Electric Circuits II

Balanced Three Phase Circuits

Lecture # 02

The material to be covered in this lecture is as follows:

- Three-phase Voltage source connections.
- The single phase equivalent circuit
- The line voltage quantities for a balanced three phase circuits.
- Complete Analysis of Y-Y circuit

After finishing this lecture, you should be able to:

- Identify the difference between Y connected source and Δ connected source.
- Extract the single phase equivalent circuit for Y-Y three phase circuit
- Deduce the line voltage quantities for positive and negatives sequences
- Conduct a complete analysis for Y-Y circuit.

Three-phase voltage source.

- The three phase source could be connected whether in Δ or in Y.

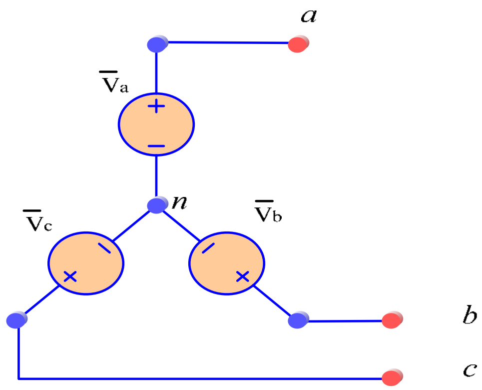


Figure 6 (a) Ideal three phase Y connected source.

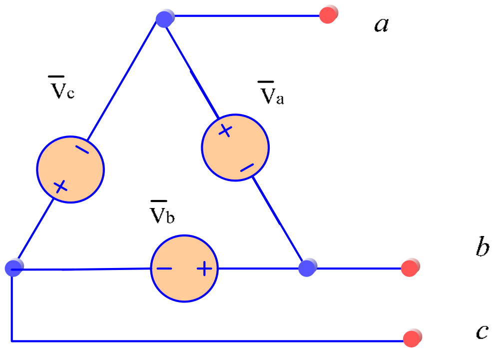


Figure 6 (b) Ideal three phase Δ connected source.

- The load can be connected in Y or in Δ and similarly to the source
- These two configurations are source connection and load connection

Three-phase voltage source (Cont)

- The load can be connected in Y or in Δ and similarly to the source .

Source Connection	Load Connection
Y	**Y**
Y	Δ
Δ	**Y**
Δ	Δ

Table 1. Source and Load Connection Types.

Three-phase voltage source (Cont)

- Three phase source model that includes winding impedance is for both connections is shown here
- The winding impedance represents the internal reactance and the internal resistance

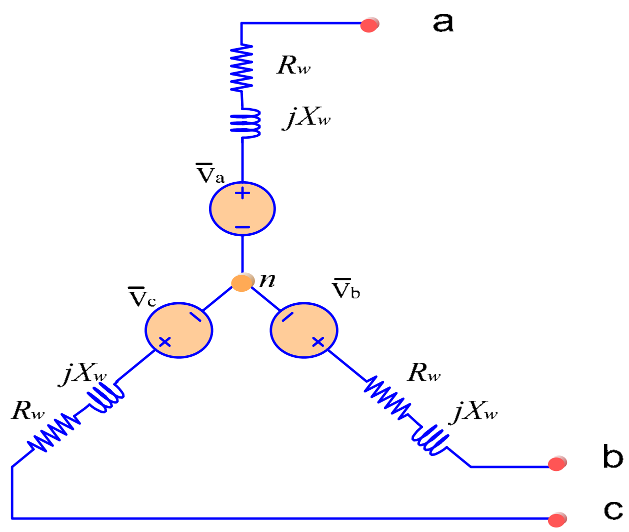


Figure 7(a) Y Connected Three Phase Source Model

Three-phase voltage source (Cont)

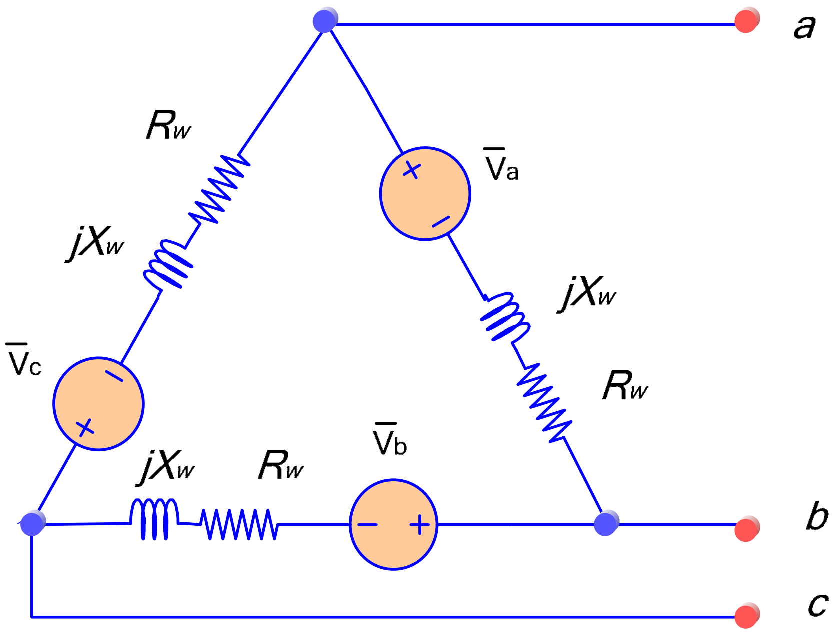


Figure 7 (b) Δ Connected Three Phase Source Model

Analysis of Y-Y circuit

- Consider the following three-phase circuit Y-Y system.
- The source if Y connected and the load is also Y, connected and in between is the transmission line or the connecting three-phase circuit.

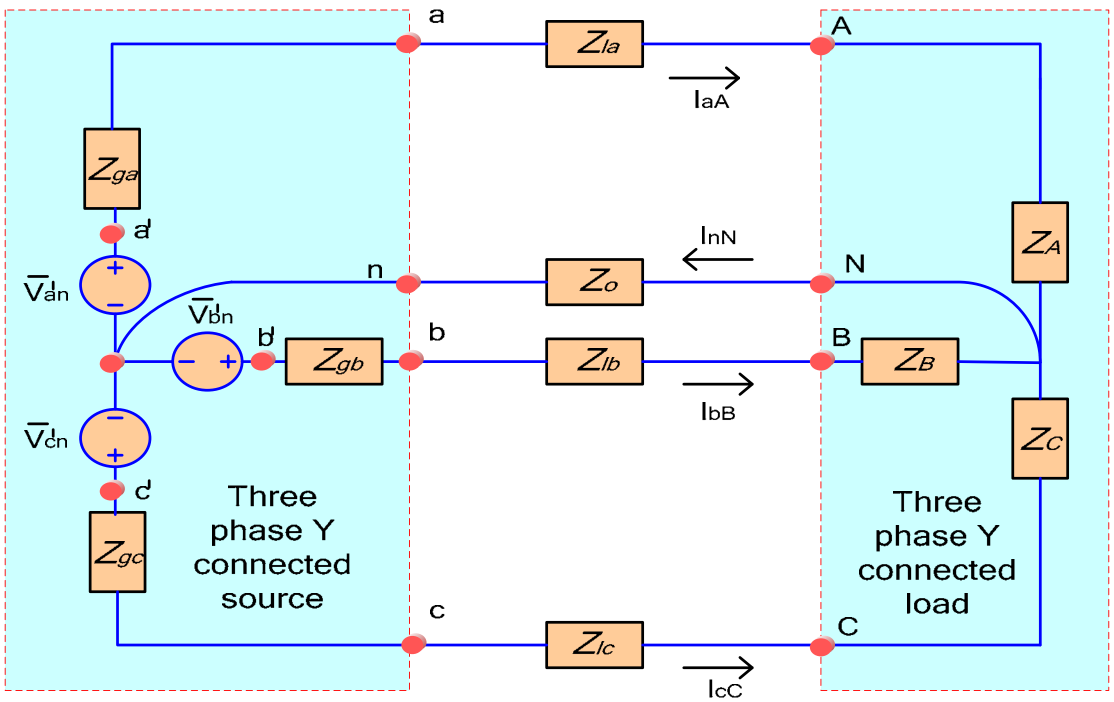


Figure 8 Three Phase Y-Y Connected System

Analysis of Y-Y circuit (cont)

- Note that the source neutral is taken as reference voltage and V_N as the voltage difference between the nodes N & n.
- Apply nodal voltage at point N then obtain the node voltage relation.

$$\frac{\overline{V}_N}{Z_0}+\frac{\overline{V}_N-\overline{V}_{b'n}}{Z_B+Z_{lb}+Z_{gb}}+\frac{\overline{V}_N-\overline{V}_{a'n}}{Z_A+Z_{la}+Z_{ga}}+\frac{\overline{V}_N-\overline{V}_{c'n}}{Z_C+Z_{lc}+Z_{gc}}=0$$

- The above equation could be applied to any Y-Y configuration.
- For a three-phase balanced circuit, the circuit should satisfy the following conditions.

❖ $Z_{ga}=Z_{gb}=Z_{gc}$ (Internal generator impedance)

❖ $Z_{la}=Z_{lb}=Z_{lc}$ (Transmission or connection line impedance)

❖ $Z_A=Z_B=Z_C$ (Load Impedance)

❖

❖ $\overline{V}_{a'n},\overline{V}_{b'n},\overline{V}_{c'n}$ $\begin{cases}\overline{\theta}_{a'n}=0^0\\ \overline{\theta}_{b'n}=-120^0\\ \overline{\theta}_{c'n}=-240^0\end{cases}$

Analysis of Y-Y circuit (cont)

- Then equation (11-x) becomes.

$$\overline{V}_N\left(\frac{1}{Z_0}+\frac{3}{Z\phi}\right)=\left(\frac{\overline{V}_{a'n}+\overline{V}_{b'n}+\overline{V}_{c'n}}{Z\phi}\right) \quad (11\text{-}2x)$$

- Where

$$Z_\phi = Z_A + Z_{la} + Z_{ga} = Z_B + Z_{lb} + Z_{gb} = Z_B + Z_{lc} + Z_{gc}$$

- Since $\overline{V}_{a'n}+\overline{V}_{b'n}+\overline{V}_{c'n}=0$ and $Z_\phi \,\&\, Z_0 \neq 0$

- Then $\overline{V}_N = 0$ and $I_0 = 0$

- If the system is balanced then the three line currents are

$$\overline{I}_{aA}=\frac{\overline{V}_{a'n}-\overline{V}_N}{Z_\phi}=\overline{V}_{a'n}/Z_\phi \qquad \overline{I}_{aB}=\frac{-\overline{V}_N+\overline{V}_{b'n}}{Z_\phi}=\overline{V}_{b'n}/Z_\phi \quad \text{and} \quad \overline{I}_{aC}=\overline{V}_{c'n}/Z_\phi$$

Analysis of Y-Y circuit (cont)

❖ The three circuits are similar; we can conduct the system calculations based on single phase, phase a circuit.

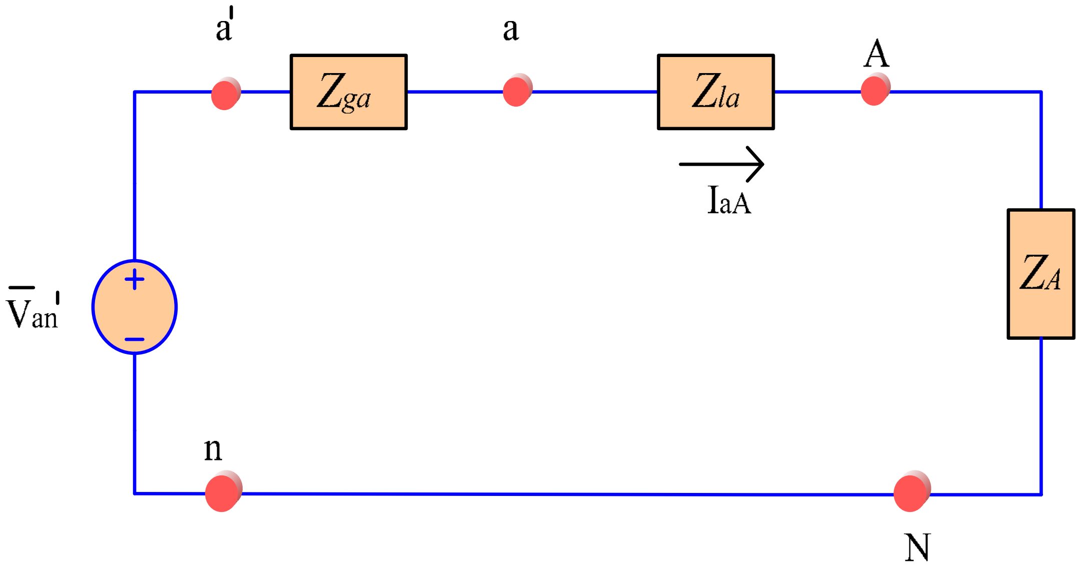


Figure 9 The equivalent single- Phase.

a) The line voltage is defined as the potential difference between any two phase voltages so

$$\bar{V}_{AB} = \bar{V}_{AN} - \bar{V}_{BN} \qquad \bar{V}_{BC} = \bar{V}_{BN} - \bar{V}_{CN} \qquad \bar{V}_{CA} = \bar{V}_{CN} - \bar{V}_{AN}$$

Analysis of Y-Y circuit (cont)

❖ When V_Φ is taken as magnitude of any phase voltage then

$$\bar{V}_{AN} = V_\phi \angle 0V \;,\; \bar{V}_{BN} = V_\phi \angle -120^0 V \;,\; \bar{V}_{BN} = V_\phi \angle -120^0 V$$

❖ So the line voltages are deduced

$$\bar{V}_{AB} = V_\phi \angle 0^0 V - V_\phi \angle -120^0 V = V_\phi - (-V_\phi \frac{1}{2}) - jV_\phi \frac{\sqrt{3}}{2}$$

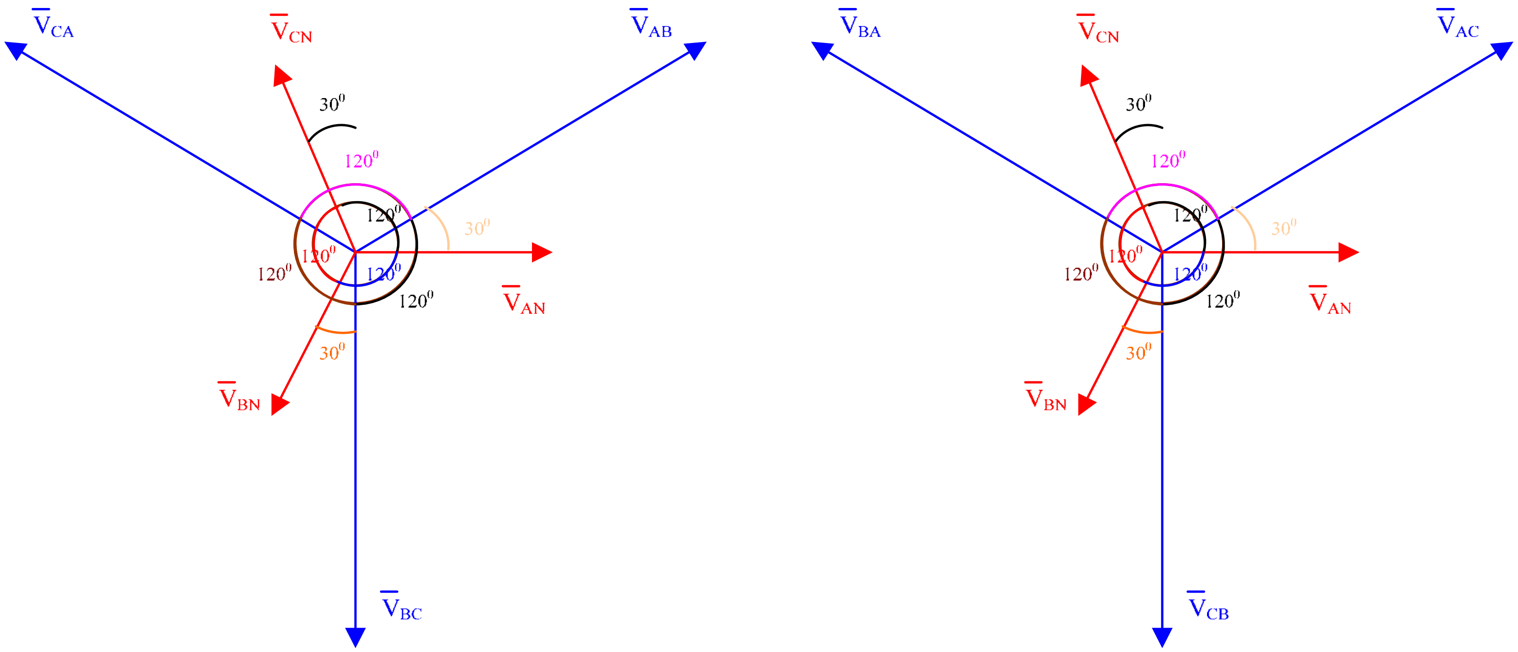
$$= V_\phi + \frac{V_\phi}{2}) - jV_\phi \frac{\sqrt{3}}{2} = \frac{3}{2} V_\phi + jV_\phi \frac{\sqrt{3}}{2} = \sqrt{3}\left(\frac{\sqrt{3}}{2} + J\frac{V_\phi}{2}\right)$$

$$\bar{V}_{AB} = \sqrt{3} V_\phi \left(\frac{\sqrt{3}}{2} + \frac{J}{2}\right) = \sqrt{3} V_\phi \angle 30^0 V$$

❖ In the same way we can show that

$$\bar{V}_{BC} = \bar{V}_{BN} - \bar{V}_{CN} = \sqrt{3} V_\phi \angle -90^0 V \quad \text{and} \quad \bar{V}_{CA} = \bar{V}_{CN} - \bar{V}_{AN} = \sqrt{3} V_\phi \angle 150^0 V$$

Analysis of Y-Y circuit (cont)



(a) (b)

Figure 10. Phasor diagrams for line and phase voltages, a) For positive sequence and b) for negative sequence.

Example:

A balanced three phase Y connected generator with positive sequence has an impedance of $0.2 + J0.5\Omega/\phi$ and an internal voltage of $120V/\phi$. The generator feeds a balanced three-phase Y connected load having an impedance of $39 + J28\Omega/\phi$, $Z_l = 0.8 + J1.5\Omega/\phi$, $\bar{V}_a$ is reference.

a) Construct the phase equivalent circuit of the system.

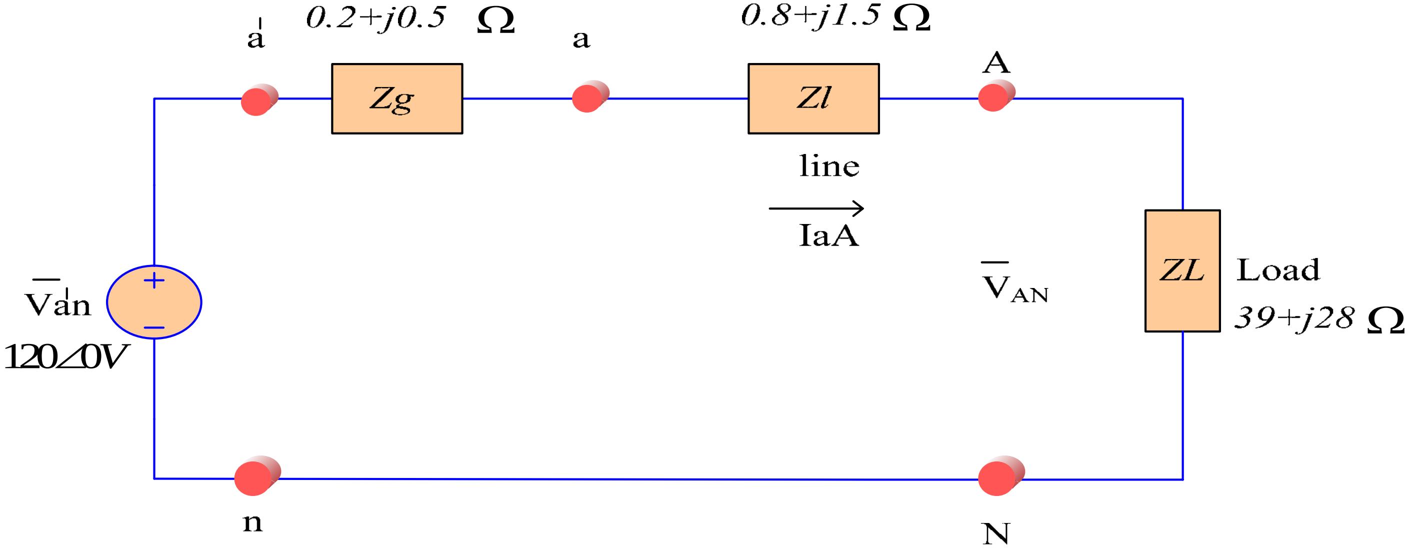


Figure 11. Single phase Equivalent Circuit

Example: (cont)

b) Calculate $\bar{I}_{aA}, \bar{I}_{bB}, \bar{I}_{cC}$.

$$\bar{I}_{aA} = \frac{\bar{V}_{a'n}}{Z_g + Z_l + Z_L} = \frac{120\angle 0}{(0.2 + j0.5) + (0.8 + j1.5) + (39 + j28)} = \frac{120\angle 0}{40 + j30}$$

$\bar{I}_{aA} = 2.4\angle -36.87^0 A$ The line current is lagging over the voltage then

$\bar{I}_{bA} = 2.4\angle -36.87^0 - 120^0 A = 2.4\angle -156.87^0 A$ and

$$\bar{I}_{cC} = 2.4\angle +83.13^0 A = 2.4\angle -36.87 - 240^0 A = 2.4\angle -276.87^0 A$$

c) Calculate the three phase voltage at the load $\bar{V}_{AN}, \bar{V}_{BN}, \bar{V}_{CN}$.

$$\bar{V}_{AN} = (39 + J28)(2.4\angle -36.9) = 115.22\angle -1.1V$$

Example: (cont)

$$\overline{V}_{BN} = 115.22\angle -121.1^0 V, \overline{V}_{CN} = 115.22\angle 118.9^0 V$$

d) Calculate the line voltages $\overline{V}_{AB}, \overline{V}_{BC}, \overline{V}_{CA}$, at the terminals of the load.

$$\overline{V}_{BN} = 115.22\angle -121.1^0 V, \overline{V}_{CN} = 115.22\angle 118.9^0 V$$

$$\overline{V}_{BC} = 199.58\angle 28.81 - 120 = 199.58\angle -91.19^0 V$$

$$\overline{V}_{CA} = 199.58\angle 148.81^0 V$$

e) Calculate the phase voltages at the phase voltage of the generator $\overline{V}_{an}, \overline{V}_{bn}, \overline{V}_{cn}$.

$$\overline{V}an = \overline{V}a'n - \overline{I}aA(Zg) = \overline{I}aA(Zl + Zl) = \overline{I}aA(Zl) + \overline{V}an$$

$$= 120 - (2.4\angle -36.87)(0.2 + J0.5) = 118.9\angle -0.32V$$

$$\overline{V}_{AN} = (2.4\angle -36.87)(0.8 + J1.5) + (115.22\angle -1.1)$$

$$\overline{V}_{bn} = 118.9\angle -120.32^0 V, V_{cn} = 118.9\angle 119.68^0 V$$

Example: (cont)

f) Calculate the line voltage V_{ab}, V_{bc}, V_{ca} at the terminal of the generator.

$$\overline{V}_{ab} = \sqrt{3}\angle 30 V_{AN} = 205.94\angle 29.68^0 V$$

$$\overline{V}_{bc} = 205.94\angle -90.32 V$$

$$\overline{V}_{ca} = 205.94\angle 149.68 V$$

Self Test:

❖ Solve the same example for negative Sequence

a) answer

No change for the single-phase equivalent circuit for negative sequence

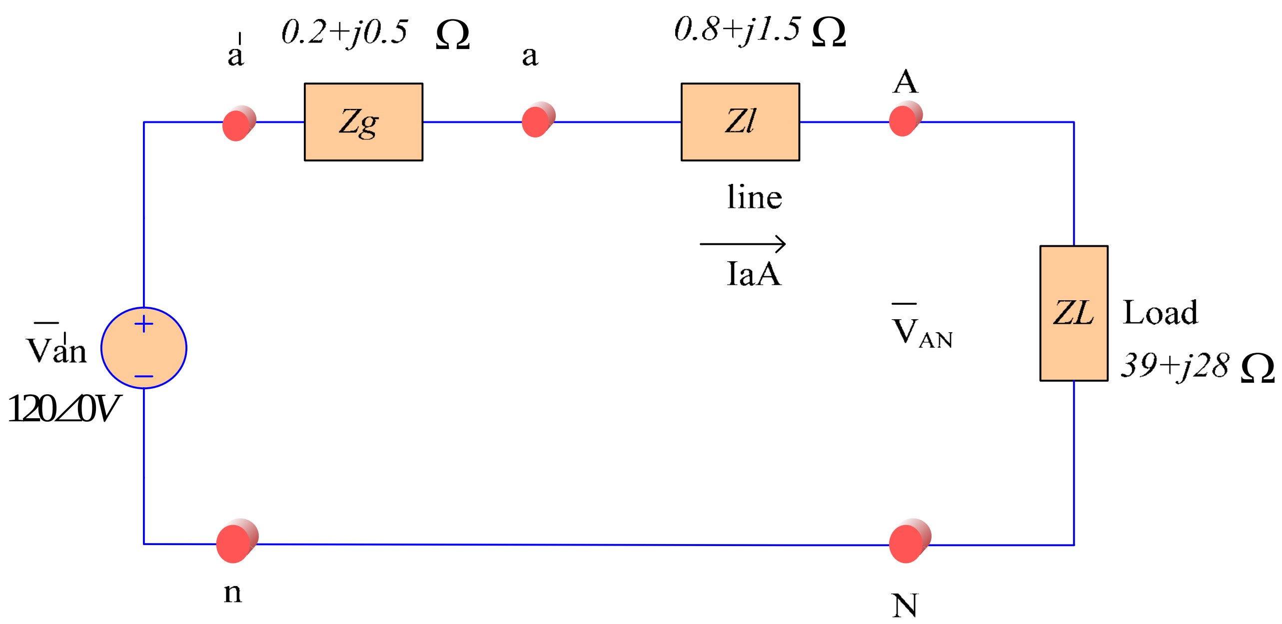


Figure12. Single phase Equivalent Circuit (for negative sequence)

Self Test(cont)

b) answer :

$$\bar{I}_{aA} = 2.4\angle -36.87^0 A$$

$$\bar{I}_{bA} = 2.4\angle 83.13^0 A$$

$$\bar{I}_{cC} = 2.4\angle -156.87^0 A$$

c) answer :

$$\bar{V}_{AN} = 115.22\angle -1.1^0\ V$$

$$\bar{V}_{BN} = 115.22\angle 118.81^0 V$$

$$\bar{V}_{CN} = 115.22\angle -121.19^0 V$$

Self Test(cont)

d) answer :

$$\overline{V}_{AB} = (\sqrt{3}\angle 30^0)V_{AN} = 199.58\angle -31.19^0 V$$

$$\overline{V}_{BC} = 199.58\angle 88.81^0 V$$

$$\overline{V}_{CA} = 199.58\angle -151.19^0 V .$$

e) Answer

$$\overline{V}_{an} = 118.9\angle -0.32^0 V$$

$$\overline{V}_{bn} = 118.9\angle 119.68^0 V$$

$$\overline{V}_{cn} = 118.9\angle -120.32^0 V$$

Self Test(cont)

f) Answer:

$$\overline{V}_{ab} = (\sqrt{3}\angle -30^0)V_{an} = 205.94\angle -30.32^0 V$$

$$\overline{V}_{bc} = 205.94\angle 89.68^0 V$$

$$\overline{V}_{ca} = 205.94\angle -150.32^0 V$$