Electric Circuits II

The Transformer

Lecture #33

The material to be covered in this lecture is as follows:

- Introduction to Transformers
- Analysis of a Linear Transformer Circuit
- Reflected Impedance

After finishing this lecture you should be able to:

- Construct the frequency domain equivalent circuit of a transformer
- Analyze a linear transformer circuit
- Calculate the reflected impedance into the primary
- Calculate the scaling factor for the reflected impedance

Introduction

- A Transformer is a magnetic device that takes advantage of the phenomenon of mutual inductance.
- A Transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.
- A Transformer may also be regarded as one whose flux is proportional to the currents in its windings.

Consider the following general circuit model for a transformer used to connect a load to a source.

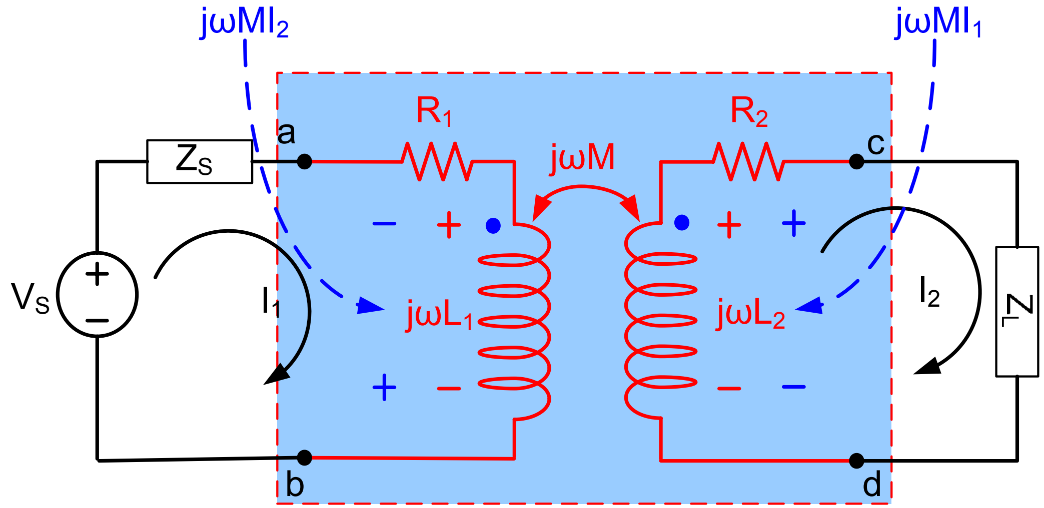

Fig.33-1 Frequency-domain circuit model for a Transformer

Introduction (cont)

Terminology

- The transformer winding connected to the source is the primary winding.
- The transformer winding connected to the load is the secondary winding.

Where:

- $R_1 \rightarrow$ The resistance of the primary winding
- $R_2 \rightarrow$ The resistance of the secondary winding
- $L_1 \rightarrow$ The self inductance of the primary winding
- $L_2 \rightarrow$ The self inductance of the secondary winding
- $M \rightarrow$ The mutual inductance
- $V_S \rightarrow$ The internal Voltage of the sinusoidal source
- $Z_S \rightarrow$ The internal impedance of the sinusoidal source
- $Z_L \rightarrow$ The Load impedance.

Analysis of a linear transformer circuit

The analysis of the circuit consists of finding I_1 and I_2 as a function of circuit parameters: R_1, R_2, L_1, L_2, M, V_S, Z_S, Z_L and ω.

Mesh 1 $$V_S = (Z_S + R_1 + j\omega L_1)I_1 - j\omega M I_2 \qquad (33\text{-}1)$$

Mesh 2 $$0 = -j\omega M I_1 + (Z_L + R_2 + j\omega L_2)I_2 \qquad (33\text{-}2)$$

For simplification, let

$$Z_{11} = R_1 + j\omega L_1 + Z_S$$ Self impedance of the primary (33-3)

$$Z_{22} = R_2 + j\omega L_2 + Z_L$$ Self impedance of the secondary (33-4)

Thus,

$$\mathbf{V}_S = \mathbf{Z}_{11}\bar{I}_1 - j\omega M I_2 \qquad (33\text{-}5)$$

$$0 = -j\omega M \mathbf{I}_1 + \mathbf{Z}_{22}\mathbf{I}_2 \qquad (33\text{-}6)$$

From (33-6) yields: $$\mathbf{I}_2 = \frac{j\omega M}{\mathbf{Z}_{22}}\mathbf{I}_1 \qquad (33\text{-}7)$$

Substitute in (33-5) yields

$$\mathbf{V}_S = \frac{\mathbf{Z}_{11}\mathbf{Z}_{22} + \omega^2 M^2}{\mathbf{Z}_{22}}\mathbf{I}_1 = \left(\mathbf{Z}_{11} + \frac{\omega^2 M^2}{\mathbf{Z}_{22}}\right)\mathbf{I}_1 \qquad (33\text{-}8)$$

Therefore

$$\mathbf{I}_1 = \frac{\mathbf{Z}_{22}}{\mathbf{Z}_{11}\mathbf{Z}_{22} + \omega^2 M^2}\mathbf{V}_S$$ Primary Current (33-9)

$$\mathbf{I}_2 = \frac{j\omega M}{\mathbf{Z}_{11}\mathbf{Z}_{22} + \omega^2 M^2}\mathbf{V}_S = \frac{j\omega M}{\mathbf{Z}_{22}}\mathbf{I}_1$$ Secondary Current (33-10)

The impedance at the terminals of the source is:

$\mathbf{Z}_{int} - \mathbf{Z}_S$ so,

$$\mathbf{Z}_{ab} = \mathbf{Z}_{11} + \frac{\omega^2 M^2}{\mathbf{Z}_{22}} - \mathbf{Z}_S = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$ Impedance looking into the primary (33-11)

Note: The impedance Z_{ab} is independent of the magnetic polarity of the transformer.

Reflected Impedance

The quantity $\mathbf{Z}_r = \dfrac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$ is called the Reflected Impedance (33-12)

This is due solely to the existence of mutual inductance.
To consider reflected impedance in more detail, we express the load impedance in rectangular form:

$$\mathbf{Z}_L = R_L + j\mathbf{X}_L \qquad (33\text{-}13)$$

where X_L is positive for inductive load and negative for capacitive load.

Z_r in rectangular form will be:

$$\mathbf{Z}_r = \frac{\omega^2 M^2\left[\left(R_L + R_2\right) - j\left(\omega L_2 + \mathbf{X}_L\right)\right]}{\left(R_L + R_2\right)^2 + \left(\omega L_2 + \mathbf{X}_L\right)^2} \qquad (33\text{-}14)$$

$$\mathbf{Z}_r = \frac{\omega^2 M^2}{\left|\mathbf{Z}_{22}\right|^2}\left[\left(R_L + R_2\right) - j\left(\omega L_2 + \mathbf{X}_L\right)\right] = \frac{\omega^2 M^2}{\left|\mathbf{Z}_{22}\right|^2}\mathbf{Z}_{22}^* \qquad (33\text{-}15)$$

Therefore the Linear Transformer reflects the conjugate of the self-impedance of the secondary circuit $\left(\mathbf{Z}_{22}\right)^*$ into the primary circuit.

The coefficient $\dfrac{\omega^2 M^2}{\left|\mathbf{Z}_{22}\right|^2}$ is called the scaling factor of the reflected impedance.

Example 33-1

Calculate the reflected impedance, the scaling factor, the primary and secondary currents for the Circuit of Fig.33-2:

Parameters of the transformer

$R_2 = 40\,\Omega, \quad X_1 = 400\,\Omega,$

$R_L = 360\,\Omega, \quad X_L = 200\,\Omega,$

$R_1 = 100\,\Omega, \; X_2 = 100\,\Omega, \quad \omega M = 80\,\Omega,$

$Z_S = 184 + j0\,\Omega, \; V_S = \dfrac{245}{\sqrt{2}}\angle 0^\circ\ \text{V}$

$M = k\sqrt{L_1 L_2}$

$M = 0.4\sqrt{(0.125)(0.5)} = 0.1 \Rightarrow \omega = 800\ \text{rad/s}$

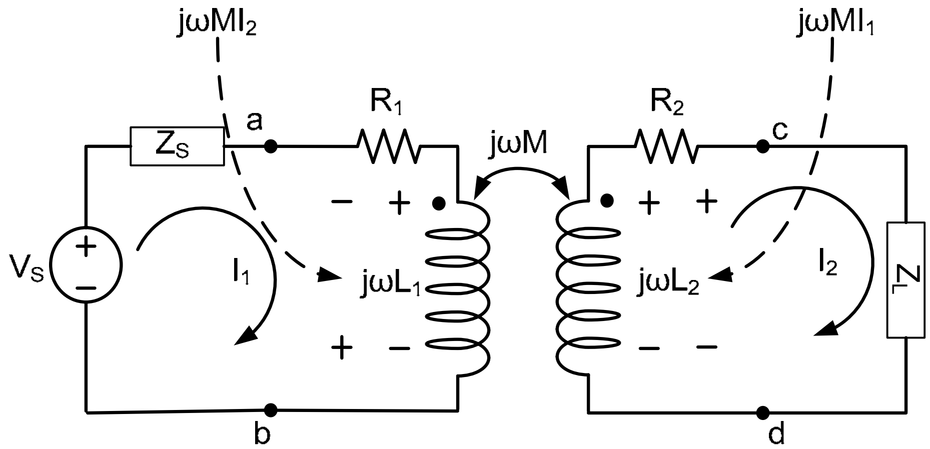

Fig.33-2 Circuit for Example 33-1

The reflected impedance:

Substituting equation (33-4) in equation (33-15) we get

$$Z_{22} = R_2 + j\omega L_2 + Z_L = (R_L + R_2) + j(X_2 + X_L) = 400 + j300$$

$|Z_{22}| = |400 + j300| = 500\Omega$ and $Z_r = \dfrac{\omega^2 M^2}{|Z_{22}|^2}\left[(R_L + R_2) - j(\omega L_2 + X_L)\right]$

Example 33-1(cont)

therefore

$$Z_r = \frac{80^2}{500^2}[400 - j300] = 1.6^2 \times 4 - j1.6^2 \times 3 = 10.24 - j7.68\ \Omega$$

The scaling factor by which $\mathbf{Z}_{22}^{*}$ is reflected is $\frac{80^2}{500^2} = \frac{64}{2500} = 0.0256$

The primary current I_1 is given by equation 33-9 as:

$$\mathbf{I}_1 = \frac{\mathbf{Z}_{22}}{\mathbf{Z}_{11}\mathbf{Z}_{22} + \omega^2 M^2}\mathbf{V}_S$$

where $Z_{11} = R_1 + j\omega L_1 + Z_S = 184 + 100 + j400 = 284 + j400\Omega$ and $Z_{22} = 400 + j300$

Therefore $\mathbf{I}_1 = \frac{400 + j300}{(284 + j400)(400 + j300) + 80^2}\frac{245}{\sqrt{2}}\angle 0^\circ$

$$= \frac{500\angle 36.87^\circ}{245280\angle 91.53^\circ + 6400}\frac{245}{\sqrt{2}}\angle 0^\circ$$

$$= \frac{0.5}{\sqrt{2}}\angle -53.13^\circ\ \text{A}$$

In the time domain, $i_1 = 0.5\cos\left(800t - 53.13^\circ\right)\ \text{A}$

Example 33-1(cont)

The secondary current $\mathbf{I}_2$ is given by equation 33-10 as:

$$\mathbf{I}_2 = \frac{j\omega M}{\mathbf{Z}_{22}}\mathbf{I}_1$$

where $Z_{22} = 400 + j300$ and $\omega M = 80$

Therefore $\mathbf{I}_2 = \frac{80\angle 90^\circ}{500\angle 36.87^\circ}\frac{0.5}{\sqrt{2}}\angle -53.13^\circ$

$$\mathbf{I}_2 = \frac{80}{500\sqrt{2}} = \frac{0.08}{\sqrt{2}}\ \mathrm{A}$$

In the time domain, $i_2 = 0.08\cos 800t\ \mathrm{A}$

Self Test:

For the following circuit of Fig.33-3

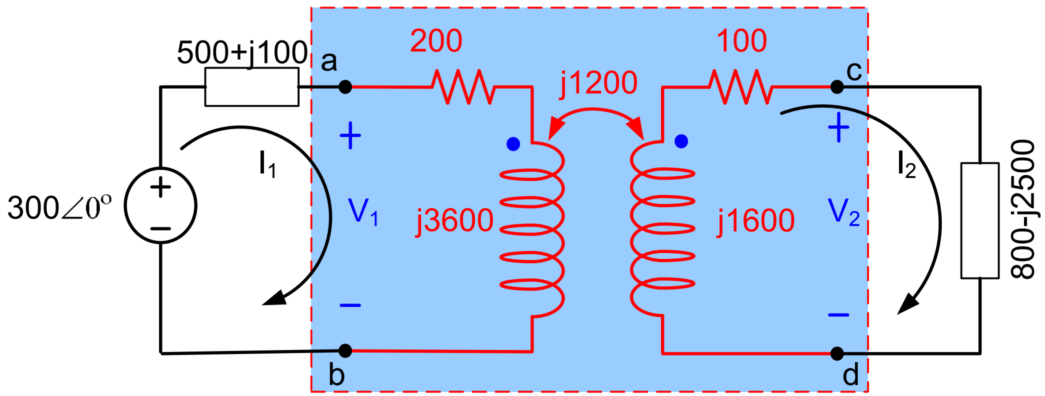

Fig.33-3 for SelfTest

- What is the self-impedance of the primary?
  a) 500+j100
  b) 700+j3700
  c) 200+j3600
  d) 1000+j4400

  Correct answer b
- What is the self-impedance of the secondary?
  a) 900-j3700
  b) 800-j2500
  c) 900-j900
  d) 100+j1600

  Correct answer c
- What is the impedance reflected into the primary?
  a) 800-j2500
  b) 100+j1600
  c) 1000+j4400
  d) 800+j800

  Correct answer d
- What is the impedance seen looking into the primary terminals?
  a) 700+j4800
  b) 200+j3600
  c) 1000+j4400
  d) 800+j800

  Correct answer c