2.2 A discrete-time signal $x(n)$ is shown in Fig. P2.2. Sketch and label carefully each of the following signals.

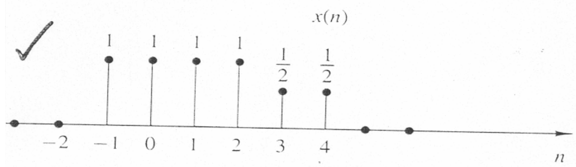

Figure P2.2

(a) $x(n-2)$ **(b)** $x(4-n)$ **(c)** $x(n+2)$ **(d)** $x(n)u(2-n)$
(e) $x(n-1)\delta(n-3)$ **(f)** $x(n^2)$ **(g)** even part of $x(n)$
(h) odd part of $x(n)$

$$x(n) = \left\{\ldots 0, 1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \ldots\right\}$$

(a)

$$x(n-2) = \left\{\ldots 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \ldots\right\}$$

(b)

$$x(4-n) = \left\{\ldots 0, \underset{\uparrow}{\frac{1}{2}}, \frac{1}{2}, 1, 1, 1, 1, 0, \ldots\right\}$$

(see 2.1(d))

(c)

$$x(n+2) = \left\{\ldots 0, 1, 1, 1, \underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{2}, 0, \ldots\right\}$$

(d)

$$x(n)u(2-n) = \left\{\ldots 0, 1, \underset{\uparrow}{1}, 1, 1, 0, 0, \ldots\right\}$$

(e)

$$x(n-1)\delta(n-3) = \left\{\ldots \underset{\uparrow}{0}, 0, 1, 0, \ldots\right\}$$

(f)

$$\begin{aligned} x(n^2) &= \{\ldots 0, x(4), x(1), x(0), x(1), x(4), 0, \ldots\} \\ &= \left\{\ldots 0, \frac{1}{2}, 1, \underset{\uparrow}{1}, 1, \frac{1}{2}, 0, \ldots\right\} \end{aligned}$$

(g)

$$\begin{aligned} x_e(n) &= \frac{x(n) + x(-n)}{2}, \\ x(-n) &= \left\{\ldots 0, \frac{1}{2}, \frac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0, \ldots\right\} \\ &= \left\{\ldots 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \ldots\right\} \end{aligned}$$

(h)

$$\begin{aligned} x_o(n) &= \frac{x(n) - x(-n)}{2} \\ &= \left\{\ldots 0, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \ldots\right\} \end{aligned}$$

Show that any signal can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the signal

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

Let

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)],$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)].$$

Since

$$x_e(-n) = x_e(n)$$

and

$$x_o(-n) = -x_o(n),$$

it follows that

$$x(n) = x_e(n) + x_o(n).$$

The decomposition is unique. For

$$x(n) = \left\{2, 3, \underset{\uparrow}{4}, 5, 6\right\},$$

we have

$$x_e(n) = \left\{4, 4, \underset{\uparrow}{4}, 4, 4\right\}$$

and

$$x_o(n) = \left\{-2, -1, \underset{\uparrow}{0}, 1, 2\right\}.$$

A. discrete-time system can be
1. Static or Dynamic
2. Linear or nonlinear
3. Time invariant or time varying
4. Causal or noncausal
5. Stable or unstable

Examine the following systems with respect to the properties above

a) $y(n) = \cos[x(n)]$
b) $y(n) = \text{Round}\,[x(n)]$, where Round denotes the integer part obtained by rounding.
c) $y(n) = |x(n)|$
d) $y(n) = x(n) + n\,x(n+1)$
e) $y(n) = \begin{cases} x(n), & \text{if } x(n) \geq 0 \\ 0, & \text{if } x(n) < 0 \end{cases}$

(a) Static, nonlinear, time invariant, causal, stable.

(b) Static, nonlinear, time invariant, causal, stable.

(c) Static, nonlinear, time invariant, causal, stable.

(d) Dynamic, linear, time variant, noncausal, unstable. Note that the bounded input $x(n) = u(n)$ produces an unbounded output.

(e) Static, nonlinear, time invariant, causal, stable.