Electric Circuits II

Two-Port Circuits
Two-Port Parameters

Lecture #42

The material to be covered in this lecture is as follows:

- Introduction to two-port circuits
- The Terminal Equations
- The Two-Port z-parameters
- The Two-Port y-parameters

After finishing this lecture you should be able to:

- Understand the Importance of Two-Port Circuits
- Relate the Current and Voltage at One Port to the Current and Voltage at the Other Port.
- Determine the Two-Port z-parameters
- Determine the Two-Port y-parameters

Introduction to Two-Port Circuits

- In analyzing some electrical systems, focusing on two pairs of terminal is convenient.
- Often, a signal is fed into one pair of terminals and then after being processed, is extracted at a second pair of terminals.
- The terminal pairs represent the points where signals are either fed in or extracted. They are referred to as ports of the system.
- Fig. 42-1 illustrates the basic two-port building block.
- Use of this building block is subject to several restrictions:
  - There can be no energy stored within the circuit
  - There can be no independent sources within the circuit
  - The current into the port must equal the current out of the port
  - All external connections must be made to either the input port or output port, no connections are allowed between the ports.

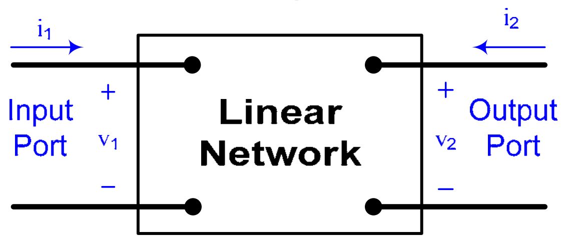

Fig. 42-1 The Two-Port Building Block

- The fundamental principle underlying two-port modeling of a system is that only the terminal variables (i_1, v_1, i_2, and v_2) are of interest.

The Terminal Equations

- In two-port network we are interested in relating the current and voltage at one port to the current and voltage at the other port. Fig. 42-1 shows the reference polarities of the terminal voltages and the reference directions of the terminal currents.
- Most general description is carried out in the s domain.
- We write all equations in the s domain, resistive networks and sinusoidal steady state solutions become special cases.
- Fig. 42-2 shows the basic building block in terms of the s-domain variables I_1, V_1, I_2, and V_2.

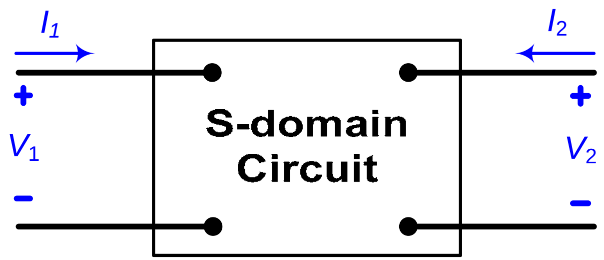

Fig. 42-2 The s-domain Two-Port Basic Building Block

- Out these four terminal variables, only two are independent. Thus we can describe a two-port network with just two simultaneous equations. However there are six ways in which to combine the four variables:

Impedance Parameters (z-parameters):

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2 \qquad (42\text{-}1)$$

Admittance Parameters (y-parameters):

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad (42\text{-}2)$$

Hybrid Parameters (h-parameters):

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2 \qquad (42\text{-}3)$$

Inverse Hybrid Parameters (g-parameters):

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2 \qquad (42\text{-}4)$$

Transmission Parameters (a-parameters):

$$V_1 = a_{11}V_2 - a_{12}I_2$$
$$I_1 = a_{21}V_2 - a_{22}I_2 \qquad (42\text{-}5)$$

Inverse Transmission Parameters (b-parameters):

$$V_2 = b_{11}V_1 - b_{12}I_1$$
$$I_2 = b_{21}V_1 - b_{22}I_1 \qquad (42\text{-}6)$$

- These six sets of equations may also be considered as three pairs of mutually inverse relations.
- The coefficients of the variables are called the parameters of the two-port circuit. We refer to the z-parameters, y-parameters, a-parameters, b-parameters, h-parameters and g-parameters of the network.

The Two-Port Parameters

z-parameters:

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

or in matrix form:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad (42\text{-}7)$$

The values of the parameters can be evaluated by setting $I_1=0$ (input port open-circuited) or $I_2=0$ (output port open-circuited). Thus,

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}, \quad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$
$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}, \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad (42\text{-}8)$$

The z-parameters are also called the open-circuit impedance parameters:

- z_{11}= Open-circuit input impedance
- z_{12}= Open-circuit transfer impedance from port 1 to port 2
- z_{21}= Open-circuit transfer impedance from port 2 to port 1
- z_{22}= Open-circuit output impedance

Example 42-1 illustrates the determination of the z-parameters for a resistive circuit.

Example 42-1

Find the z-parameters for a resistive circuit shown in Fig. 42-3

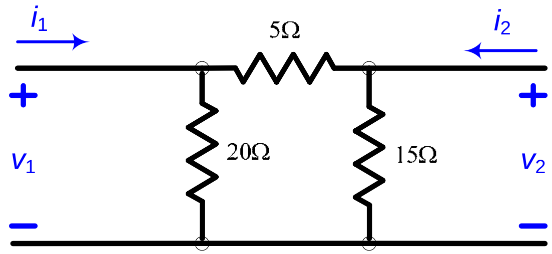

Fig. 42-3 The Circuit for Example 42-1

Solution:

To obtain z_{11} and z_{21} we connect a voltage V_1 (or a current source I_1) to port 1 with port 2 open circuited as in Fig. 42-4a.

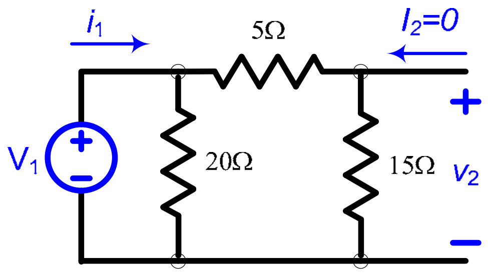

Fig. 42-4a Circuit for finding z_{11} an z_{21}

$$z_{11}=\left.\frac{V_1}{I_1}\right|_{I_2=0}=\frac{20\times 20}{40}=10\Omega,$$

When I_2 is zero, $V_2=\frac{V_1}{15+5}15=0.75V_1$ therefore $z_{21}=\left.\frac{V_2}{I_1}\right|_{I_2=0}=\frac{0.75V_1\times 20}{V_1/10}=7.5\Omega$

To obtain z_{12} and z_{22} we connect a voltage V_2 (or a current source I_2) to port 2 with port 1 open circuited as in Fig. 42-4b.

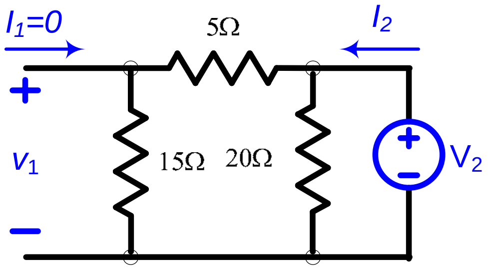

Fig. 42-4b Circuit for finding z_{12} an z_{22}

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{15\times 25}{40} = 9.375\Omega,$$

When I_1 is zero, $V_1 = \frac{V_2}{5+20}(20) = 0.8V_2$ and $I_2 = \frac{V_2}{9.375}$ hence $z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{0.8V_2}{V_2/9.375} = 7.5\Omega$

Note that each of these parameters is the ratio of a voltage to a current and therefore is an impedance with the dimension of ohms; this is why they are called z-parameters.

When $z_{11} = z_{22}$, the two-port network is said to be symmetrical.

When the two-port network is linear and has no dependent sources, the transfer impedances are equal (z_{12}=z_{21}), and the two-port network is said to be reciprocal.

y-parameters:

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

or in matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (42\text{-}9)$$

The values of the parameters can be evaluated by setting $V_1=0$ (input port short-circuited) or $V_2=0$ (output port short-circuited). Thus,

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}, \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$
$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}, \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \qquad (42\text{-}10)$$

The y-parameters are also called the short-circuit admittance parameters:

- y_{11}= Short-circuit input admittance
- y_{12}= Short -circuit transfer admittance from port 2 to port 1
- y_{21}= Short -circuit transfer admittance from port 1 to port 2
- y_{22}= Short -circuit output admittance

Example 42-2 illustrates the determination of the y-parameters for a resistive circuit.

Example 42-2

Obtain the y-parameters for the resistive circuit shown in Fig. 42-5

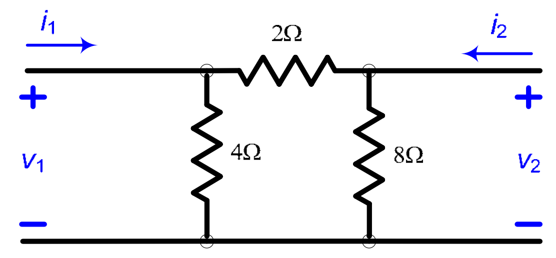

Fig. 42-5 The Circuit for Example 42-2

Solution:

To obtain y_{11} and y_{21} we connect a current I_1 (or a voltage source V_1) to input port 1 with output port 2 short circuited as in Fig. 42-6a.

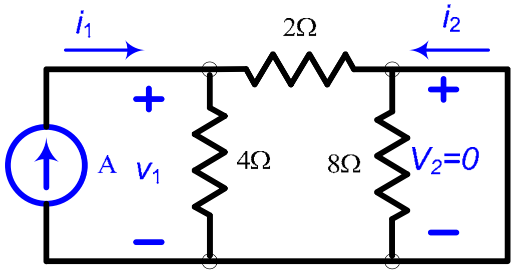

Fig. 42-6a Finding y_{11} and y_{21} for Example 42-2

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{I_1}{I_1(4\|2)}\bigg|_{V_2=0} = \frac{I_1}{\frac{4}{3}I_1}\bigg|_{V_2=0} = 0.75S,$$

When V_2 is zero, $-I_2 = \frac{4}{4+2}I_1 = \frac{2}{3}I_1$ and $V_1 = \frac{4}{3}I_1$ hence $y_{21} = \frac{I_2}{V_1}\bigg|_{I_2=0} = \frac{-\left(\frac{2}{3}\right)I_1}{\left(\frac{4}{3}\right)I_1}\bigg|_{I_2=0} = -0.5S$

To obtain y_{12} and y_{22} we connect a current source I_2 (or a voltage source V_2) to port 2 with port 1 short circuited as in Fig. 42-6b.

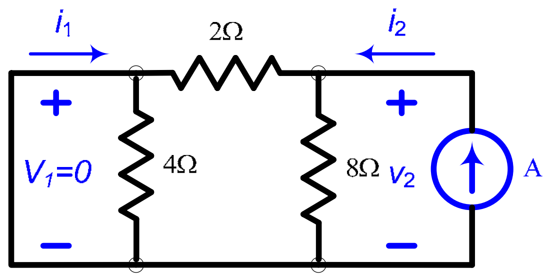

Fig. 42-6b Finding y_{12} and y_{22} for Example 42-2

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \left.\frac{I_2}{I_2(8\|2)}\right|_{V_2=0} = \left.\frac{I_2}{\frac{8}{5}I_2}\right|_{V_1=0} = 0.625S$$

When V_1 is zero, $-I_1 = \frac{I_2}{8+2}(8) = 0.8I_2$ and $V_2 = \frac{8I_2}{5}$ hence $y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \left.\frac{-0.8I_2}{1.6I_2}\right|_{V_1=0} = -0.5S$

Note that each of these parameters is the ratio of a current to a voltage and therefore is an admittance with the dimension of siemens; this is why they are called y-parameters.

Self Test 42:

a) Determine the z-parameters for the circuit in Fig. 42-7
b) Determine the y-parameters for the circuit in Fig. 42-8

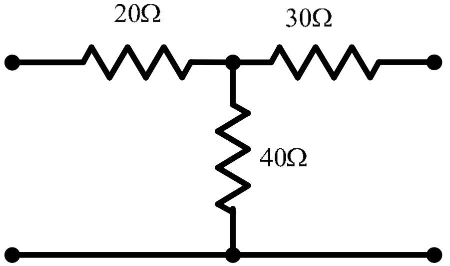

Fig. 42-7 Circuit for self test 42a

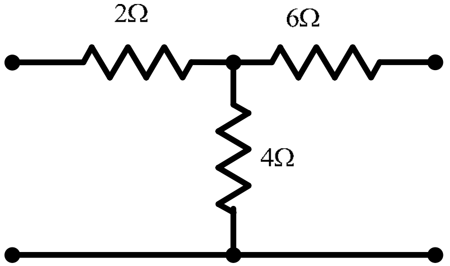

Fig. 42-8 Circuit for self test 42b

Answer:

a) $z_{11} = 60\Omega \quad z_{12} = 40\Omega \quad z_{22} = 70\Omega \quad z_{21} = 40\Omega$

b) $y_{11} = 0.2273S \quad y_{12} = y_{21} = -0.0909S \quad y_{22} = 0.1364S$