King Fahd University of Petroleum and Minerals
Electrical Engineering Department
EE 207 Homework # 1 Solutions September 19,1998

Problem 1.9:

The fundamental periods of the signals are:

a) $\sin 50\pi t$: $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{50\pi} = \frac{1}{25}$ s $\Rightarrow f_1 = 25$ Hz

b) $\cos 60\pi t$; $T_2 = \frac{2\pi}{60\pi} = \frac{1}{30}$ s $\Rightarrow f_2 = 30$ Hz

c) $\cos 70\pi t$: $T_3 = \frac{2\pi}{70\pi} = \frac{1}{35}$ s $\Rightarrow f_3 = 35$ Hz

d) $\sin 50\pi t + \cos 60\pi t$:

Find largest f_0 such that $f_1 = 25\text{Hz} = n_1 f_0 = 5 \times 5$
$f_2 = 30\text{Hz} = n_2 f_0 \quad 6 \times 5$

$\Rightarrow f_0 = 5\text{ Hz} \Rightarrow T_0 = \frac{1}{5}$ s.

e) $\sin 50\pi t + \cos 70\pi t$

$f_1 = 25\text{Hz} = n_1 f_0 = 5 \times 5$

$f_3 = 35\text{Hz} = n_3 f_0 = 7 \times 5$

$\Rightarrow f_0 = 5\text{Hz} \Rightarrow T_0 = \frac{1}{5}$ s

Problem 1.13:

a) Signals written as real parts of rotating phasors:

$\sin 50\pi t = \cos(50\pi t - \pi/2) = \mathrm{Re}[e^{j(50\pi t - \pi/2)}]$

$\cos 60\pi t = \mathrm{Re}[e^{j60\pi t}]$

$\cos 70\pi t = \mathrm{Re}[e^{j70\pi t}]$

$\sin 50\pi t + \cos 60\pi t = \mathrm{Re}[e^{j(50\pi t - \pi/2)} + e^{j60\pi t}]$

$\sin 50\pi t + \cos 70\pi t = \mathrm{Re}[e^{j(50\pi t - \pi/2)} + e^{j70\pi t}]$

b) Signals written in terms of complex conjugate rotating phasors:

$$\sin 50\pi t = \cos(50\pi t - \pi/2) = \frac{1}{2}e^{j(50\pi t - \pi/2)} + \frac{1}{2}e^{-j(50\pi t - \pi/2)}$$

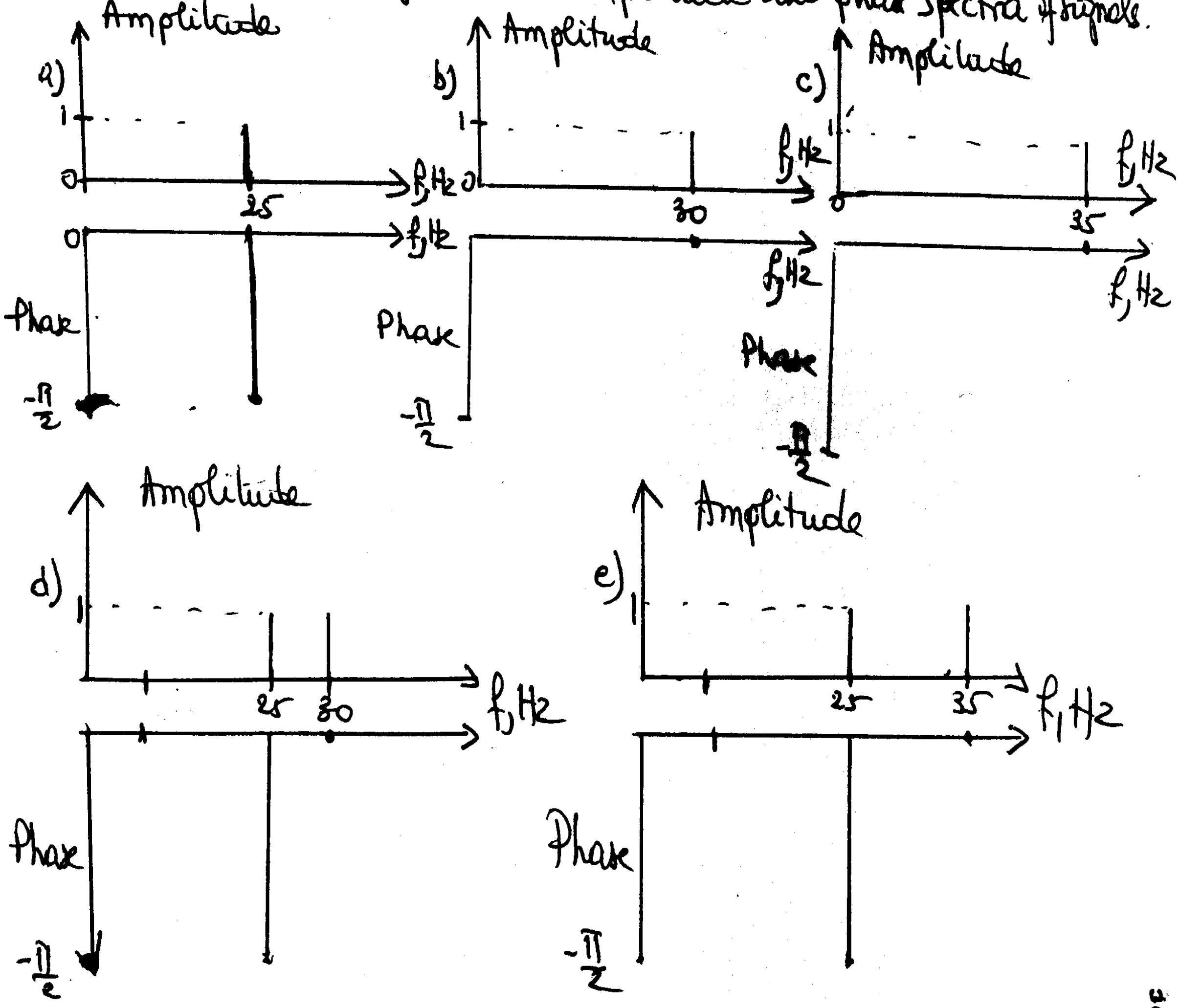
$$\cos 60\pi t = \frac{1}{2}e^{j60\pi t} + \frac{1}{2}e^{-j60\pi t}$$

$$\cos 70\pi t = \frac{1}{2}e^{j70\pi t} + \frac{1}{2}e^{-j70\pi t}$$

$$\sin 50\pi t + \cos 60\pi t = \frac{1}{2}e^{j(50\pi t - \pi/2)} + \frac{1}{2}e^{-j(50\pi t - \pi/2)} + \frac{1}{2}e^{j60\pi t} + \frac{1}{2}e^{-j60\pi t}$$

$$\sin 50\pi t + \cos 70\pi t = \frac{1}{2}e^{j(50\pi t - \pi/2)} + \frac{1}{2}e^{-j(50\pi t - \pi/2)} + \frac{1}{2}e^{j70\pi t} + \frac{1}{2}e^{-j70\pi t}$$

c) Sketch of single-sided amplitude and phase spectra of signals.

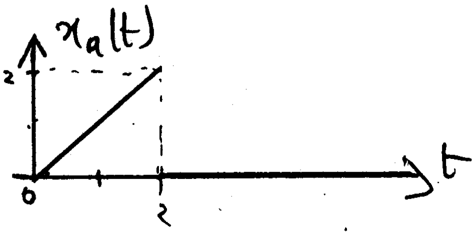


d) To get the double-sided amplitude spectra, halve the lines of the single-side spectra above and take the mirror image through the vertical axis. To get the double-side phase spectra, take the anti-mirror image of the phase spectra shown above through the vertical axis.

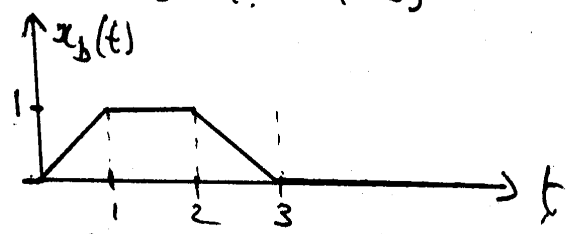
Problem 1.18:

Plot of signals.

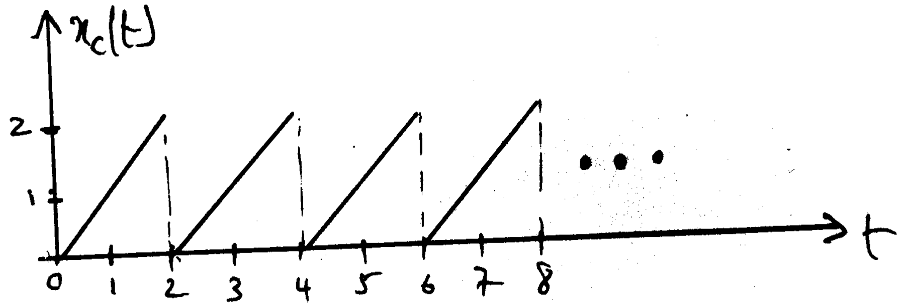
a) $x_a(t) = r(t)u(2-t)$



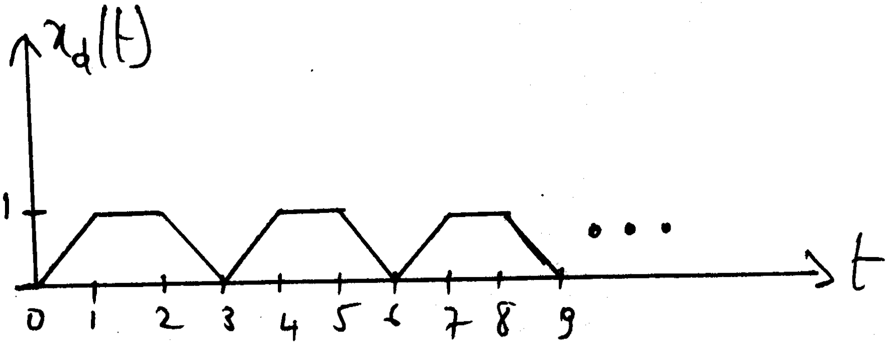
b) $x_b(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$



c) $x_c(t) = \sum_{n=0}^{\infty} x_a(t-2n)$



d) $x_d(t) = \sum_{n=0}^{\infty} x_b(t-3n)$



Problem 1.22 (c)(d):

c) $x_3(t) = r(t) - r(t-1) - r(t-3) + r(t-4)$

d) $x_4(t) = r(t) - 2u(t-1) - r(t-2)$

Problem 1.36:

Average powers of signals.

Average power of $A\cos(\omega t + \theta) = \frac{A^2}{2}$

a) $P_a = \frac{1^2}{2} = \frac{1}{2}\,W$

b) $P_b = \frac{1^2}{2} = \frac{1}{2}\,W$

c) $P_c = \frac{1^2}{2} = \frac{1}{2}\,W$

d) $P_d = \frac{1^2}{2} + \frac{1^2}{2} = 1\,W$

e) $P_e = \frac{1^2}{2} + \frac{1^2}{2} = 1\,W$.

Problem 1-38.

(a) Power:

$$P_a = \lim_{T\to\infty}\frac{1}{2T}\left[\int_0^1 1^2 dt + \int_1^2 6^2 dt + \int_2^T 4^2 dt\right]$$

$$= \lim_{T\to\infty}\frac{1}{2T}[1 + 36 + 16(T-2)]$$

$$= 0 + 0 + \lim_{T\to\infty}\frac{16T}{2T} - 0 = 8\ W$$

(b) Energy:

$$E_b = \int_0^1 1^2 dt + \int_1^2 6^2 dt = 37\ J$$

(c) Energy:

$$E_c = \int_0^\infty e^{-10t} dt = \frac{-e^{-10t}}{10}\Big|_0^\infty = \frac{1}{10}\ J$$

(d) Power;

$$P_d = \lim_{T\to\infty}\frac{1}{2T}\int_0^T (e^{-5t}+1)^2 dt$$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_0^T (e^{-10t}+2e^{-5t}+1) dt$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[\frac{e^{-10t}}{-10} + \frac{2e^{-5t}}{-5} + t\right]_0^T = \frac{1}{2}\ W$$

(e) Power. Similarly to (d), it can be shown that P_e = 1/2 W.
(f) Neither. It can be shown that both the energy and power are infinite.
(g) Power:

$$P_g = \lim_{T\to\infty}\frac{1}{2T}\left[\int_0^1 t^2 dt + \int_1^T 1^2 dt\right] = \frac{1}{2}\ W$$

(h) Neither. First try energy:

$$E = \int_0^{\infty} [t^{-1/4} u(t-3)]^2 dt$$

$$= \int_3^{\infty} t^{-1/2} dt$$

$$= \frac{t^{1/2}}{1/2} \Big|_3^{\infty} \to \infty$$

Now try power:

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [t^{-1/4} u(t-3)]^2 dt$$

$$= \lim_{t\to\infty} \frac{1}{2T} \int_3^{\infty} t^{-1/2} dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \frac{t^{1/2}}{1/2} \Big|_3^{T}$$

$$= \lim_{T\to\infty} \frac{T^{1/2} - 3^{1/2}}{T} \to 0$$