Electrical Engineering Department

EE 207 – Signals and Systems
Assignment Solution No. 3

Problem 3-4

This is an even square wave with zero average value (symmetric about the t-axis). Because of the zero average value, $a_0 = 0$. Because of it being even, $b_n = 0$, all n. To find a_n, evaluate (3-15):

$$a_n = \frac{2}{T_0}\left[\int_{-T_0/2}^{-T_0/4} (-A)\cos(n\omega_0 t)\,dt + \int_{-T_0/4}^{T_0/4} A\cos(n\omega_0 t)\,dt + \int_{T_0/4}^{T_0/2} (-A)\cos(n\omega_0 t)\,dt\right]$$

$$= \frac{2A}{T_0}\left[-\frac{\sin(n\omega_0 t)}{n\omega_0}\Big|_{-T_0/2}^{-T_0/4} + \frac{\sin(n\omega_0 t)}{n\omega_0 t}\Big|_{-T_0/4}^{T_0/4} - \frac{\sin(n\omega_0 t)}{n\omega_0}\Big|_{T_0/4}^{T_0/2}\right].$$

$$= \frac{2A}{T_0}\left[\frac{4\sin\left(n\frac{2\pi}{T_0}\frac{T_0}{4}\right)}{n\omega_0} - \frac{2\sin\left(n\frac{2\pi}{T_0}\frac{T_0}{2}\right)}{n\omega_0}\right], \quad \omega_0 = \frac{2\pi}{T_0}$$

$$= \frac{4A}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$= \begin{cases} 0, & n \text{ even} \\ (-1)^{(n-1)/2}\dfrac{4A}{n\pi}, & n \text{ odd} \end{cases}$$

(d) For an even triangular signal:

$$X_n = \frac{2}{T_0}\int_0^{T_0/2} x_\Delta(t)\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T_0}\int_0^{T_0/2} A\left(1 - \frac{4t}{T_0}\right)\cos(n\omega_0 t)\,dt$$

$$= \frac{2A}{T_0}\left[\frac{\sin(n\omega_0 t}{n\omega_0}\Big|_0^{T_0/2} - \frac{4}{T_0}\int_0^{T_0/2} t\cos(n\omega_0 t)\,dt\right]$$

The first term in the brackets is zero after substitution of the limits; the second term we must integrate by parts or look up in a tables:

$$X_n = \frac{2A}{T_0}\left(-\frac{4}{T_0}\right)\left[\frac{t\sin(n\omega_0 t)}{n\omega_0}\Big|_0^{T_0/2} - \int_0^{T_0/2}\frac{\sin(n\omega_0 t)}{n\omega_0}dt\right]$$

The first term in the brackets again evaluates to zero after substitution of the limits. The remaining integral evaluates to

$$X_n = -\frac{4A}{n\pi T_0}\frac{\cos(n\omega_0 t)}{n\omega_0}\Big|_0^{T_0/2} = \begin{cases} \dfrac{4A}{n^2\pi^2}, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}.$$

Problem 3-8

(a) Using Parseval's theorem, we have for the time domain computation of the average power:

$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} [2\cos(\omega_0 t)\sin(2\omega_0 t)]^2 dt; \quad T_0 = 1/5000\ s; \ \omega_0 = 2\pi/T_0\ rad/s$$

Using appropriate trigonometric identies and noting that cross terms integrate to zero, we obtain

$$P_{ave} = \frac{1}{2} + \frac{1}{2}\left(\frac{1}{4}\right) = \frac{3}{4}\ watts$$

In the frequency domain,

$$P_{ave} = \sum_{n=-\infty}^{\infty} |X_n|^2$$

Using Euler's theorem and uniqueness of the Fourier series, we find the Fourier coefficients to be

$$X_1 = X_{-1} = \frac{1}{2};\ X_3 = X_{-3} = -\frac{1}{4};\ X_5 = X_{-5} = -\frac{1}{4}$$

All other Fourier coefficients are zero. Thus, the power summed in the frequency domain is

$$P_{ave} = \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{4}\ watts$$

(b) Only the fundamental (5000 Hz) component is passed by the telephone system. Therefore,

$$P_{out} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\ watt$$

and

$$\frac{P_{out}}{P_{ave}} = \frac{2}{3} = 67\ \%$$

Problem 3-12

(a) The exponential Fourier series coefficients are given by

$$X_n = \frac{1}{2}\int_{-1}^{1} e^{-|t|}e^{-j2\pi n t/2}dt = \int_{0}^{1} e^{-t}\cos(\pi n t)dt = \frac{1-(-1)^n e^{-1}}{1+(\pi n)^2} .$$

The second integral follows because of the eveness of exp(-|*t*|). Evaluation of the coefficients and substitution into the exponential series results in

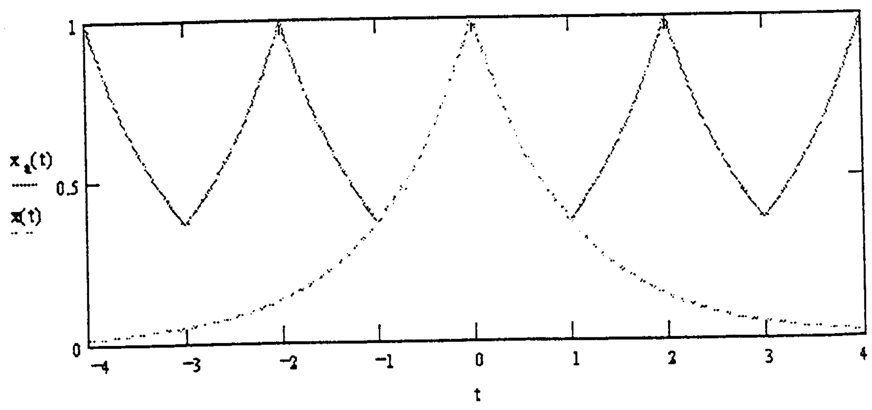
$$x(t) = \cdots + \frac{1+e^{-1}}{1+9\pi^2}e^{-j3\pi t} + \frac{1-e^{-1}}{1+4\pi^2}e^{-j2\pi t} + \frac{1+e^{-1}}{1+\pi^2}e^{-j\pi t} + (1-e^{-1})$$

$$+ \frac{1+e^{-1}}{1+\pi^2}e^{j\pi t} + \frac{1-e^{-1}}{1+4\pi^2} + \frac{1+e^{-1}}{1+9\pi^2}e^{j9\pi t} + \cdots$$

(b) The trigonometric Fourier series is

$$x(t) = (1-e^{-1}) + 2\frac{1+e^{-1}}{1+\pi^2}\cos(\pi t) + 2\frac{1-e^{-1}}{1+4\pi^2}\cos(2\pi t) + \cdots$$

(c) The signal that the Fourier series sums to is shown below, along with the double-sided exponential being approximated. Note that the two are identical in the interval [-1, 1].



Problem 3-17

Property	a	b	c	d	e	f
Real coefficients		X			X	
Imaginary coefficients	X			X		
Complex coefficients			X			X
Even-indexed coefficients = 0	X	X	X	X	X	X
$X_0 = 0$	X	X	X	X	X	X

Problem 3-21

(a) See Figure 3-10(a), except that the nulls of the sinc-function are at multiples of 500 Hz and the lines are spaced by 125 Hz.
(b) See Figure 3-10(b), except that the nulls of the sinc-function are at multiples of 1000 Hz and the lines are spaced by 125 Hz.
(c) See Figure 3-10(c), except that the nulls of the sinc-function are at multiples of 500 Hz and the lines are spaced by 62.5 Hz.