EE 207-03 Additional Examples on Convolution

① Find $f_1(t) * f_2(t)$ if $f_1(t) = 2e^{-4t}\,u(t)$ and $f_2(t) = 5\cos 3t\; u(t)$

Solution

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_2(\tau)\, f_1(t-\tau)\, d\tau = \int_{-\infty}^{\infty} f_1(\tau)\, f_2(t-\tau)\, d\tau$$

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} 5\cos 3\tau\, u(\tau) \cdot 2e^{-4(t-\tau)}\, u(t-\tau)\, d\tau$$

$u(t-\tau) = 1$ for $(t-\tau) \geq 0$ or $\tau \leq t$

$u(t-\tau) = 0$ for $(t-\tau) < 0$ or $\tau > t$

$u(\tau) = 1$ for $\tau \geq 0$

$u(\tau) = 0$ for $\tau < 0$

$$\Rightarrow f_1(t) * f_2(t) = \int_0^t 5\cos 3\tau \cdot 2e^{-4(t-\tau)}\, d\tau$$

$$= 10e^{-4t} \int_0^t e^{4\tau} \cos 3\tau\, d\tau$$

$$= \frac{10e^{-4t}}{16+9} \left[e^{4\tau} (4\cos 3\tau + 3\sin 3\tau) \right]_0^t$$

$$= 1.6\cos 3t + 1.2\sin 3t - 1.6e^{-4t}, \quad t \geq 0$$

OR $\boxed{f_1(t) * f_2(t) = (1.6\cos 3t + 1.2\sin 3t - 1.6e^{-4t})\, u(t)}$

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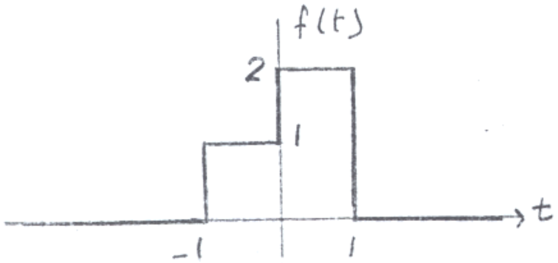
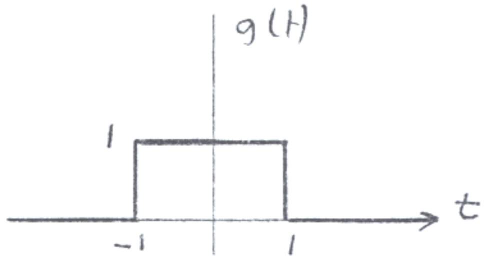
② Find $f(t) * g(t)$, where $f(t)$ and $g(t)$ are as shown below





Solution

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(\lambda)\, g(t-\lambda)\, d\lambda \qquad (1)$$

By inspection of the given graph, it is known that

$f(\lambda) = 0$ for $-\infty < \lambda < -1$, $f(\lambda) = 1$ for $-1 \le \lambda < 0$

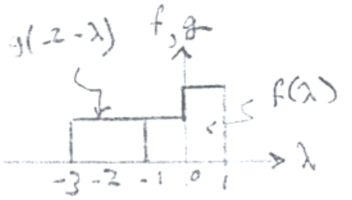
$f(\lambda) = 2$ for $0 \le \lambda < 1$, $f(\lambda) = 0$ for $1 < \lambda$

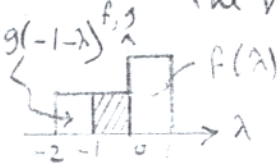
$g(t-\lambda) = 0$ for $-\infty \le (t-\lambda) < -1$, $g(t-\lambda) = 1$ for $-1 \le (t-\lambda) \le 1$

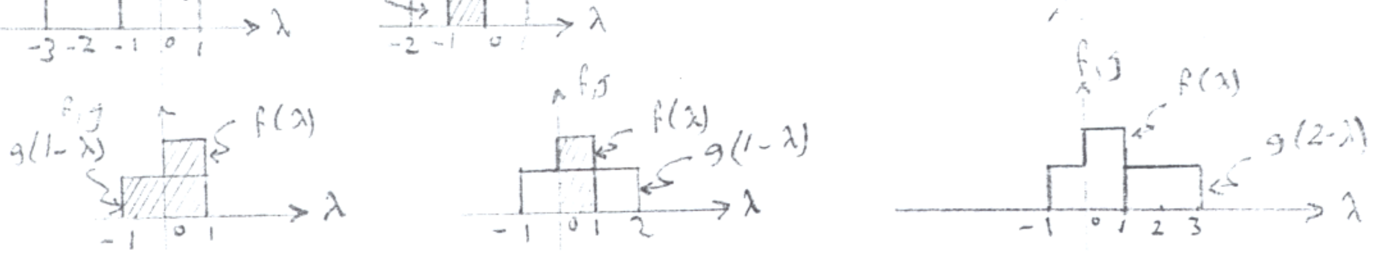
$g(t-\lambda) = 0$ for $1 < (t-\lambda)$

When equation (1) is calculated, by inspection there are following significant check points

$t = -2$	$g(t-\lambda)$ touches $f(\lambda)$
$t = -1$	a half of $g(t-\lambda)$ overlaps with $f(\lambda)$
$t = 0$	entire range of $g(t-\lambda)$ coincides with $f(\lambda)$
$t = 1$	a left half of $g(t-\lambda)$ overlaps with $f(\lambda)$
$t = 2$	the left edge of $g(t-\lambda)$ touches with the right edge of $f(\lambda)$







So, at $t = -2$,

$$f(-2) * g(-2) = \int_{-\infty}^{\infty} f(\lambda)\, g(-2-\lambda)\, d\lambda = 0$$

at $t = -1$

$$f(-1) * g(-1) = \int_{-\infty}^{\infty} f(\lambda) g(-1-\lambda)\, d\lambda = \int_{-1}^{0} 1 \times 1\, d\lambda = 1$$

at $t = 0$

$$f(0) * g(0) = \int_{-\infty}^{\infty} f(\lambda) g(0-\lambda) d\lambda = \int_{-1}^{0} 1 \times 1\, d\lambda + \int_{0}^{1} 2 \times 1\, d\lambda = 3$$

at $t = 1$

$$f(1) * g(1) = \int_{-\infty}^{\infty} f(\lambda) g(1-\lambda)\, d\lambda = \int_{0}^{1} 2 \times 1\, d\lambda = 2$$

at $t = 2$

$$f(2) * g(2) = \int_{-\infty}^{\infty} f(\lambda)\, g(2-\lambda) d\lambda = 0$$