EE 207 HW Solution #5

Problem 5-1

(a) Use superposition and the Laplace transforms of the unit step and decaying exponential to get

$$X(s) = \frac{1}{s} - \frac{1}{s+2} = \frac{2}{s(s+2)}$$

(b) Use superposition and the Laplace transform of an exponential to get

$$X(s) = \frac{1}{s+2} - \frac{1}{s+10} = \frac{8}{(s+2)(s+10)}$$

(c) Use superposition, time delay, and the Laplace transform of a unit step to get

$$X(s) = \frac{1}{s} - \frac{1}{s}e^{-10s} = \frac{1 - e^{-10s}}{s}$$

(d) Use superposition, time delay, and the Laplace transform of a unit impulse to get

$$X(s) = 1 - e^{-10s}$$

Problem 5-4

The Fourier transforms of (b) and (d) do not exist. In both cases, the path of integration for (5-3) cannot be chosen as the $j\omega$ axis since there are singularities on it. In the case of (c), the Laplace transform does not represent $x(t)$ for $t < 0$ since $x(t) \neq 0$ for $t < 0$.

Problem 5-8

Find the integro-differential equation for the current by applying KVL:

$$(2+3)i(t) + \frac{1}{10}\int_{-\infty}^{t} i(\lambda)d\lambda = 0,\ t > 0 \text{ or } 5i(t) + \frac{1}{10}\int_{-\infty}^{t} i(\lambda)d\lambda = 0,\ t > 0$$

Laplace transform this equation using the initial condition $v(0-) = -10$ V to get

$$5I(s) + \frac{I(s)}{10s} = \frac{10}{s} \text{ or } I(s) = \frac{2}{s + 1/50}$$

Therefore, the current is

$$i(t) = 2e^{-t/50}u(t)$$

Problem 5-10

(a) Expand as shown below, using s-shift and pairs 4 and 5 of Table 5-5:

$$X_1(s) = \frac{s+10}{s^2+8s+20} = \frac{s+10}{(s+4)^2+4} = \frac{s+4}{(s+4)^2+4} + 3\frac{2}{(s+4)^2+4}$$

Thus,

$$x_1(t) = [\cos(2t) + 3\sin(2t)]e^{-4t}u(t)$$

(b) Follow the same procedure as in part (a):

$$X_2(s) = \frac{s+3}{s^2+4s+5} = \frac{s+3}{(s+2)^2+1} = \frac{s+2}{(s+2)^2+1} + \frac{1}{(s+2)^2+1}$$

The inverse Laplace transform is

$$x_2(t) = [\cos(t) + \sin(t)]e^{-2t}u(t)$$

(c) This rational fraction may be expanded as

$$X_3(s) = \frac{s}{s^2+6s+18} = \frac{s}{(s+3)^2+9} = \frac{s+3}{(s+3)^2+9} - \frac{3}{(s+3)^2+9}$$

The time domain waveform is

$$x_3(t) = [\cos(3t) - \sin(3t)]e^{-3t}u(t)$$

(d) This function of s may be written as

$$X_1(s) = \frac{10}{s^2+10s+34} = \frac{10}{3}\frac{3}{(s+5)^2+9}$$

When inverse Laplace transformed, this gives the time domain waveform $x_4(t) = (10/3)\sin(3t)e^{-5t}u($

Problem 5-13

(a) Initial value:

$$\lim_{t \to 0^+} x_1(t) = \lim_{s \to \infty} s \frac{s + 10}{s^2 + 3s + 2} = 1$$

Final value:

$$\lim_{t \to \infty} x_1(t) = \lim_{s \to 0} s \frac{s + 10}{s^2 + 3s + 2} = 0$$

(b) Initial value:

$$\lim_{t \to 0^+} x_2(t) = \lim_{s \to \infty} s \frac{5}{s^3 + s^2 + 9s + 9} = 0$$

Final value - factor the denominator as

$$X_2(s) = \frac{5}{(s + 1)(s^2 + 9)}$$

The factor $s^2 + 9$ corresponds to a sinusoid in the time domain. As a result, the final value does not exist due to the oscillatory term.

(c) The initial value does not exist. The reason that the limit of $sX_3(s)$ doesn't exist as $s \to \infty$ is because of the impulse in the inverse Laplace transform of $X_3(s)$.

Final value:

$$\lim_{t \to \infty} x_3(t) = \lim_{s \to 0} s \frac{s^2 + 5s + 7}{s^2 + 3s + 2} = 0$$

(d) Initial value and final values, respectively:

$$\lim_{t \to 0^+} x_4(t) = \lim_{s \to \infty} s \frac{s + 3}{s^2 + 2s} = 1; \ \lim_{t \to \infty} x_4(t) = \lim_{s \to 0} s \frac{s + 3}{s^2 + 2s} = \frac{3}{2}$$

5.15

(b) The Laplace transforms of the given signals are

$$X_1(s) = \frac{s}{s^2+5^2} \text{ and } X_2(s) = \frac{3}{s^2+3^2}$$

The Laplace transform of their convolution is

$$Y(s) = \frac{s}{s^2+5^2}\frac{3}{s^2+3^2} = \frac{3}{16}\frac{s}{s^2+9} - \frac{3}{16}\frac{s}{s^2+25}$$

Inverse Laplace transformation gives

$$y(t) = \frac{3}{16}\left[\cos(3t) - \cos(5t)\right]u(t)$$

(d) The Laplace transforms of the given signals are

$$X_1(s) = \frac{3}{s^2+3^2} \text{ and } X_2(s) = \frac{e^{-5s}}{s}$$

The Laplace transform of their convolution is

$$Y(s) = \frac{3}{s^2+3^2}\frac{e^{-5s}}{s} = \frac{1}{3}\left[\frac{1}{s} - \frac{s}{s^2+3^2}\right]e^{-5s}$$

Inverse Laplace transformation gives

$$y(t) = \frac{1}{3}\{1 - \sin[3(t-5)]\}\,u(t-5)$$

which follows by superposition and time delay.

Problem 5-18

(a) Expand in partial fractions as

$$X(s) = \frac{5}{s+2} + \frac{2}{s+3} + \frac{1}{s^2+4}$$

The time-domain waveform is

$$x(t) = \left[5e^{-2t} + 2e^{-3t} + \frac{1}{2}\sin(2t)\right]u(t)$$

(b) The partial fraction expansion for this Laplace transform is

$$X(s) = \frac{1}{s+1} + \frac{1}{s+3} + \frac{1}{(s+1)^2+4}$$

Use superposition and appropriate transform pairs to get

$$x(t) = \left[e^{-t} + e^{-3t} + \frac{1}{2}e^{-t}\sin(2t)\right]u(t)$$

Problem 5-27

(a) By KVL, with $v_c(0\text{-}) = -V$ with respect to the current reference, we have

$$\frac{1}{C}\int_0^t i(\lambda)d\lambda - V + Ri(t) = 0 \text{ or } \frac{1}{sC}I(s) - \frac{V}{s} + RI(s) = 0 \text{ or } I(s) = \frac{V/R}{s + 1/RC}$$

Using the Laplace transform pair for an exponential, we obtain

$$i(t) = \frac{V}{R}e^{-t/RC}u(t)$$

(b) The initial conditions are $I(0\text{-}) = 0$ and $v_c(0\text{-}) = -V$ with respect to the current reference. Using KVL, we obtain

$$\frac{1}{C}\int_0^t i(\lambda)d\lambda - V + L\frac{di(t)}{dt} = 0 \text{ or } \frac{1}{sC}I(s) - \frac{V}{s} + sLI(s) = 0 \text{ or } I(s) = \frac{(V/L)\sqrt{LC}}{(s^2 + 1/LC)\sqrt{LC}}$$

Using the inverse Laplace transform of a sine, we obtain

$$i(t) = V\sqrt{\frac{C}{L}}\sin\left(\frac{t}{\sqrt{LC}}\right)u(t)$$