Electric Circuits II

Balanced Three Phase Circuits

Lecture # 03

The material to be covered in this lecture is as follows:

- ❖ Transformation from Delta to Why.
- ❖ Transformation from Why to Delta.
- ❖ The single-phase equivalent circuit.
- ❖ The line and phase current quantities for a Delta circuit.
- ❖ Complete Analysis of Y-Δ circuit.

After finishing this lecture, you should be able to:

- ❖ Transform the three-phase circuit Y to Δ and vice versa.
- ❖ Extract the single-phase equivalent circuit for Y-Δ three-phase circuit.
- ❖ Deduce the line and phase current quantities for Δ connection.
- ❖ Identify the difference between delta and why currents and voltage relations.
- ❖ Conduct a complete analysis for Y-Δ circuit.

Transformation from Delta to Why

- Starting from a Delta connection to reach Wye Connection

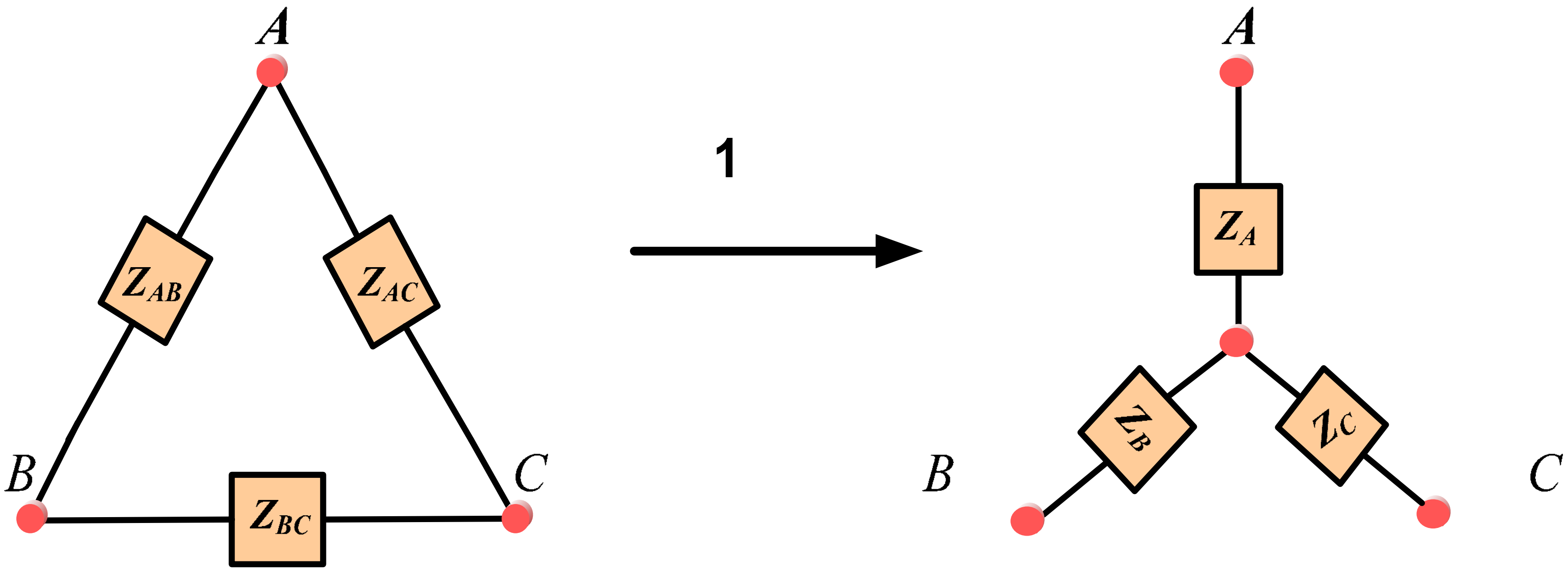

Figure 13 Delta to Wye Transformation circuits

Transformation from Delta to Why (Cont)

$$Z_A = \frac{Z_{AB}.Z_{AC}}{Z_{AB} + Z_{BC} + Z_{AC}}$$

$$Z_B = \frac{Z_{AB}.Z_{BC}}{Z_{AB} + Z_{BC} + Z_{AC}}$$

$$Z_C = \frac{Z_{AC}.Z_{BC}}{Z_{AB} + Z_{BC} + Z_{AC}}$$

- ❖ In case where $$Z_{AB} = Z_{AC} = Z_{BC} = Z$$
- ❖ Then $$Z_A = \frac{Z^2}{3Z} = \frac{1}{3} Z$$
- ❖ So we can deduce that $$Z_Y = \frac{1}{3} Z_\Delta$$

Transformation from Why to Delta

- Starting from a Why connection to reach Delta Connection

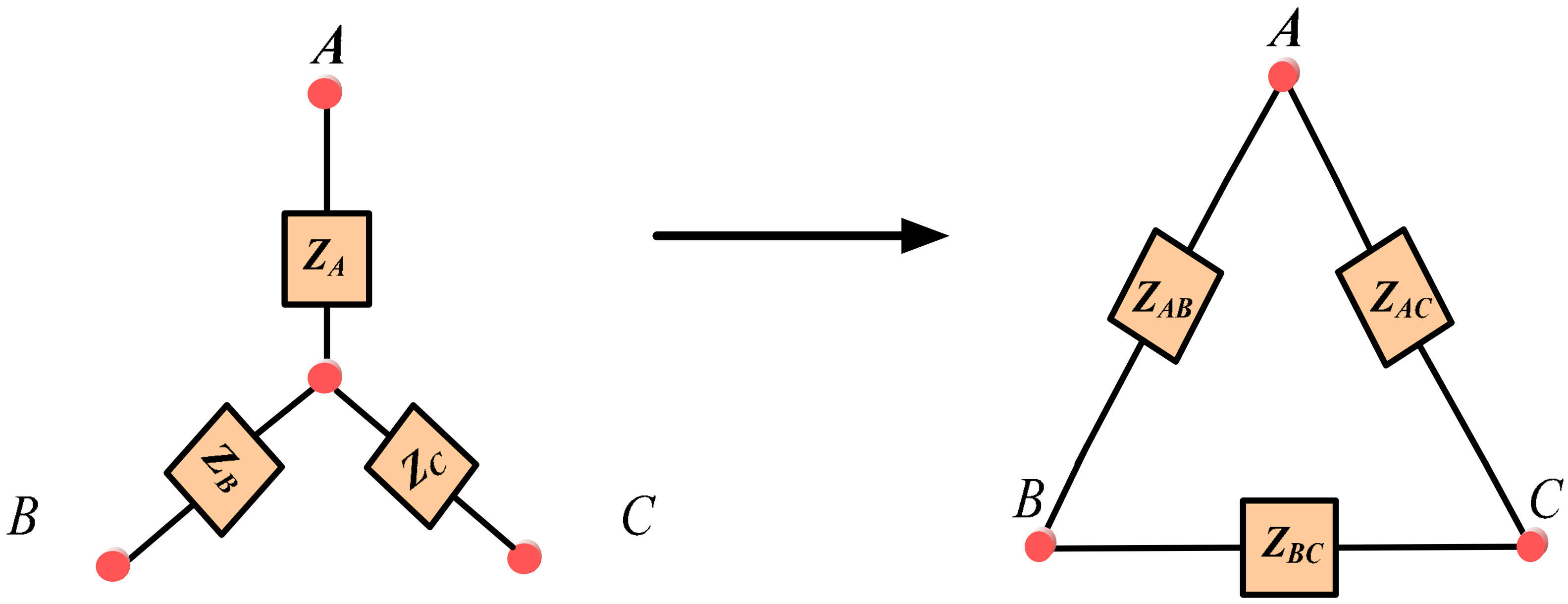

Figure 14 Wye to Delta to Transformation circuits

Transformation from Why to Delta (Cont)

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_C}$$

$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_A}$$

$$Z_{AC} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_B}$$

- In case where $Z_A = Z_B = Z_C = Z$
- Then $Z_{AB} = \frac{3Z^2}{Z} = 3Z$
- So we can deduce that $Z_{AB} = \frac{3Z^2}{Z} = 3Z_Y$

The single-phase equivalent circuit

- The source is Y connected and the load is Delta connected and balanced. So

$$Z_{AB} = Z_{BC} = Z_{CA} = Z_{\Delta}$$

- the conversion of or transformation of Δ to Y transformation gives

$$Z_Y = \frac{Z_\Delta}{3}$$

- After replacing the delta connected load by the equivalent Y connected load, the circuit can be analyzed by the equivalent single phase equivalent circuit.

- After replacing the delta connected load by the equivalent Y connected load, the circuit can be analyzed by the equivalent single phase equivalent circuit.

The single-phase equivalent circuit (cont)

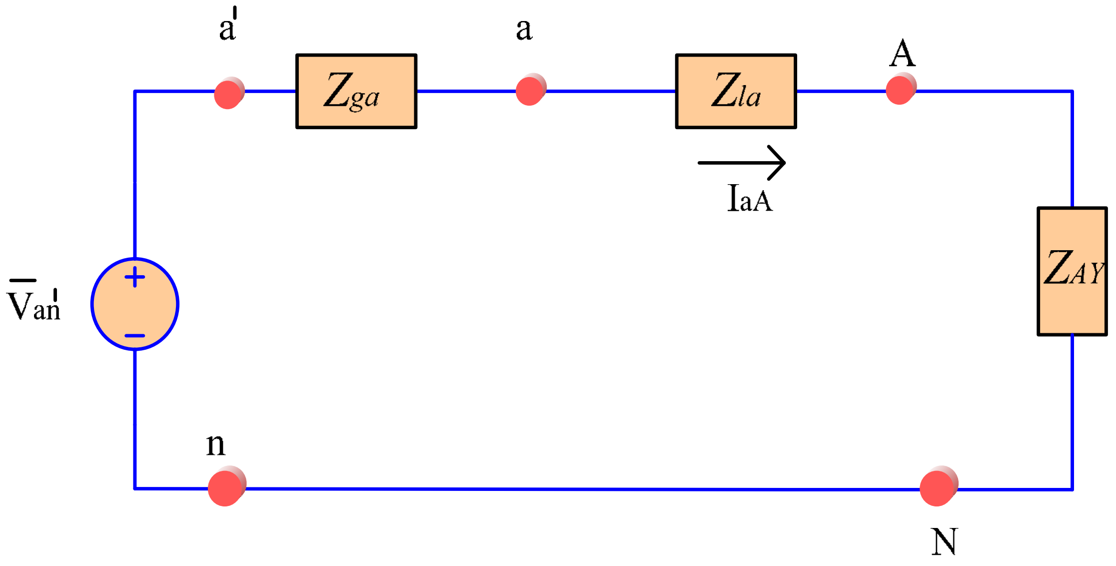

Figure 15 Delta to Wye Load Transformation Single Phase Equivalent Circuit

- This circuit is used to calculate phase voltages, phase currents, line voltages and line current.
- Generators phase and line voltages, in each of the original delta load.

The line and phase current quantities for a Delta circuit

❖ When positive phase sequence is assumed and $I\phi$ is taken for the phase reference current.

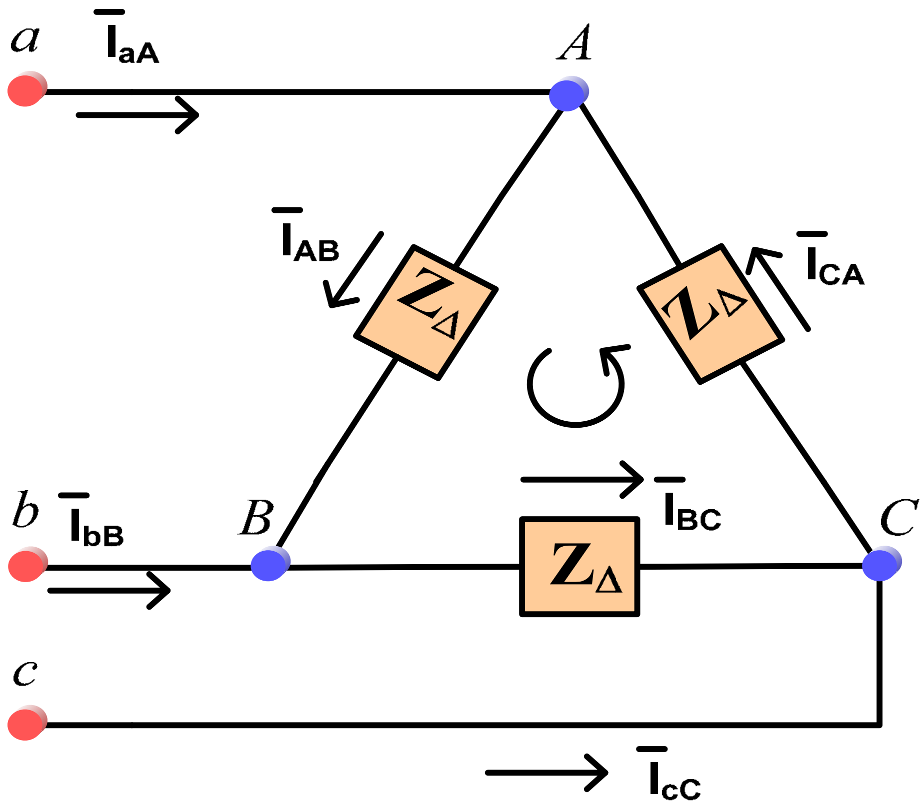

Figure 16 Delta balanced load connections

The line and phase current quantities for a Delta circuit (cont)

- Then
$$\bar{I}_{AB} = I_{\phi}\angle 0^0\, A$$
$$\bar{I}_{BC} = I_{\phi}\angle -120^0\, A$$
$$\bar{I}_{CA} = I_{\phi}\angle +120^0\, A$$

- By KCL at node A we get $\bar{I}_{aA} = \bar{I}_{AB} - \bar{I}_{CA} = I_{\phi}\angle 0^0 - I_{\phi}\angle +120^0 = \sqrt{3} I_{\phi}\angle -30^0 = \sqrt{3}\angle -30^0 \bar{I}_{AB}$

- By KCL at node B we get $\bar{I}_{cB} = \bar{I}_{BC} - \bar{I}_{AB} = \sqrt{3}\angle -30^0 \bar{I}_{BC} = \sqrt{3} I_{\phi}\angle -150^0$

- By KCL at node C we get $\bar{I}_{cC} = \bar{I}_{CA} - \bar{I}_{BC} = \sqrt{3}\angle -30^0 \bar{I}_{CA} = \sqrt{3} I_{\phi}\angle 90^0\, A$

The line and phase current quantities for a Delta circuit (cont)

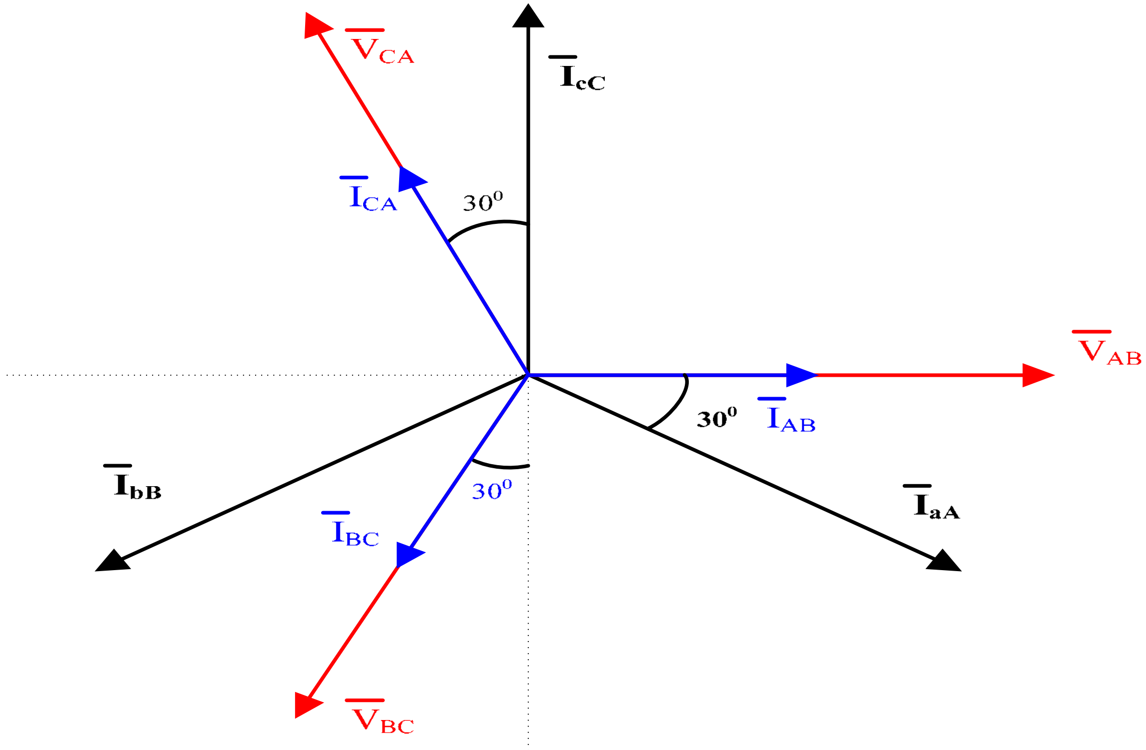

Figure 17 Phasor Diagram for Delta balanced currents and voltages

Complete Analysis of Y-Δ circuit.

❖ The analysis is conducted through the following application example.

Example:

❖ Y connected source load feeds Δ connected load through a distribution line having impedance of

$0.3 + j0.9\Omega/\phi, Z_\Delta = 118.5 + j85.8\Omega/\phi$ et $V_{a'n} = 120\angle 0 V/\phi$ is taken as reference.

❖ The internal per phase impedance is $0.2 + j0.5\Omega/\phi$.

a) Construct the single-phase equivalent circuit.

$$Z_y = \frac{Z_\Delta}{3} = (118.5 + j85.8)/3 = 39.5 + j28.6 = 146.3\angle 35.9/3$$

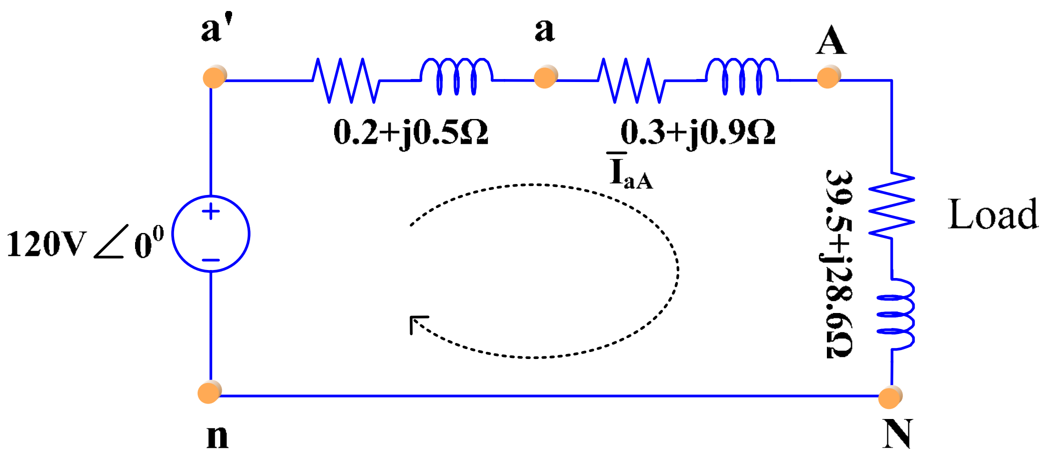

Figure 18 Single Phase Equivalent Converted Circuit .

b) Calculate the line currents $\bar{I}_{aA}, \bar{I}_{bB}, \bar{I}_{cC}$.

$$\bar{I}_{aA} = \frac{120\angle 0V}{(0.2+0.3+39.5)+j(0.5+0.9+28.6)} = \frac{120\angle 0V}{40+j30A}$$

$$\bar{I}_{aA} = 2.4\angle -36.87^0 A, \bar{I}_{bB} = 2.4\angle -156.87^0 A, \bar{I}_{cC} = 2.4\angle -83.13^0 A$$

Example(cont)

c) Calculate the phase voltages at the load terminals.

$$\overline{V}_{AN} = (39.5 + J28.6)(2.4\angle -36.87) = 117.04\angle -0.96^0 V \;,\; \overline{V}_{BN} = 117.04\angle -120.96^0 V$$

$$\overline{V}_{CN} = 117.04 = \angle -119.04A = 117.04 = \angle -240.96^0 V$$

$$\overline{V}_{AB} = \sqrt{3}\angle 30^0 \overline{V}_{AN} = 202.72\angle 29.04^0 V$$

$$\overline{V}_{BC} = 202.72\angle -90.96^0 V, \overline{V}_{CA} = 202.72\angle 149.04^0 V$$

d) Calculate the phase current of the load.

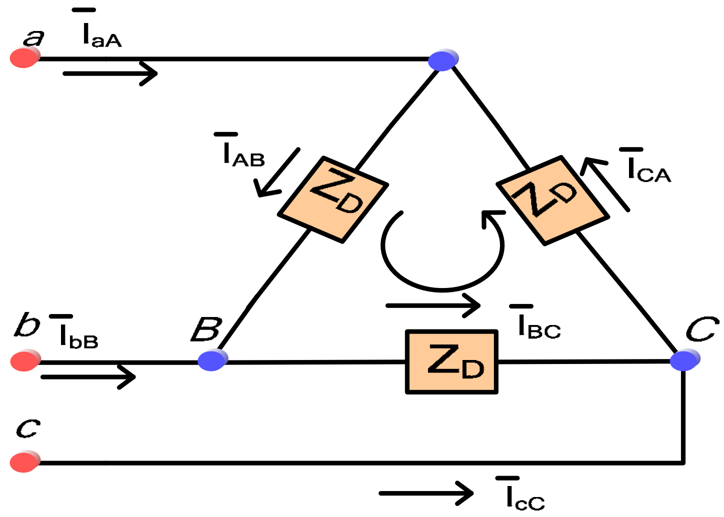

Figure 19 Three Phase Delta Connected Load.

Example(cont)

$$\bar{I}_{aA} = 1/\sqrt{3}\angle 30^0 \bar{I}_{AB} = 1.39\angle -6.87^0 A$$

$$\bar{I}_{BC} = 1.39\angle -6.87 - 120^0 = 1.39\angle -126.87^0 A$$

$$\bar{I}_{CA} = 1.39\angle 113.13^0 A$$

In an other way

$$\bar{I}_{AB} = V_{AB}/Z_{\phi\Delta} = \frac{202.72\angle 29.04}{118.5 + J85.8} = 1.39\angle -6.87^0 A$$

e) Calculate the line voltage at the source terminals.

$$V_{an} = (Z_l + Z_y)\bar{I}_{aA} = (39.8 + j29.5)(2.4\angle -36.87^0)$$

$$V_{an} = 118.90\angle -0.32^0 \underline{V}, V_{bn} = 118.9\angle -0.32 - 120$$

$$\bar{V}_{an} = 118.90\angle -120.32 - 120 = \bar{V}_{cn} = 118.9\angle -240.32^0 V$$

$$\bar{V}_{ab} = \sqrt{3}\angle 30 \bar{V}_{an} = 205.94\angle 29.68^0 V$$

$$\bar{V}_{bc} = 205.94\angle -90.32^0 V, \bar{V}_{ca} = 205.94\angle 149.68^0 V$$

Self test:

To analyse a Y-Δ connected balanced circuit we have to ?

a) Include the Delta phase load without any changes in the single-phase equivalent circuit.

b) Include the transformed Delta to Y phase load in the single-phase equivalent circuit.

c) Include three times the Delta phase load value in the single phase equivalent circuit.

Answer is b)

Self test:

Which options are true for a set of balanced delta variables?

a) Each phase voltage leads its corresponding line voltage by 30^o.

b) Each phase current leads its corresponding line current by 30^o.

c) Each phase current is higher in magnitude than its corresponding line current.

d) For the single-phase equivalent circuit, the line load voltage has to be used between A and N.

f) Each line voltage magnitude is equal each phase voltage magnitude.

Answer b) and f)