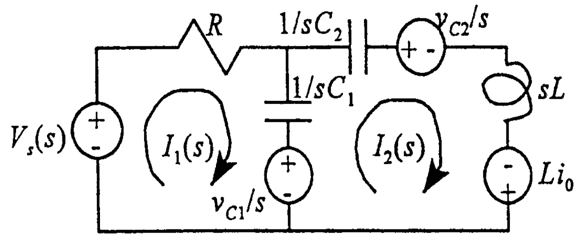
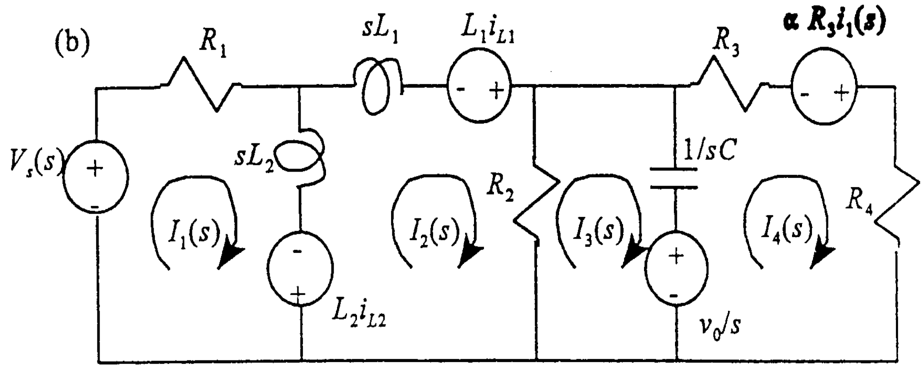
Problem 6-1

(a)



(b)



Problem 6-2

(a) The loop equations are:

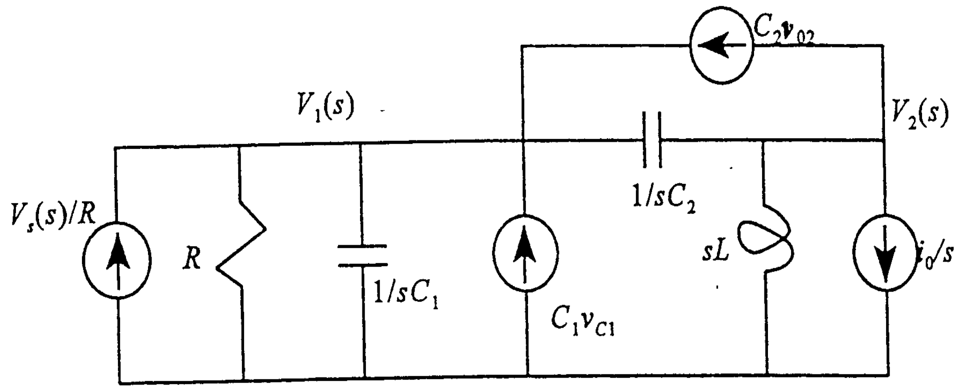
$$\begin{bmatrix} R + 1/sC_1 & -1/sC_1 \\ -1/sC_1 & 1/sC_1 + 1/sC_2 + sL \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_s(s) - v_{C_1}/s \\ v_{C_1}/s - v_{C_2}/s + Li_0 \end{bmatrix}$$

(b) The loop equations for this case are:

$$\begin{bmatrix} R_1 + sL_2 & -sL_2 & 0 & 0 \\ -sL_2 & sL_1 + sL_2 + R_2 & -R_2 & 0 \\ 0 & -R_2 & R_2 + 1/sC & -1/sC \\ -\alpha R_3 & 0 & -1/sC & 1/sC + R_3 + R_4 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \\ I_4(s) \end{bmatrix} = \begin{bmatrix} V_s(s) + L_2 i_{L_2} \\ -L_2 i_{L_2} + L_1 i_{L_1} \\ -v_0/s \\ v_0/s \end{bmatrix}$$

Problem 6-3

Do a Thevinin to Norton transformation of all voltage sources. For example, the circuit of part (a) becomes:



Problem 6-4

(a) The matrix equations for the nodal analysis are:

$$\begin{bmatrix} 1/R + sC_1 + sC_2 & -sC_2 \\ -sC_2 & sC_2 + 1/sL \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} V_s(s)/R + C_1v_{C_1} + C_2v_{C_2} \\ -C_2v_{C_2} - i_0/s \end{bmatrix}$$

(b) The nodal analysis matrix equations are:

$$\begin{bmatrix} 1/R_1 + 1/sL_1 + 1/sL_2 & -1/sL_{1_1} & 0 \\ -1/sL_1 & 1/sL_1 + sC + 1/R_2 + 1/R_3 & -1/R_3 \\ 0 & -1/R_3 & 1/R_3 + 1/R_4 \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} = \begin{bmatrix} V_s(s)/R_1 - i_{L_1}/s - i_{L_2}/s \\ i_{L_1}/s + Cv_0 - \alpha I_1(s) \\ \alpha I_1(s) \end{bmatrix}$$

Note that

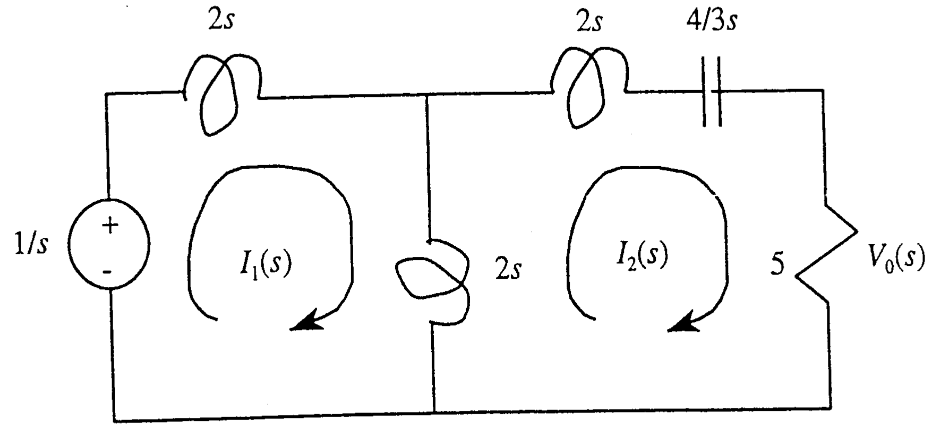
$$I_1(s) = [V_s(s) - V_1(s)]/R_1$$

When substituted for $I_1(s)$ on the right-hand side and rearranging, we obtain the following matrix equation:

$$\begin{bmatrix} 1/R_1 + 1/sL_1 + 1/sL_2 & -1/sL_{1_1} & 0 \\ -1/sL_1 - \alpha/R_1 & 1/sL_1 + sC + 1/R_2 + 1/R_3 & -1/R_3 \\ \alpha/R_1 & -1/R_3 & 1/R_3 + 1/R_4 \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} = \begin{bmatrix} V_s(s)/R_1 - i_{L_1}/s - i(0-)/s \\ i_{L_1}/s + Cv_0 - \alpha V_s(s)/R_1 \\ \alpha V_s(s)/R_1 \end{bmatrix}$$

Problem 6-6

Replace the coupled coils with a T-equivalent circuit:



Writing KVL equations around each loop, we obtain

$$\begin{bmatrix} 4s & -2s \\ -2s & 4s + \dfrac{4}{3s} + 5 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 1/s \\ 0 \end{bmatrix}$$

Solve for $I_2(s)$ and then find the output voltage:

$$V_0(s) = 5I_2(s) = \frac{5}{6\left(s^2 + \frac{5}{3}s + \frac{4}{9}\right)} = \frac{5}{6(s+1/3)(s+4/3)} = \frac{A}{s+1/3} + \frac{B}{s+4/3}$$

The expansion coeffiients are

$$A = (s+1/3)V_0(s)|_{s=-1/3} = \frac{5}{6} \text{ and } B = (s+4/3)V_0(s)|_{s=-4/3} = -\frac{5}{6}$$

Thus the output current is

$$v_0(t) = \frac{5}{6}\left[e^{-t/3} - e^{-4t/3}\right]u(t)$$

Problem 6-10

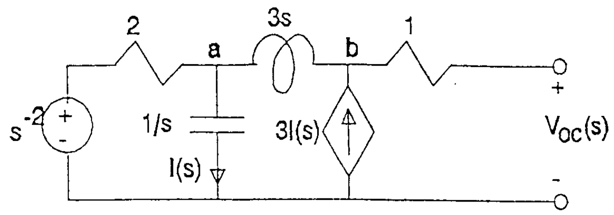
(a) Use the fact that

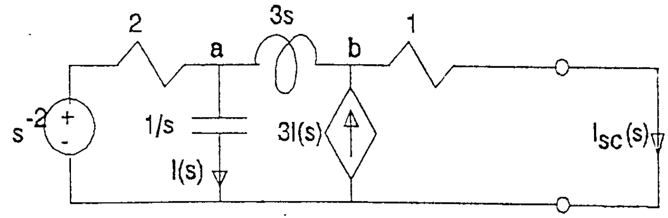
$$Z_{eq}(s) = \frac{V_{oc}(s)}{I_{sc}(s)}$$

The first circuit shown can be used to compute V_{oc}. Write KCL equations at nodes a and b:

$$\frac{1}{2}\left(V_a - \frac{1}{s^2}\right) + I + \frac{V_a - V_b}{3s} = 0, \quad V_1 = \frac{I}{s}$$

$$\frac{V_b - V_a}{} = 3I = 3sV_a$$





Solve the second equation for V_b in terms of V_b and substitute into the first equation after substituting for $I(s)$:

$$V_a\left(\frac{1}{2}+\frac{1}{3s}+s\right)-\frac{V_b}{3s}=\frac{1}{2s^2},\quad or\ V_a\left(\frac{1}{2}+\frac{1}{3s}+s-\frac{1}{3s}-3s\right)=\frac{1}{2s^2}$$

Thus

$$V_a=\frac{1}{s^2(1-4s)}\quad and\ V_2=V_{oc}=\frac{1+9s}{s^2(1-4s)}$$

Now use the lower circuit to find I_{sc}. Write KCL at nodes a and b:

$$\frac{V_a-1/s^2}{2}+sV_a+\frac{V_a-V_b}{3s}=0$$

$$\frac{V_b-V_a}{3s}-3I'+V_b=0,\quad I'=sV_a$$

(Note that all voltages and currents are different in the short circuit case than in the open circuit case - in particular, the current through the capacitor.) Substitute for I' in the second equation and solve for V_a in terms of V_b. This gives

$$V_a=\frac{1+3s}{1+9s^2}V_b$$

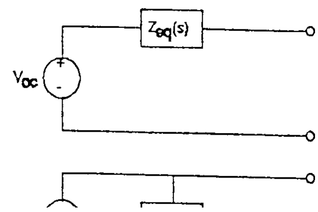
Collect terms in the first equation and substitute for V_a:

$$V_b=\frac{1+9s^2}{s^2(3-s+6s^2)}=I_{sc}=\frac{V_b}{1}$$

Therefore

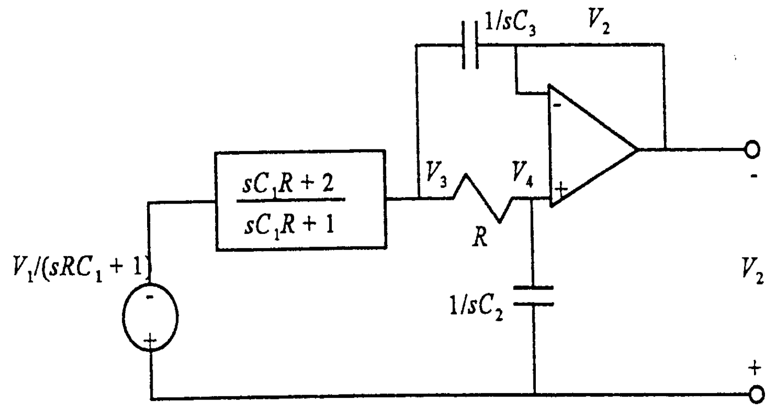
$$Z_{eq}=\frac{V_{oc}}{I_{sc}}=\frac{6s^2-s+3}{1-4s}$$

The Thevinin and Norton equivalent circuits are shown below:



Problem 6-17

(a) Do a Thevenin-to-Norton equivalent of the input, and a parallel combination of R and $1/sC_1$; then combine $R \| 1/sC_1$ and the second R in series to finally end up with the Laplace-equivalent circuit shown below:



Write a KCL equation at node 3:

$$\frac{V_3 - V_1/(sC_1R + 1)}{(sC_1R + 2)/(sC_1R + 1/R)} + \frac{V_3 - V_2}{R} + sC_3(V_3 - V_2) = 0 \qquad (1)$$

Write a KCL equation at node 4:

$$\frac{V_4 - V_3}{R} + sC_2V_4 = 0 \qquad (2)$$

Because of the ideal operational amplifier, $V_2 = V_4$. We may use (3) to replace V_4 by V_2 in (1) and (2). Then from (2) with the substitution, we obtain

$$V_3 = (1 + sC_2R)V_2 \qquad (3)$$

Replace V_3 in (1) using (3). We then solve for V_2/V_1 to get

$$H(s) = \frac{1}{R^3C_1C_2C_3s^3 + 2R^2C_2(C_1 + C_3)s^2 + R(C_1 + 3C_2)s + 1}$$

or

$$H(s) = \frac{1}{R^3C_1C_2C_3} \frac{1}{s^3 + \frac{2}{R}\left(\frac{1}{C_1} + \frac{1}{C_3}\right) + \frac{1}{R^2C_3}\left(\frac{3}{C_1} + \frac{1}{C_2}\right)s + \frac{1}{R^3C_1C_2C_3}}$$

With the parameter values given, the following MATLAB program (echo on) gives the values of th denominator polynomial coefficients, the roots or poles, and compares the complex poles obtaine with what should be the case for the complex poles of a Butterworth filter with the real roo matching.

```
%     Solution for Problem 6-17
%
C1 = 0.0022*10^(-6);
C2 = 0.00033*10^(-6);
C3 = 0.0056*10^(-6);
R = 10000;
A = (2/R)*(1/C1+1/C3)
A =
  1.2662e+005
B = (1/(R^2*C3))*(3/C1+1/C2)
B =
  7.8463e+009
C = 1/(R^3*C1*C2*C3)


C =
 2.4597e+014
p = [1 A B C];
s_poles = roots(p)
s_poles =
 1.0e+004 *
 -6.4078
 -3.1273 + 5.3484i
 -3.1273 - 5.3484i
s_poles_butter = s_poles(1)*(cos(pi/3)+i*sin(pi/3))
s_poles_butter =
-3.2039e+004- 5.5493e+004i
```

Note that the poles of the designed filter and the true Butterworth filter match up fairly well. One could adjust the R and C values to give a closer match.