooth well-behaved boundary of a circle guarantees a trouble-free generation of the bi-harmonic navigation field. Moreover, the Matlab PDE toolbox [10] has the ability to automatically accommodate the presence of a circular boundary in a BVP by simply providing both the location of its center and its radius. The circular building primitives have the form:

$$\Gamma_o(x,y,x_i,y_i,r) = \{x,y:[(x-x_i)^2 + (y-y_i)^2]^{1/2} = r\} \qquad (5)$$

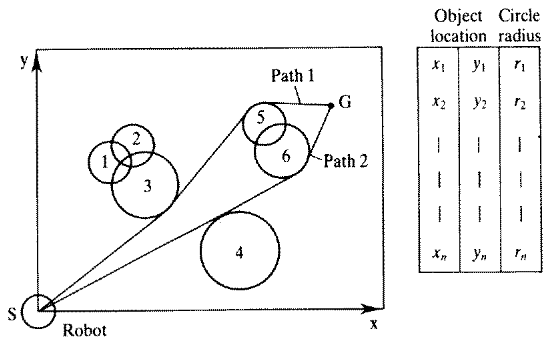

Figure-3: circles as building blocks of representations.

B. The generating BVP:

The BVP used to generate the steering field is similar to the one in (4); however, the boundary conditions are set differently,

$$\nabla^4 V(x,y) \equiv 0 \qquad x,y \in \Omega$$

and

$$(\nabla V)(\nabla V)^t = \lambda[\nabla \cdot Q(x,y)] \cdot I + G[J(Q(x,y)) + J^t(Q(x,y))]$$

subject to:

$$Q = -c\,\vec{n}\Big|_{\Gamma} \quad \text{and} \quad Q = c\,\vec{n}\Big|_{\beta} \tag{6}$$

where C is a positive constant and $\vec{n}$ is a surface unit vector normal to β. The vector at a point P used to steer motion toward the target is:

$$F(P) = \frac{\alpha}{\sqrt{u(P)^2 + v(P)^2}} \begin{bmatrix} u(P) \\ v(P) \end{bmatrix} \tag{7}$$

where α is a positive constant.

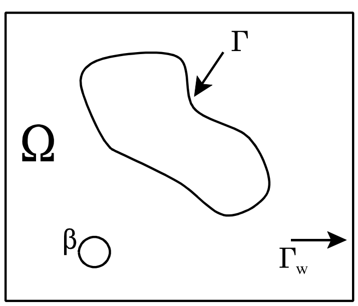

Figure-4: Geometry of the environment

C. The algorithm:

Figure-5 below shows the flowchart used to implement the sensor-based navigator. The core of the algorithm is a BVP solved numerically by the finite element method (FEM). Since FEM can only handle finite domains, the exterior boundaries of the environment (Γ_w) are assumed to be *a priori* known and are used to initialize the obstacle set, Γ, in the algorithm.

IV. Results

In this section simulation is used to test the capabilities of the planner. The environment the planner is presented with is a 12×12 cluttered room separated into two parts by a divider that has an opening. The planner is required to move a point object from one part of the room to the other. The sensor used here is a local, narrow beam range sensor that can detect objects within a circular region of radius 1.2. The radius of the circular building blocks is 0.15. The starting point of motion is: x=0, y=0, and the target point is: x=4.5, y=10.5.

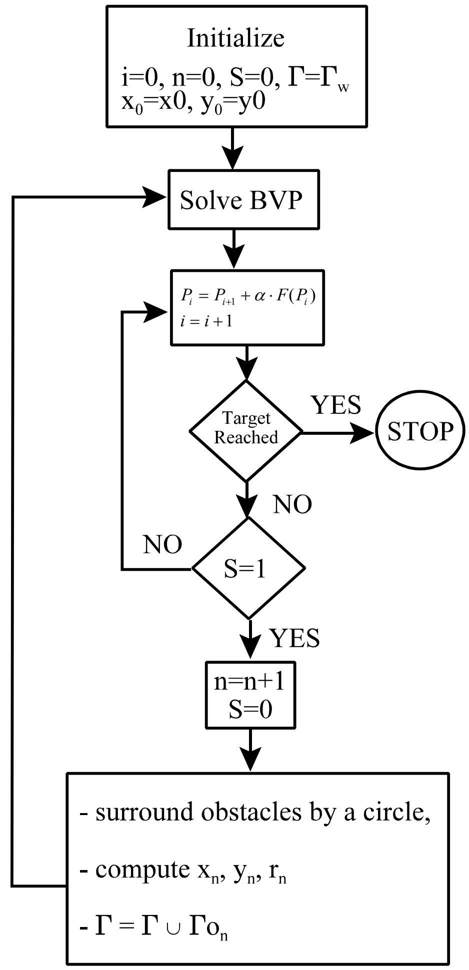

Figure-5: Flow chart of the navigation algorithm.

The planner starts steering motion not knowing the contents of the environment. All what it knows is its destination and the exterior boundaries of the room. As can be seen from figure-6, the planner manages to lay a trajectory to the target that avoids the obstacles relying only on the data its sensors provide. For this case, the sensors of the planner detected the presence of obstacles twenty seven times. Each time it had to adjust the steering field so that the presence of the newly acquired data is accommodated. The navigation field corresponding to the first attempt is shown in figure-7.

Equipped with the knowledge it gained from its first engagement of the environment, the planner tries a second time to reach its target from the same start point. As can be seen from figure-8, the planner was able again to reach its target making use of the experience it accumulated from the

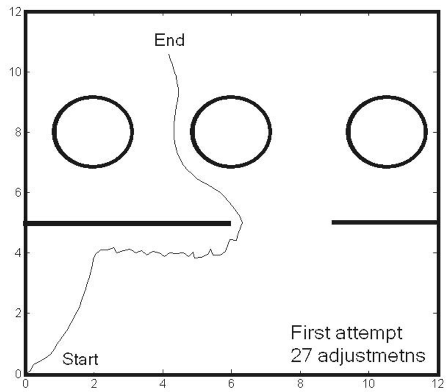

Figure-6: Trajectory, first attempt.

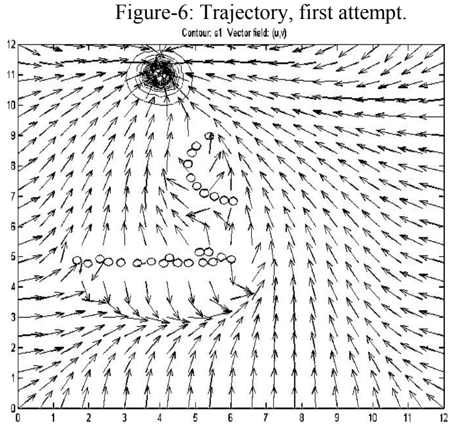

Figure-7: Steering field, first attempt

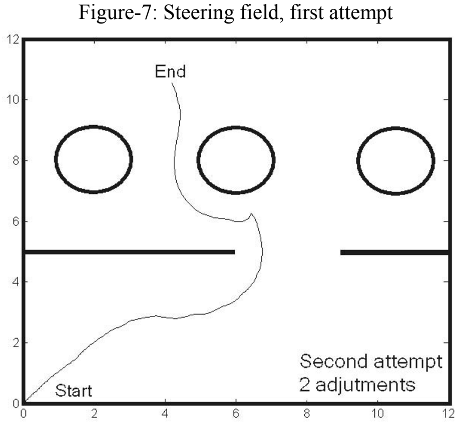

Figure-8: Trajectory, second attempt.

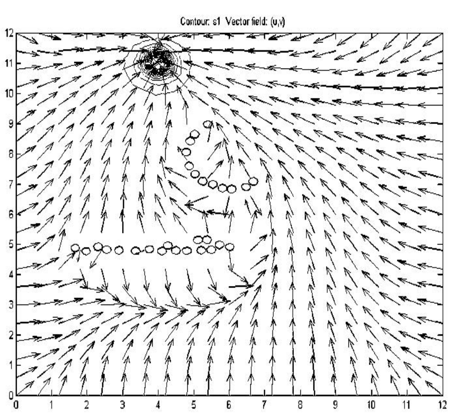

Figure-9: Steering field, second attempt

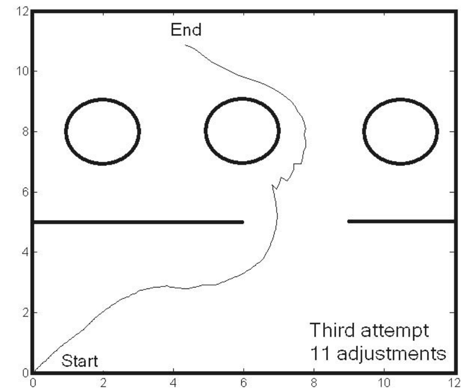

Figure-10: Trajectory, third attempt.

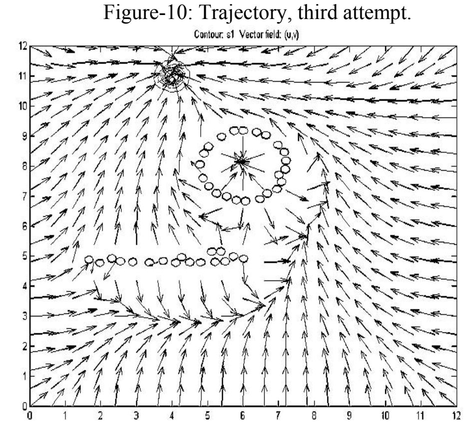

Figure-11: Steering field, third attempt

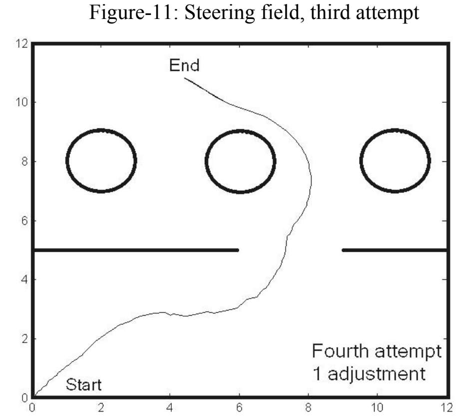

Figure-12: Trajectory, fourth attempt.

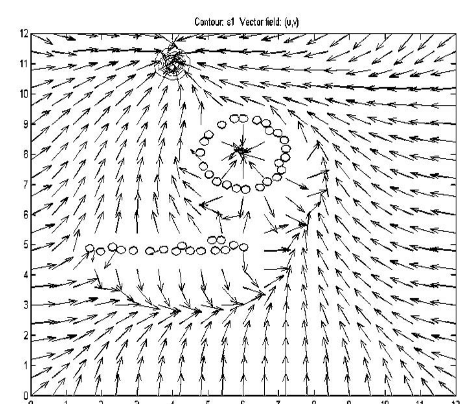

Figure-13: Steering field, fourth attempt previous attempt. This experience was reflected in reducing the computational burden required for the adjustment of the steering field. Also the quality of the path significantly improved. The sensors picked the presence of obstacles along the trajectory on which motion is heading and adjusted the steering field and the heading only twice. The navigation field corresponding to the second attempt is shown in figure-9. The paths and guidance fields from the third and fourth attempts are shown in figures-10-13.

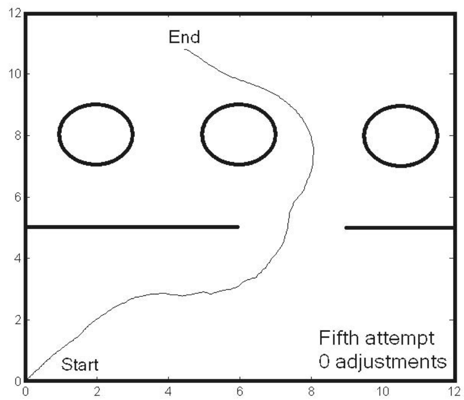

Figure-14: Trajectory, fifth attempt.

A fifth attempt by the planner to reach its target from the same starting point yielded no new sensory data or field adjustment. This is an indication that the planner has acquired a necessary and sufficient level of environmental data to lay a conflict-free trajectory to the target. Although the generated path was encoded using a necessary and sufficient level of information, its characteristics both differential and integral are acceptable; actually the quality is very close to that of a path obtained under full *a priori* information. Figure14 shows the trajectory generated during the fifth attempt.

V. Conclusions

In this paper the bi-harmonic potential field approach to planning is modified to operate in a sensor-based manner that would allow it to plan motion in a unknown environment. The reported method is a part of ongoing work to build a new class of intelligent motion controllers that have a good chance of meeting the demands a realistic environment may present an agent with. The behavior of agents equipped with such controllers is goal-oriented, context-sensitive (i.e. meaningfully react to the events happening in their external environment), and intelligent. Future work on the bi-harmonic approach will focus on adding more capabilities to this method such as directional sensitivity.

Acknowledgment

The authors would like to thank KFUPM for its support of this work. The first author would like to thank Naji Almusabi, Mayowa Qasim and Hassan Elmorra for their helpful comments.

Appendix

A. *Definition*: Let V(X) be a smooth (at least twice differentiable), scalar function ($V(X): R^N \rightarrow R$). A point Xo is called a critical point of V if the gradient vanishes at that point ($\nabla V(Xo)=0$); otherwise, Xo is regular. A critical point is Morse, if its Hessian matrix (H(Xo)) is nonsingular. V(X) is Morse if all its critical points are Morse [15].

B. *Proposition*: If V(X) is a harmonic function defined in an N-dimensional space (R^N) on an open set Ω, then the Hessian matrix at every critical point of V is nonsingular, i.e. V is Morse.

*Proof***:** There are two properties of harmonic functions that are used in the proof:

1- a harmonic function (V(X)) defined on an open set Ω contains no maxima or minima, local or global in Ω. An extrema of V(X) can only occur at the boundary of Ω,

2- if V(X) is constant in any open subset of Ω, then it is constant for all Ω.

Other properties of harmonic functions may be found in [12,13].

Let Xo be a critical point of V(X) inside Ω. Since no maxima or minima of V exist inside Ω, Xo has to be a saddle point. Let V(X) be represented in the neighborhood of Xo using a second order Taylor series expansion:

$$V(X) = V(Xo) + \nabla V(Xo)^T (X - Xo) + \frac{1}{2}(X - Xo)^T H(Xo)(X - Xo) \qquad |X-X0|<<1. \qquad (8)$$

Since Xo is a critical point of V, we have:

$$V' = V(X) - V(Xo) = \frac{1}{2}(X - Xo)^T H(Xo)(X - Xo) \qquad |X-X0|<<1. \qquad (9)$$

Notice that adding or subtracting a constant from a harmonic function yields another harmonic function , i.e. V\` is also harmonic. Using eigenvalue decomposition [16], V\` may be written as:

$$V' = \frac{1}{2}(X - Xo)^T U^T \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \lambda_N \end{bmatrix} U(X - Xo) \qquad (10)$$

$$= \frac{1}{2}\xi^T \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \lambda_N \end{bmatrix} \xi = \frac{1}{2}\sum_{i=1}^{N} \lambda_i \xi_i^2$$

where U is an orthonormal matrix of eigenvectors, λ's are the eigenvalues of H(Xo), and $\xi=[\xi_1\ \xi_2\ ..\xi_N]^T = U(X-Xo)$. Since V\` is harmonic, it cannot be zero on any open subset Ω; otherwise, it will be zero for all Ω, which is not the case. This can only be true if and only if all the λ_i's are nonzero. In other words, the Hessian of V at a critical point Xo is nonsingular. This makes the harmonic function V also a Morse function.

It ought to be mentioned that a navigation function defined in [14] is a special case of a harmonic potential field. According to [14] a navigation function must satisfy the following properties:

1- it is smooth (at lest C^2),
2- it contains only one minimum located at the target point,
3- it is a Morse function,
4- it is maximal and constant on Γ.

A harmonic function (V) is C^∞ and Morse. Harmonic functions are extrema-free in Ω. Their maxima and minima can only happen at the boundary of Ω. In the harmonic approach Γ and the target point (X_T) are treated as the bounary of Ω. Through applying the appropriate boundary conditions, the minimum of V is forced to occur on X_T. Also by the application of the Drichlet boundary conditions, the value of V is forced to be maximal and constant at Γ. As mentioned earlier, the Drichlet conditions are one of many settings [17,18] used in constructing a harmonic potential that may be used for navigation.

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