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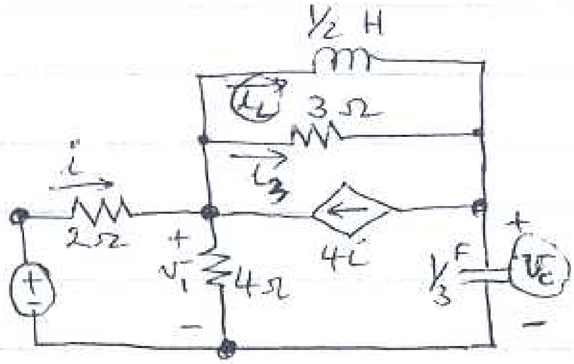
EE205 Major II

Problem I



state variables i_L & v_c

$$\frac{1}{2}\frac{di_L}{dt} + v_c - v_1 = 0$$

$$\frac{di_L}{dt} = -2v_c + 2v_1$$

$$\frac{1}{3}\frac{dv_c}{dt} + 4i = i_L + i_3$$

$$\frac{dv_c}{dt} = 3i_L + 3i_3 - 12i$$

we have $\quad 2i + v_1 = v_s$

$$3i_3 - v_1 = v_c$$

$i + 4i = i_L + i_3 - \frac{v_1}{4}$ ie $20i - 4i_3 - 4i_L + v_1 = 0$

or $\quad 20i - 4i_3 + v_1 = 4i_L$

$$\Delta = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 3 & -1 \\ 20 & -4 & +1 \end{vmatrix} = -74$$

$$i = \begin{vmatrix} v_s & 0 & 1 \\ -v_c & 3 & -1 \\ 4i_L & -4 & 1 \end{vmatrix} \Big/ -74 = \frac{6i_1}{37} - \frac{2}{37}v_c + \frac{7}{74}v_s$$

$$i_3 = \begin{vmatrix} 2 & v_s & 1 \\ 0 & -v_c & -1 \\ 20 & 4i_L & 1 \end{vmatrix} \Big/ -74 = \frac{-8}{74}i_1 - \frac{22}{74}v_c + \frac{20}{74}v_s$$

$$v_1 = \begin{vmatrix} 2 & 0 & v_s \\ 0 & 3 & -v_c \\ 20 & -4 & 4i_L \end{vmatrix} \Big/ -74 = -\frac{24}{74}i_1 + \frac{8}{74}v_c + \frac{60}{74}v_s$$

$$\frac{di_L}{dt} = -2v_c + 2v_1 = \frac{-24}{37}i_1 - \frac{66}{37}v_c + \frac{60}{37}v_s$$

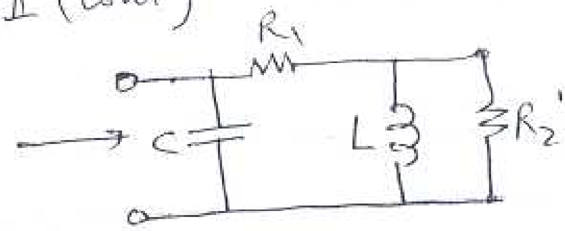
$$\frac{dv_c}{dt} = 3i_L + 3\left(\frac{-8}{74}i_1 - \frac{22}{74}v_c + \frac{20}{74}v_s\right) - 12\left(\frac{6}{37}i_1 - \frac{2}{37}v_c + \frac{7}{74}v_s\right)$$

$$= \frac{27}{37}i_1 - \frac{9}{37}v_c - \frac{12}{37}v_s$$

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_c \end{bmatrix} = \begin{bmatrix} -\frac{24}{37} & -\frac{66}{37} \\ \frac{27}{37} & -\frac{9}{37} \end{bmatrix}\begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{60}{37} \\ -\frac{12}{37} \end{bmatrix} v_s$$

EE205 MII (cont)

Prob II ⓐ



$$Y = j\omega C + \frac{1}{Z_1}, \quad Z_1 = R_1 + \frac{j\omega L R_2}{R_2 + j\omega L}$$

$$= j\omega C + \frac{R_2 + j\omega L}{R_1R_2 + j\omega L(R_1+R_2)}$$

$$= \frac{j\omega C[R_1R_2 + j\omega L(R_1+R_2)] + R_2 + j\omega L}{R_1R_2 + j\omega L(R_1+R_2)}$$

$$= \quad " \quad \times \frac{R_1R_2 - j\omega L(R_1+R_2)}{R_1R_2 - j\omega L(R_1+R_2)}$$

$$= \frac{R_1R_2[R_2 - \omega^2 LC(R_1+R_2)] + \omega^2 L(R_1+R_2)(L+R_1R_2C) + j\omega(L+R_1R_2C)R_1R_2 - j\omega L(R_1+R_2)[R_2 - \omega^2 LC(R_1+R_2)]}{(R_1R_2)^2 + [\omega L(R_1+R_2)]^2}$$

At resonance $\omega(L+R_1R_2C)R_1R_2 - \omega L(R_1+R_2)[R_2 - \omega^2 LC(R_1+R_2)] = 0$

ie for nonzero ω:

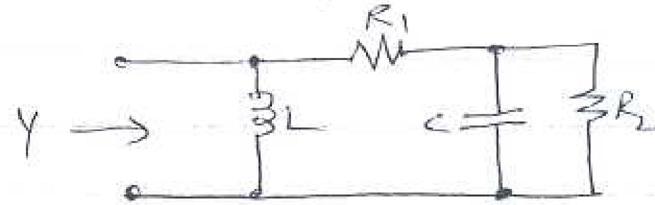
$$R_1R_2(L+R_1R_2C) - LR_2(R_1+R_2) + \omega^2 L^2 C(R_1+R_2)^2 = 0$$

ie $\omega^2 L^2 C(R_1+R_2)^2 + (R_1R_2)^2 C - LR_2^2 = 0$

or $\omega^2 = \dfrac{LR_2^2 - (R_1R_2)^2 C}{L^2 C(R_1+R_2)^2} = \left(\dfrac{R_2}{R_1+R_2}\right)^2 \left(\dfrac{1}{LC} - \dfrac{R_1^2}{L^2}\right)$

$$\therefore \quad \omega_r = \frac{R_2}{R_1+R_2}\sqrt{\frac{1}{LC} - \frac{R_1^2}{L^2}}$$

Prob II (b)



$$Y = \frac{1}{j\omega L} + \frac{1}{Z_1} \quad , \quad Z_1 = R_1 + \frac{R_2}{1+j\omega C R_2} = \frac{R_1+R_2+j\omega C R_1 R_2}{1+j\omega C R_2}$$

$$= \frac{(R_1+R_2+j\omega C R_1R_2) + j\omega L(1+j\omega C R_2)}{j\omega L(R_1+R_2+j\omega C R_1 R_2)}$$

$$= \frac{(R_1+R_2+j\omega C R_1R_2) + j\omega L(1+j\omega C R_2)}{-\omega^2 LCR_1R_2 + j\omega L(R_1+R_2)} \times \frac{-\omega^2 LCR_1R_2 - j\omega L(R_1+R_2)}{-\omega^2 LCR_1R_2 - j\omega L(R_1+R_2)}$$

$$= \frac{\Re + j\omega(L+CR_1R_2)(-\omega^2 LCR_1R_2) - j\omega L(R_1+R_2)(R_1+R_2-\omega^2 LCR_2)}{\text{Real denominator.}}$$

at resonance (for non zero ω.

$$(CR_1R_2+L)(-\omega^2 LCR_1R_2) - L(R_1+R_2)(R_1+R_2-\omega^2 LCR_2) = 0$$

or $$-\omega^2 CR_1R_2(L+CR_1R_2) + \omega^2 LCR_2(R_1+R_2) = (R_1+R_2)^2$$

$$\omega^2\left[(CR_1R_2)^2 + R_1R_2LC - LCR_2(R_1+R_2)\right] = -(R_1+R_2)^2$$

or $$\omega^2\left[(CR_1R_2)^2 - LCR_2^2\right] = -(R_1+R_2)^2$$

ie $$\omega^2 = \frac{(R_1+R_2)^2}{LCR_2^2 - (R_1R_2C)^2}$$

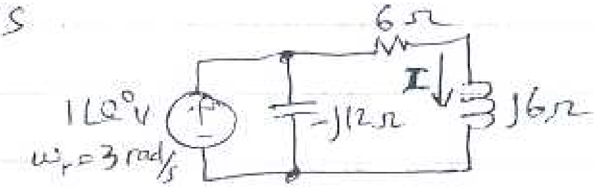
$$\therefore \omega_r = \frac{R_1+R_2}{R_2}\sqrt{\frac{1}{LC - R_1^2C^2}}$$

EE205 MII (cont)

Prob II (c)

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R_1^2}{L^2}}$$ – when there is no R_2 – (open circuit)

$$= \sqrt{18-9} = 3 \text{ rad/s}$$



$$I = \frac{1\angle 0}{6+j6} = \frac{1}{6\sqrt{2}}\angle -45^\circ \text{ A}$$

$$w_C(t) = \frac{1}{2} C v^2(t) = \frac{1}{2}\,\frac{1}{36}\cos^2 3t = \frac{1}{72}\left[\frac{1}{2}(1+\cos 6t)\right]$$

$$w_L(t) = \frac{1}{2} L i^2(t) = \frac{1}{2}\,2\left[\frac{1}{72}\cos^2(3t-45)\right] = \frac{1}{72}\cos^2(3t-45^\circ)$$

$$= \frac{1}{72}\left[\frac{1}{2}\left(1+\cos(6t-90)\right)\right]$$

$$w_C(t) + w_L(t) = \frac{1}{144}\left[1+\cos 6t + 1 + \sin 6t\right]$$

$$= \frac{1}{144}\left[2+\sqrt{2}\cos(6t-45)\right]$$

$$W_{max} = \frac{2+\sqrt{2}}{144} \text{ J}$$

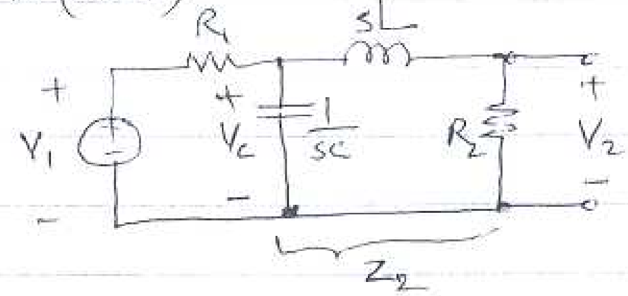
$$P_R = \frac{1}{2} I^2 R = \frac{1}{2}\,\frac{1}{72}\,6 = \frac{1}{24} \text{ W}, \quad T = \frac{2\pi}{3}$$

$$P_R T = \frac{2\pi}{3}\cdot\frac{1}{24} = \frac{2\pi}{72} \text{ J}$$

$$Q = 2\pi \frac{W_{max}}{P_R T} = 2\pi \frac{\frac{2+\sqrt{2}}{144}}{\frac{2\pi}{72}} = \frac{2+\sqrt{2}}{2} = 1 + \frac{1}{\sqrt{2}} = \underline{\underline{1.707}}$$

EE205 MII (Cont)

Prob III



$$Z_2 = \frac{(R_2+sL)\frac{1}{sC}}{R_2+sL+\frac{1}{sC}} = \frac{R_2+sL}{s^2LC+sCR_1+1}$$

$$\frac{V_c}{V_1} = \frac{\frac{R_2+sL}{\cancel{s^2LC+sCR_1+1}}}{\frac{sL+R_2+s^2LCR_1+sCR_1R_2+R_1}{\cancel{s^2LC+sCR_1+1}}} = \frac{sL+R_2}{s^2(R_1LC)+s(R_1R_2C+L)+(R_1+R_2)}$$

zero at $s = -\frac{R_2}{L}$

Poles at solutions of quadratic equation s_1 & s_2.

$$V_2 = \frac{R_2}{sL+R_2} V_c$$

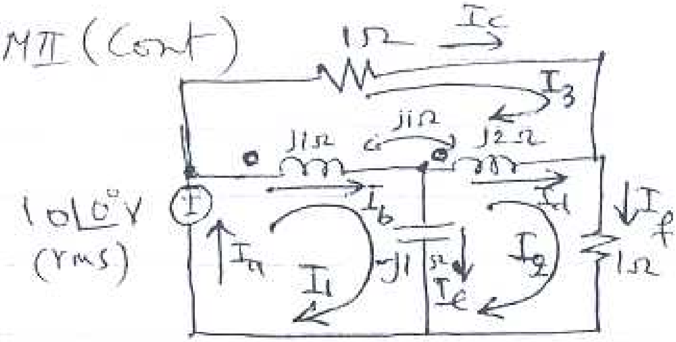
$$\frac{V_2}{V_1} = \frac{V_2}{V_c}\cdot\frac{V_c}{V_1} = \frac{R_2}{\cancel{sL+R_2}}\cdot\frac{\cancel{sL+R_2}}{s^2(R_1LC)+s(R_1R_2C+L)+(R_1+R_2)}$$

$$= \frac{R_2}{s^2(R_1LC)+s(R_1R_2C+L)+(R_1+R_2)}$$

no zeros & same poles as $\frac{V_c}{V_1}$

EE 205 MII (Cont)

Prob IV (a)



$$10\angle 0^\circ = j1(I_1-I_3) + j1(I_2-I_3) - j1(I_1-I_3)$$

$$10\angle 0^\circ = j2 I_2 - j2 I_3$$

$$0 = j2(I_2-I_3) + j(I_1-I_3) - j(I_2-I_1) + I_2$$

$$0 = j2 I_1 + (1+j) I_2 - j3 I_3$$

$$0 = I_3 + j2(I_3-I_2) - j(I_1-I_3) + j(I_3-I_1) - j(I_2-I_3)$$

$$0 = -j2 I_1 - j3 I_2 + (1+j5) I_3$$

$$\Delta = \begin{vmatrix} 0 & j2 & -j2 \\ j2 & 1+j & -j3 \\ -j2 & -j3 & 1+j5 \end{vmatrix} = -j2\left[j2(1+j5) - (j3)(j2)\right] + (-j2)\left[j2(-j3) - (1+j)(-j2)\right]$$

$$= 8$$

$$I_2 = -10\left[j2(1+j5) + 6\right]/8 = \frac{40-j20}{8} = (5-j2.5)\,A$$

$$I_1 = \frac{10}{8}\left[(1+j)(1+j5) + 9\right] = \frac{50+j60}{8} = (6.25+j7.5)\,A$$

$$I_3 = \frac{10}{8}\left[6 + j2(1+j)\right] = (5+j2.5)\,A$$

$\therefore$ $I_a = I_1 = (6.25+j7.5)\,A$, $I_d = I_2 - I_3 = -j5\,A$

$I_b = I_1 - I_3 = (1.25+j5)\,A$, $I_e = I_1 - I_2 = (1.25+j10)\,A$

$I_c = I_3 = (5+j2.5)\,A$, $I_f = I_2 = (5-j2.5)\,A$

EE 205 MII (cont)

Prob IV (b)

$V_a = 10V \quad S_a = -V_a I_a^* = -10(6.25 - j7.5) = (-62.5 + j75)\,VA$

$V_b = j(-I_b) + j I_d = j1.25\,V \Rightarrow S_b = V_b I_b^* = j1.25(1.25 - j5) = (6.25 + j1.56)\,VA$

$V_c = 1\, I_c = (5 + j2.5)\,V \Rightarrow S_c = V_c I_c^* = (31.25 + j0)\,VA$

$V_d = j2 I_d + j I_b = (5 + j1.25)\,V \Rightarrow S_d = V_d I_d^* = (-6.25 + j25)\,VA$

$V_e = -j I_e = (10 - j1.25)\,V \Rightarrow S_e = V_e I_e^* = (-j101.56)\,VA$

$V_f = 1 I_f = (5 - j2.5)V \Rightarrow S_f = V_f I_f^* = (5 - j2.5)(5 + j2.5) = (31.25 + j0)\,VA$

c) $\sum P_{dev} = 62.5\,W, \quad \sum P_{diss} = 31.25 + 31.25 = 62.5\,W$

d) $\sum Q_{dev} = 101.56\,VAR$

$\sum Q_{abs} = 75 + 1.56 + 25 = 101.56\,VAR$