Electric Circuits II

Frequency Selective Circuits (Filters)
Low Pass Filter

Lecture #36

The material to be covered in this lecture is as follows:

- Introduction to the Low Pass filter
- The Series RL Circuit- Qualitative Analysis
- Defining The Cutoff Frequency
- The Series RL Circuit- Quantitative Analysis

After finishing this lecture you should be able to:

- Understand the Behavior of Low-Pass Filters
- Understand the cutoff frequency.
- Relate cutoff frequency to component values
- Analyze Qualitatively And Quantitatively A Low-Pass Filter
- Design A Simple Low-Pass Filter

Introduction to Low Pass Filter

- Filters that will be considered here and in following lectures are passive filters.
- We will examine in this and next lecture two circuits that behave as low-pass filters Fig. 36-1
    - Series RL Circuit
    - Series RC Circuit

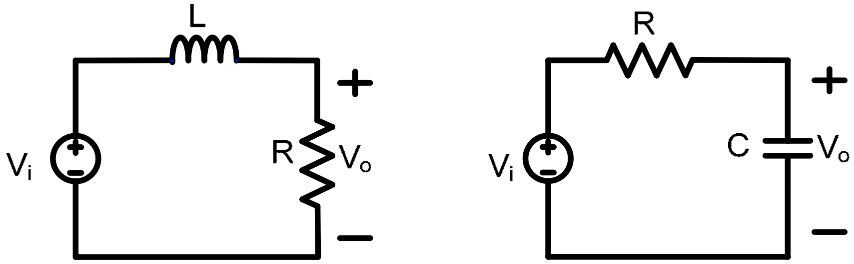

Fig. 36-1 Two Circuits that behave as Low-Pass filter

- Discover what characteristics of these circuits determine the cutoff frequency.
- Note across which element the output signal is taken for the above two circuits

The Series RL Circuit – Qualitative Analysis

- Let us start with a Low-Pass Filter, series RL Circuit shown in Fig. 36-2a
- Assume the circuit's input is a sinusoidal voltage source with varying frequency
- The circuit's output is the voltage across R.
- Behavior of R will not change with changing frequency
- Behavior of inductor L will change with changing frequency

- Recall that the impedance of an inductor is $j\omega L$ at low frequencies the inductor's impedance is very small compared with the resistor's impedance,
- The inductor effectively functions as a short circuit. The term low frequencies thus refers to any frequencies for which $\omega L \ll R$
- The equivalent circuit for $\omega = 0$ is shown in Fig. 36-2b
- At this frequency $V_i = V_o$ both in magnitude and phase angle

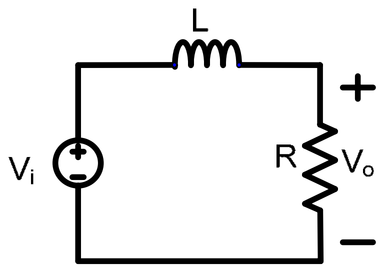

Fig. 36-2a A Series RL Low-Pass filter

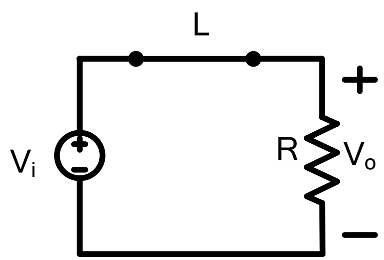

Fig. 36-2b A Series RL Low-Pass filter at $\omega = 0$

The Series RL Circuit – Qualitative Analysis (cont)

- Increasing the frequency causes the inductor's impedance to increase.
- Increasing the inductor's impedance causes a corresponding increase in the magnitude of the voltage drop across L
- Increasing the inductor's impedance causes a corresponding decrease in the magnitude of V_o the output voltage drop across R
- Increasing the inductor's impedance also introduces a shift in phase angle between V_L the voltage across the inductor and I_L the current in the inductor
- This results in a phase angle difference between the input and output voltage, V_o lags V_i as the frequency increases this phase lag approaches 90°
- The inductor effectively functions as an open circuit. The term high frequencies thus refers to any frequencies for which $\omega L \gg R$
- The equivalent circuit for $\omega = \infty$ is shown in Fig. 36-2c

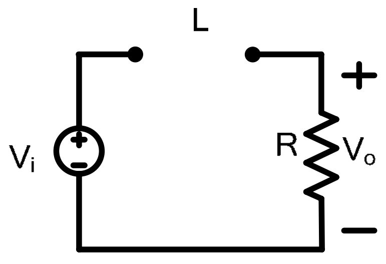

Fig. 36-2c A Series RL Low-Pass filter at $\omega = \infty$

The Series RL Circuit – Qualitative Analysis (cont)

- ✓ Based on the behavior of the output voltage magnitude at high frequencies $V_o = 0$ at $\omega = \infty$ this series RL circuit selectively passes low-frequency inputs to the output.
- ✓ This circuit response is shown in Fig. 36-3
- ✓ The ideal filter exhibits a discontinuity in magnitude at the cutoff frequency ω_C
- ✓ Not possible to use real components to construct a circuit that has an abrupt transition in magnitude. The change is gradually for a real circuit. So what do we mean by cutoff frequency?

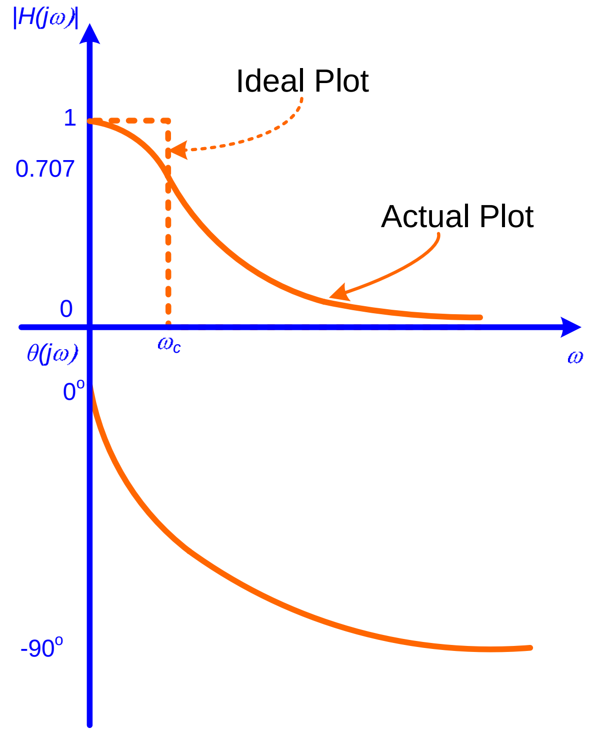

Fig. 36-3 Frequency Response for Series RL Circuit in Fig. 36-2a

Defining the cutoff frequency

- The transition between Passband and Stopband is not abrupt so there is not a single frequency ω_C but a band of transition.
- Therefore we need to define the cutoff frequency.

The definition for cutoff frequency widely used by electrical engineers is the frequency for which the transfer function magnitude is decreased by the factor $\frac{1}{\sqrt{2}}$ from its maximum value.

$$\left|H\left(j\omega_C\right)\right| = \frac{1}{\sqrt{2}} H_{\max} \qquad (36\text{-}1)$$

Recall that the average power delivered by any circuit to a load is proportional to V_L^2 where is the amplitude of the voltage drop across the load:

$$P = \frac{1}{2}\frac{V_L^2}{R} \qquad (36\text{-}2)$$

Define P_{max} as the value of the average power delivered to a load when the magnitude of the load voltage is maximum:

$$P_{max} = \frac{1}{2}\frac{V_{Lmax}^2}{R} \qquad (36\text{-}3)$$

Defining the cutoff frequency (cont.)

If we vary the frequency of the source voltage V_i the load voltage is a maximum when the magnitude of the circuit's transfer function is also a maximum:

$$V_{L\max} = H_{\max}\left|V_i\right| \tag{36-4}$$

When the frequency is equal to ω_C the magnitude of the load voltage is:

$$\left|V_L\left(j\omega_C\right)\right| = \left|H\left(j\omega_C\right)\right|\left|V_i\right|$$

$$= \frac{1}{\sqrt{2}} H_{\max}\left|V_i\right|$$

$$= \frac{1}{\sqrt{2}} V_{L\max} \tag{36-5}$$

Substituting equ. (36-5) into equ. (36-2)

$$P(j\omega_c) = \frac{1}{2}\frac{V_L^2(j\omega_c)}{R}$$

$$= \frac{1}{2}\frac{\left(\frac{1}{\sqrt{2}}V_{L\max}\right)^2}{R}$$

Defining the cutoff frequency (cont.)

$$= \frac{1}{2}\frac{V_{L\max}^2/2}{R}$$

$$= \frac{P_{\max}}{2} \qquad (36\text{-}6)$$

Therefore at the cutoff frequency ω_C, the average power delivered by the circuit is one half the maximum average power. Thus ω_C is also called the half-power frequency, or the corner frequency.

The Series RL Circuit – Quantitative Analysis

- Consider the circuit of Fig. 36-4

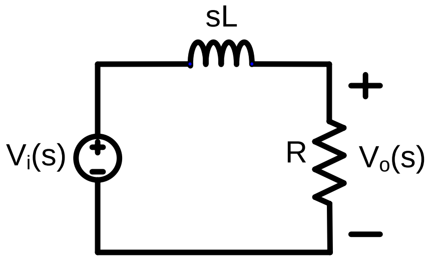

Fig. 36-4 s-domain equivalent for the Circuit of Fig. 36-2a

The voltage transfer function for this circuit is: $$H(s)=\frac{\frac{R}{L}}{s+\frac{R}{L}} \qquad (36\text{-}7)$$

To study the frequency response we replace $s=j\omega$ in equation (36-7): $$H(j\omega)=\frac{\frac{R}{L}}{j\omega+\frac{R}{L}} \qquad (36\text{-}8)$$

The Series RL Circuit – Quantitative Analysis (cont)

Equation (36-8) expressed in polar form gives the magnitude and phase:

$$\left|H(j\omega)\right| = \frac{\frac{R}{L}}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}} \qquad (36\text{-}9)$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right) \qquad (36\text{-}10)$$

Close examination of these equations provide quantitative support for the plots seen in Fig. 36-3. We can now compute ω_C in terms of the circuit elements:

Recall from (36-1) that $\left|H(j\omega_C)\right| = \frac{1}{\sqrt{2}} H_{\max}$, for low-pass filter, $H_{\max} = \left|H(j0)\right|$ as seen in Fig. 36-3 thus: $\left|H(j\omega_C)\right| = \frac{1}{\sqrt{2}}|1| = \frac{\frac{R}{L}}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}$ (36-11), solving we get: $\omega_C = \frac{R}{L}$ (36-12).

Example 36.1

Choose values for R and a commonly available value of L=100 mH in the circuit in Fig. 36-2a, such that the circuit could be used in an EG (electrograph) to filter noise above 10 Hz and pass the electric signals from the heart at or near 1 Hz. Compute the magnitude of V_o at 1 Hz, 10 Hz, and 60 Hz to see how well the filter performs, take V_i=1 V.

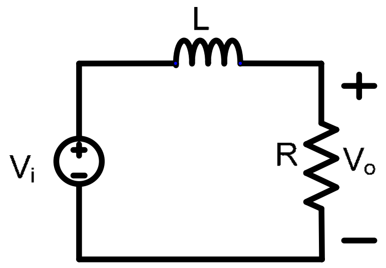

Fig. 36-2a A Series RL Low-Pass filter

Answer:

The problem is to design a filter with a cutoff frequency of f_c=10 Hz.

$$\omega_C = 2\pi(10) = 20\pi \, {}^{rad}\!/\!_{s}$$

Solving for R from equ. (36-12) $R = \omega_C L = 20\pi\left(100\times10^{-3}\right) = 6.8\Omega$ and $|V_o| = |H(j\omega)||V_i|$ we get:

$$\left|V_o(\omega)\right| = \frac{\frac{R}{L}}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}\left|V_i\right| = \frac{20\pi}{\sqrt{\omega^2 + 400\pi^2}}\left|V_i\right|$$

For f_c=1 Hz, $\left|V_o\right| = 0.995V$ almost equal to the input voltage as expected

For f_c=10 Hz, $\left|V_o\right| = 0.707V$ equal to $= \frac{1}{\sqrt{2}}V_i$ as 10 Hz is the cutoff frequency.

For f_c=60 Hz, $\left|V_o\right| = 0.164V$ very small compared to the input so the filter is performing well.

Self Test 36:

A series RL low-pass filter with a cutoff frequency of $\omega_C = 2kHz$ is needed. Using $R = 5k\Omega$, compute:

a) L;

b) $H(j\omega)$ at 50 kHz; and

c) $\theta(j\omega)$ at 50 kHz

Answer:

a) 0.40 H
b) 0.04;
c) -87.71^o