Electrical Engineering Department

EE207 - Signals and Systems
Assignment Solution - No.2

Problem 2-1

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Problem 2-2
(a) Firstorder (b) First Order (differntiat eonce to get rid of the integral)

(c) Zero order (d) Second order (e) Second order

Problem 2-3

(a), (b), (c), and (e) are fixed; and (d) is not because of the time varying coefficient

Problem 2-4

Only (c) and (e) are nonlinear. Superpostion will not hold in (e) because of the term +10.
As an example to show linearity, consider (d):

$$\frac{d_1 y(t)}{dt} = t^2 y_1(t) = \int_{-\infty}^{t} x_1(\lambda)d\lambda$$
$$\frac{d_2 y(t)}{dt} = t^2 y_2(t) = \int_{-\infty}^{t} x_2(\lambda)d\lambda$$

Multiply the first equation by a constant, say a, and the second by another constant, say b;

add to obtain

$$\frac{d}{dt}[ay_1 + by_2] + t^2[ay_1(t) + by_2(t)] = \int_{-\infty}^{t} [ax_1(\lambda) + bx_2(\lambda)]d\lambda$$

This is of the same form as the original equation.

Problem 2-10

(a) using Kirchoff's voltage equation and Ohm's law, the appropriate equations are:
$$x(t) = L\frac{di(t)}{dt} + y(t)$$
$$y(t) = Ri(t)$$
$$\frac{di(t)}{dt} = \frac{1}{R}\frac{dy(t)}{dt}$$

Substitute the last equation into the first and rearrange:

$$\frac{dy(t)}{dt} + \frac{R}{L}y(t) = \frac{R}{L}x(t)$$

(b) The proff is similar to that of Problem 2-4

(c) Consider

$$\frac{dy(t-\tau)}{dt} = \frac{dy(t')}{dt'}\frac{dt'}{dt} \qquad \text{where } t' = t-\tau$$

Thus

$$\frac{dy(t-\tau)}{dt} + \frac{R}{L}y(t-\tau) = \frac{R}{L}x(t-\tau)$$

which shows that the system is fixed.

(d) Note that the solution to the homogeneous equation is

$$y_H(t) = Ae^{-Rt/L} \qquad , t > 0$$

Assume a complete solution of this form where A is time varying. Substitute into the differential equation of part (a) to obtain

$$A(t) = \int_0^t \frac{R}{L}x(\lambda)e^{R\lambda/L}d\lambda + A_0$$

The inductor current being zero at t=0 implies that $A_0 = 0$, so the solution to the differential equation can be written as

$$y(t) = \int_0^t \frac{R}{L}x(\lambda)e^{\left[-\frac{R}{L}x(t-\lambda)\right]}d\lambda$$

Problem 2-11

Property	a	b	c	d	e	f	g
Linear	X		X		X		
Causal	X	X	X	X		X	
Fixed	X	X		X	X	X	X
Dynamic	X	X	X	X	X		

Problem 2-23

By KVL around the loop,

$$x(t) = R_1 i(t) + \frac{1}{C}\int_{-\infty}^{t} i(\lambda)d\lambda + R_2 i(t)$$

but $i = y/R_2$. Substitute this into the integro-differential equation and differentiate once to get

$$\frac{R_1 + R_2}{R_2}\frac{dy(t)}{dt} + \frac{1}{R_2 C}y(t) = \frac{dx(t)}{dt}$$

To find the impulse response, it is easiest to find the step response and differentiate it. The solution to the homogeneous equation is

$$a(t) = Ae^{-t/(R_1+R_2)C} \qquad ,t>0$$

Where $a(t)$is used to denote the step response. With the input a step, the right hand side of the differential equation is a unit impulse. To get the requires initial condition, integrate the differential equation through zero:

$$\frac{R_1 + R_2}{R_2}\int_{0^-}^{0^+}\frac{da(t)}{dt}dt + \frac{1}{R_2 C}\int_{0^-}^{0^+}a(t)dt = \int_{0^-}^{0^+}\delta(t)dt$$

To match the right hand side, the integrand of the first term on the left hand side must contain a unit impulse, and therefore the second term on the left hand side is proportional to a unit step. Hence the integral of the second term through zero is zero. The first term is a perfect differential, and of course the right hand side integral evaluates to unity. Thus we obtain

$$\frac{R_1 + R_2}{R_2}[a(0^+) - a(0^-)] = 1$$

or $$a(0^+) = \frac{R_2}{R_1 + R_2}$$

as the required initial condition, and the step response is

$$a(t) = \frac{R_2}{R_1 + R_2}e^{-t/(R_1+R_2)C}u(t)$$

Differentiating, we obtain for the impulse response

$$h(t) = \frac{R_2}{R_1 + R_2}\left[\delta(t) - \frac{1}{(R_1 + R_2)C}e^{-t/(R_1+R_2)C}u(t)\right]$$

Problem 2-32

(a) use KCL at the output node to obtain

$$\frac{x(t) - y(t)}{R_1} = \frac{y(t)}{R_2} + C\frac{dy(t)}{dt}$$

When rearranged, this gives the answer given in the problem statement.
(b) The homogeneous equation for the impulse response is

$$R_1 C\frac{dh(t)}{dt} + \left(1 + \frac{R_1}{R_2}\right)y(t) = x(t)$$

Assume a solution of the form

$$h(t) = Ae^{pt}$$

Substitute into the differential equation to get the characteristic equation:

$$R_1 Cp + \left(1 + \frac{R_1}{R_2}\right) = 0$$

Thus, p is given by

$$p = -\frac{1}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = -\frac{R_1 R_2 C}{R_1 + R_2} = -\tau$$

To get the required initial condition, integrate the differential equation, with impulse forcing function, through zero:

$$R_1 C\int_{0^-}^{0^+} \frac{dh(t)}{dt}dt + \left(1 + \frac{R_1}{R_2}\right)\int_{0^-}^{0^+} h(t)dt = \int_{0^-}^{0^+} \delta(t)dt = 1$$

The second term on the left hand side is discontinuous at time zero, but contains no delta function; the first term must contain a delta function to balance the right hand side. Since the latter is a perfect differential, we can integrate it. The former integrates to zero through time zero. Thus we have

$$R_1 C[h(0^+) - h(0^-)] = 1 \qquad \text{or} \qquad h(0^+) = \frac{1}{R_1 C}$$

Substituting the results for $h(0^+) = A$ and p, we obtain the result given in the problem statement for the impulse response.
(c) Integrate the impulse response to obtain the given answer.
(d) Note that the input can be written as

$$x(t) = u(t) - u(t-1)$$

and use superpostion to obtain the given answer.
(e) Duhamel's integral simply tells us to integrate the step response. This gives

$$y_r(t) = \int_0^t \frac{R_2}{R_1 + R_2}(1 - e^{-\lambda/\tau})d\lambda \qquad , t > 0$$

Integrating and putting the limits gives the answer.
(f) Using superposition,

$$y(t) = y_r(t) - 2y_r(t-1) + y_r(t-2)$$

Probelm 2-36

From Problem 2-21 and the condition for BIBO satbility, we have

$$\int_{-\infty}^{\infty}|h(t)|dt = \int_{-\infty}^{\infty}\left|\delta(t) - \frac{R}{L}e^{-Rt/L}u(t)\right|dt \leq \int_{-\infty}^{\infty}\delta(t)dt + \frac{R}{L}\int_{-\infty}^{\infty}e^{-Rt/L}dt = 1 + 1 = 2 < \infty$$

Therefore, the system is BIBO stable