EE 380- 1 Control Engineering I, Major-2

Wednesday, January 2nd, 2008, 6:00 PM - 7:30 PM Dr. Ahmad A. Masoud

Q1 (5 marks): A stable, third order system with no zeros, a dominant complex pole pair, and a quickly fading pole has the following unit step response characteristics:

settling time (Ts) = 34 sec, maximum percent overshoot (δ) = 25%, steady state error (ess) = 0.333

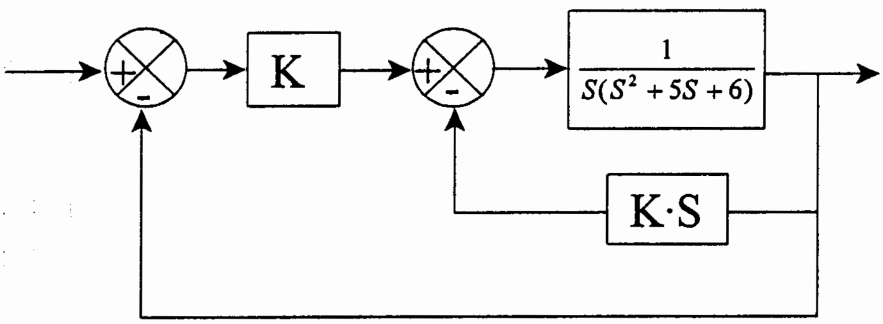
Determine the transfer function of the system.

Q2 (5 marks): Consider the system with fourth order transfer function shown blow. The transfer function is dependant on the two parameters (A,B). Use the Routh-Horowitz test to DRAW the sub-region in parameter space for which the system is stable:

$$\frac{1}{S^4 + 10 \cdot S^3 + A \cdot S^2 + 50 \cdot S + B}$$

Q3 (5 marks): Consider the system shown below with free parameter K,

1- determine all the features needed for sketching the root locus (3.5 marks)
2- sketch the rootlocus (1.5 marks)



EE 380 Major-2 ①

Q.1 A third order system has the general form (no zeros)

$$\frac{\omega_n^2/a}{(\frac{1}{a}s+1)(s^2+2\zeta\omega_n s+\omega_n^2)}$$

Since we have two dominant poles & one quickly fading pole, the second order design rules may be used to find ω_n & ζ

$$\delta = .25 = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \Rightarrow \zeta \cong .4$$

$$T_s = 34 \simeq \frac{4}{\zeta \cdot \omega_n} \Rightarrow \omega_n \cong .29$$

now $e_{ss} = .333 = 1/3$

$$\therefore \frac{1}{3} = 1 - \lim_{s\to 0} s\left(\frac{\omega_n^2/a}{(1/a\, s+1)(s^2+2\zeta\omega_n s+\omega_n^2)}\right)\frac{1}{s}$$

$$\frac{1}{3} = 1 - \frac{\omega_n^2/a}{\omega_n^2} = 1 - \frac{1}{a} \Rightarrow a = 1.5$$

&

$$H(s) = \frac{.042}{\frac{2}{3}s^3 + 1.133 s^2 + .242 s + .063}$$

Q2. $H(s) = \dfrac{1}{s^4 + 10s^3 + As^2 + 50s + B}$

s^4	1	A	B
s^3	10	50	0
s^2	$A-5$	B	0
s^1	$\dfrac{-10B+50A-250}{A-5}$	0	0
s^0	B	0	0

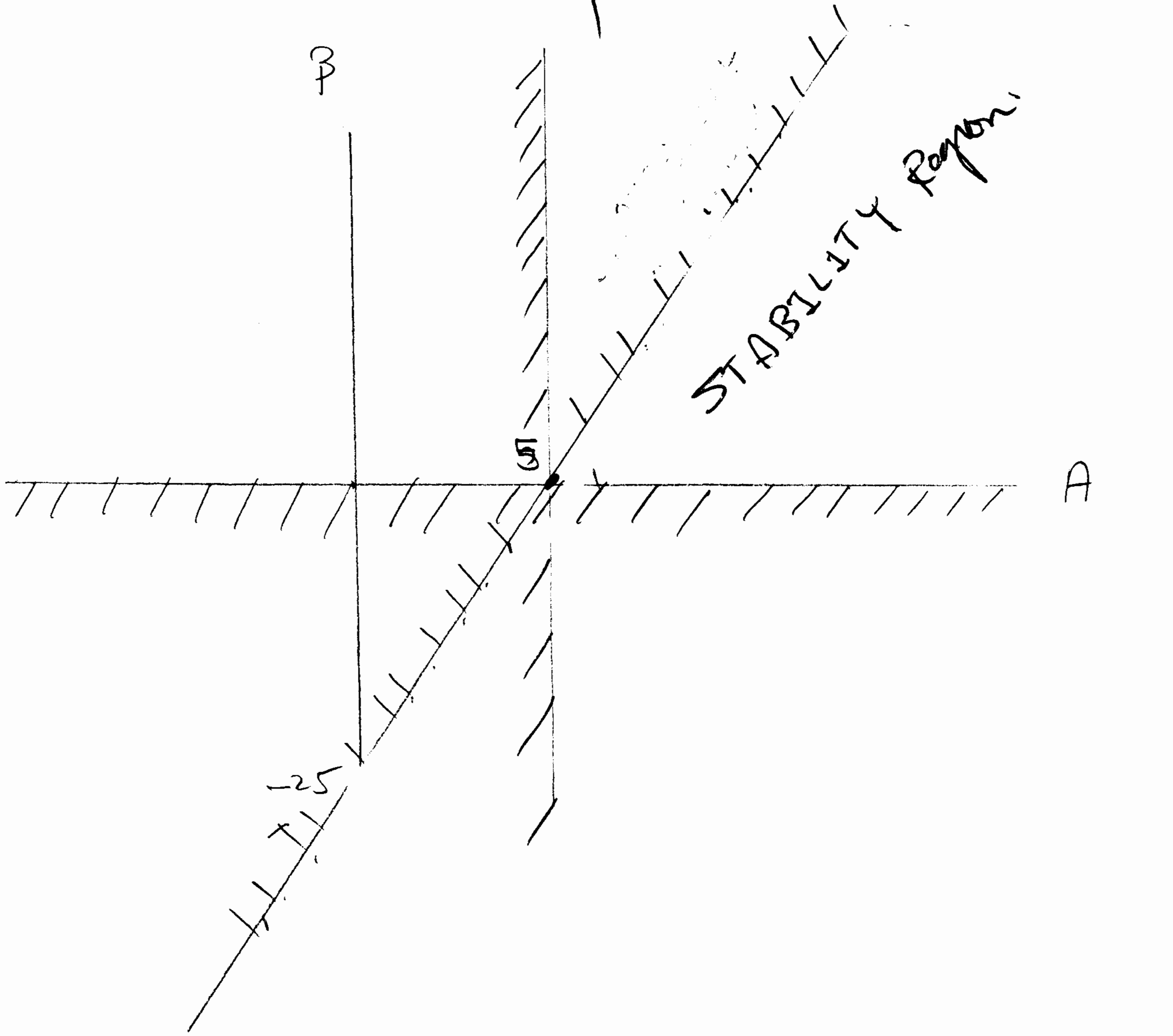
Conditions are

$A-5>0$ AND

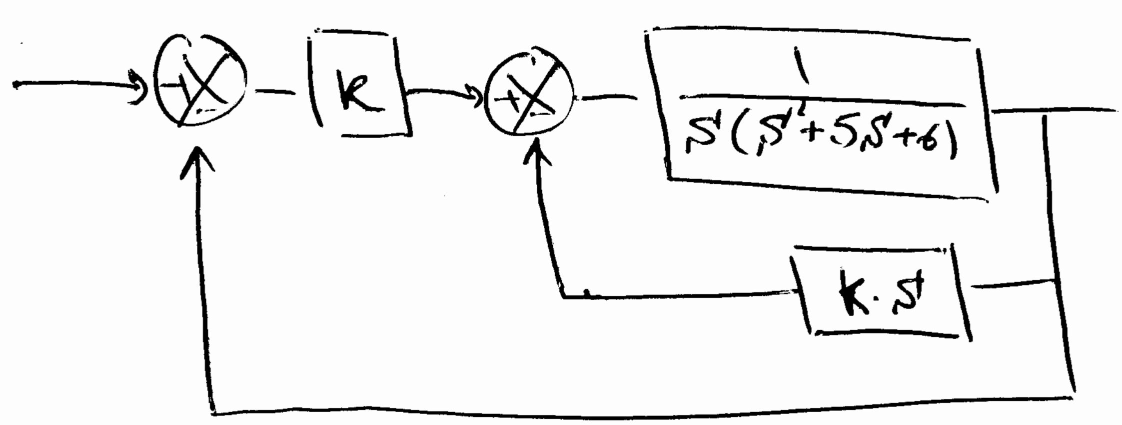
$-10B+50A-250>0$ AND

$B>0$.

$10B-50A+250=0$



Q3



$$T.F = \frac{K}{S^3 + 5S^2 + (6+K)S + K}$$

$$C.E. \Rightarrow S^3 + 5S^2 + (6+K)S + K = 0$$

$$R.L.E \Rightarrow 1 + K\frac{S+1}{S(S+2)(S+3)} = 0.$$

# of branches = 3

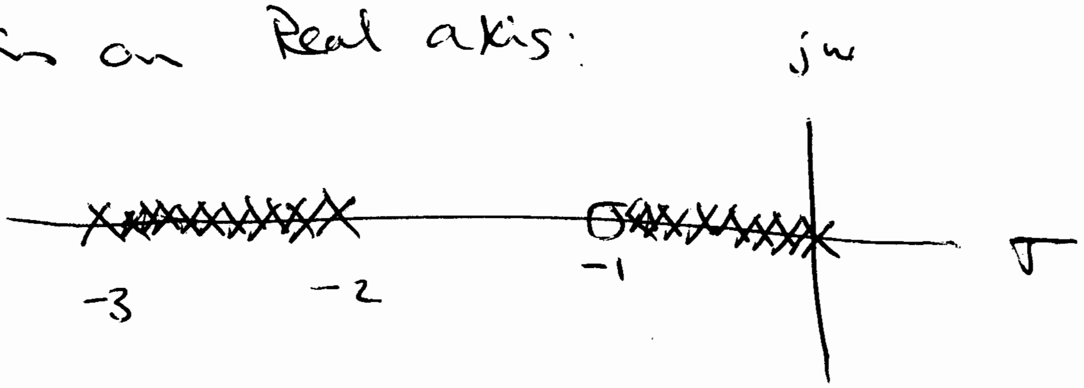
start : $S = 0, \; S = -2, \; S = -3$

End : $S = -1, \; S \to \infty, \; S \to \infty$

asymptotes: 2 , $\sigma_c = \frac{(0-2-3)-(-1)}{2} = -2.0$

$\theta_0 = 90 = \pi$, $\theta_1 = \frac{3\pi}{2}$.

Locations on Real axis:



BAP.

$$\frac{d}{ds}\frac{s+1}{s(s+2)(s+3)} = 0 \qquad \therefore \frac{d}{ds}\frac{s+1}{s^3+5s^2+6s}$$

$\Rightarrow$ $s^3+4s^2+5s+3=0$

$\therefore$ solution: -2.4656 ✓ BAP

$-.7672+j.792$ ×

$-.7672-j.792$ ×

Intersection with $j\omega$-axis

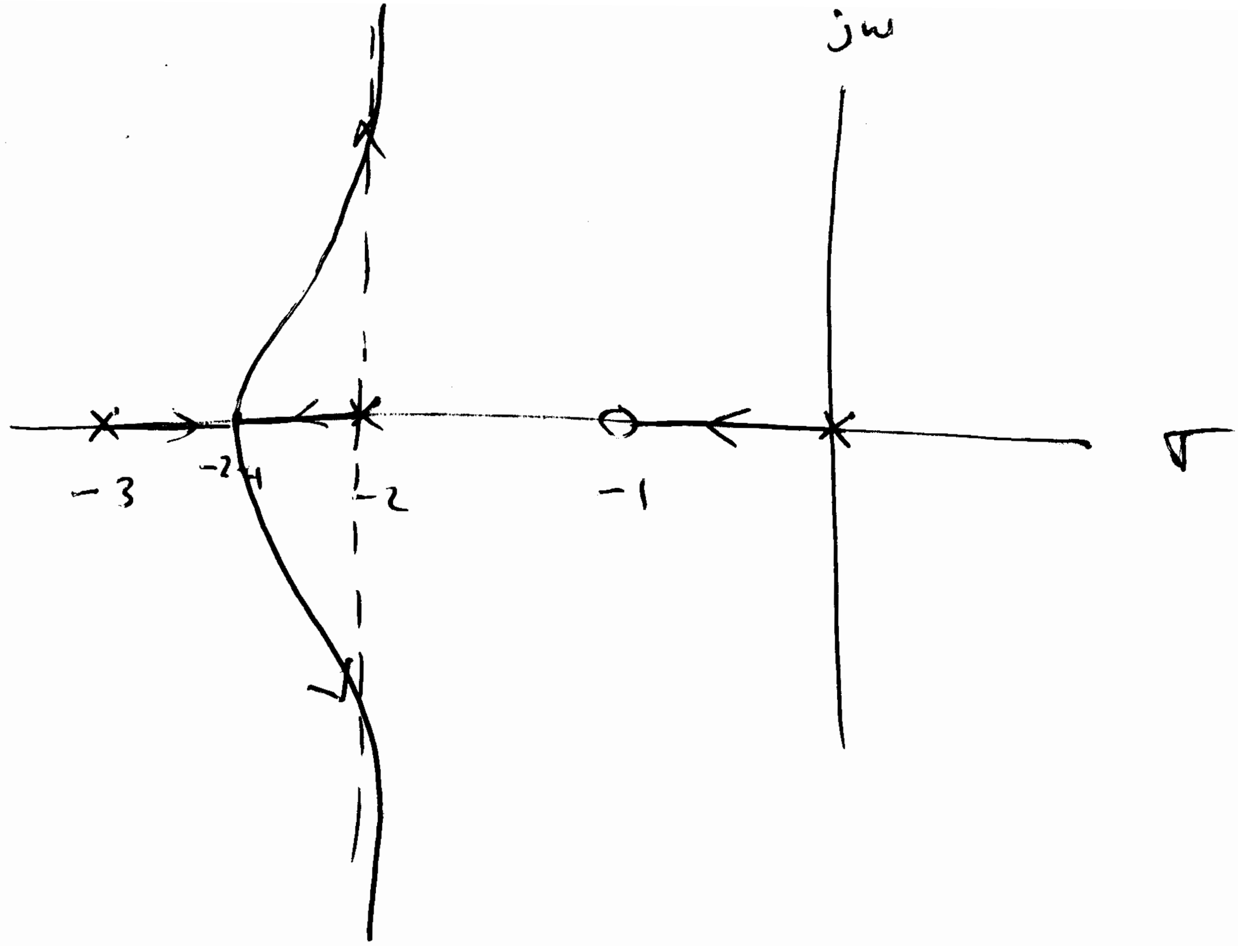
CE: $s^3+5s^2+(6+K)s+K=0$

s^3	1	$6+K$
s^2	5	K
s^1	$\frac{30+4K}{5}$	0
s^0	K	

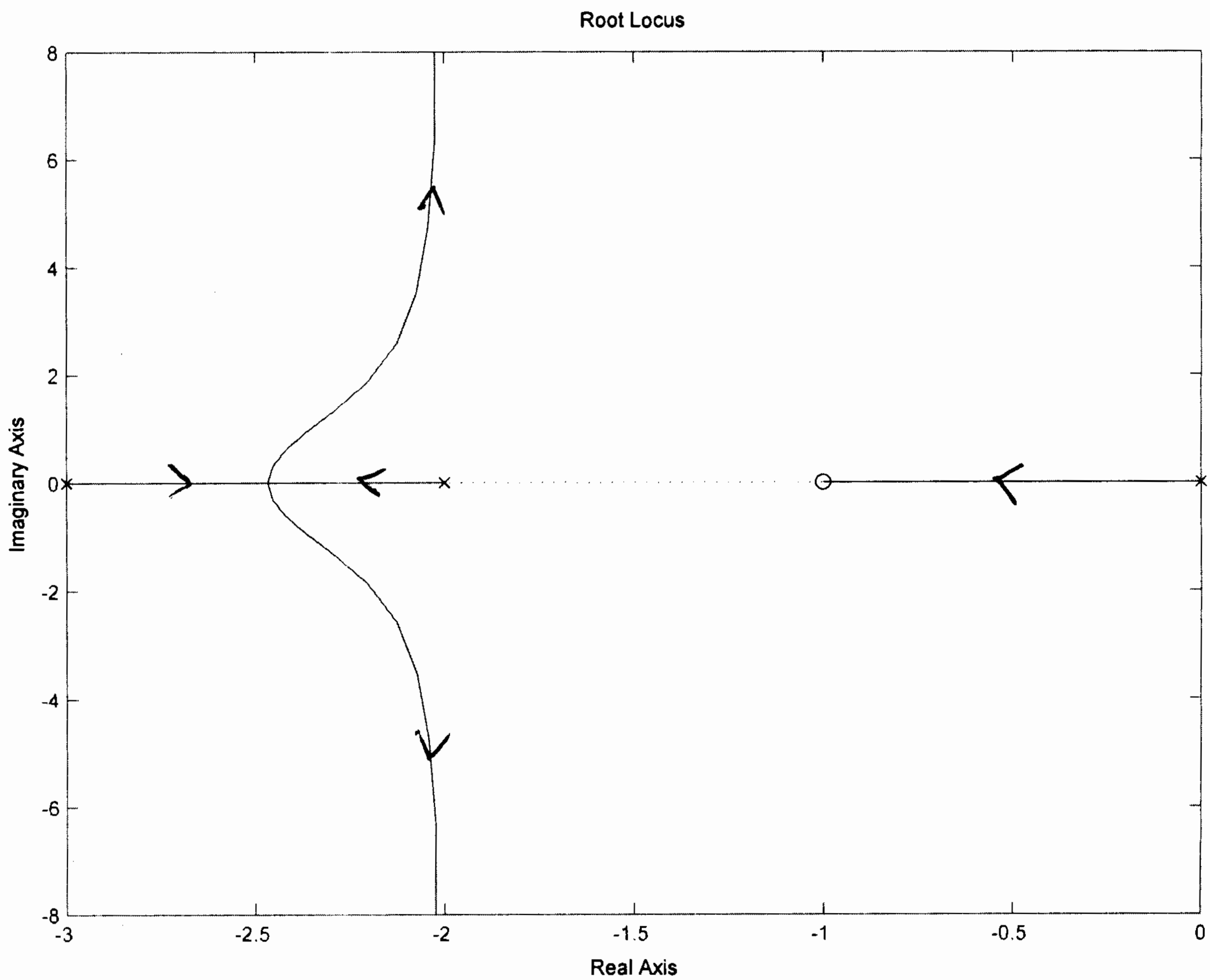
No value for K will make all the elements of the first row positive and only one zero

$\therefore$ No intersection with $j\omega$-axis

We have enough to attempt a sketch


jω
σ
-3
-2.4
-2
-1

Exact:


Root Locus
Imaginary Axis
Real Axis
8
6
4
2
0
-2
-4
-6
-8
-3
-2.5
-2
-1.5
-1
-0.5
0