Multichannel adaptive forward error-correction system

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Abstract: A new multichannel adaptive error-correction system has been investigated for use over noisy time-varying channels. Information is fed at a fixed overall rate over a number of parallel channels, with the amount of information per channel being determined on the basis of channel quality. The system is shown to be superior to that obtainable with a fixed-rate forward error-correction systems, and in addition the proposed system is shown to have a performance superior to that obtainable with classical diversity systems operating at the same rate.

1 Introduction

There are severe problems in communicating over time-varying channels such as those that are subject to multipath interference and those where the level of noise may vary significantly over a period of time. A typical channel giving rise to difficulties of this kind is the mobile radio channel. Here the aim is to achieve as high a throughput rate as possible at acceptable levels of performance/reliability, or vice versa, and to do this at affordable bandwidths, costs and complexity.

Over the years a number of systems have been proposed for combating the effects of the time-varying nature of such channels. The approaches that have been developed make use of facilities such as channel quality estimates (CQE), feedback links, and the availability of independent multichannels. In the absence of these facilities very little can be done to overcome the effects of time variation of the channel. One possible approach to the problem is to use a fixed-rate forward error correction (FEC) which has been designed to handle worst-case conditions. Generally, with this approach a poor throughput rate of communication is the result [1]. On the other hand, however, if it is possible to monitor the channel in some way then it becomes feasible to adjust one or other of the transmission parameters, such as the power, and/or the transmission rate, so as to match the transmission to the prevailing channel conditions [2–6]. In the case of duplex operation, the state of the forward channel can be inferred to some degree by making measurement on the return channel. In the case of simplex working, however, information has to be communicated back to the transmitter using a separate feedback channel.

One popular approach which relies on the use of a feedback channel but does not require CQE information is the so-called automatic repeat request (ARQ) technique [7]. In a basic ARQ system, a high-rate error-detecting code is used. When an error is detected in a received codeword a negative acknowledgment is received back at the transmitter which then repeats the codeword. The process is repeated until the codeword is successfully received and positively acknowledged. A properly designed ARQ system can be very reliable, but under poor channel conditions the throughput falls rapidly as a direct consequence of the large number of retransmissions that are needed. However, the number of retransmissions can be reduced if extra redundancy for forward error correction (FEC) is added to the transmitted sequence so that some of the erroneous codewords are corrected and accepted. Such ARQ/FEC systems are referred to as hybrid ARQ systems. In one class of these systems, often called type-I hybrid ARQ, the amount of redundancy is fixed, i.e. a fixed-rate forward error correcting code is used [7]. However, to achieve optimum performance over time-varying channels, the error-correcting capability should respond to the present channel conditions. This forms the basis of type-II hybrid ARQ [8], and the question of how encoding and/or decoding should be adapted has received considerable attention [9–19] and continues to do so.

Another approach is that based on the use of a number of independent channels for transmitting the *same* information signal. Such systems are generally referred to as diversity systems. Upon receiving the several copies of the transmitted signal the receiver may either select the best one, i.e. that with the highest signal-to-noise ratio (selective diversity), or base its decision on a simple summation of the signals (equal-gain combining) [20]. If it is possible, however, to obtain reliable CQE then the received information signals can be combined in an optimum way (maximal-ratio combining) [21]. In a generalisation of this approach [22–24], the information signal is first coded using FEC, and the resulting codeword is then sent as in a normal diversity system, with appropriate error decoding being performed on the received signal after selection or diversity combining is carried out.

Recently Benelli [25] has suggested using a number of independent channels in a different manner. He proposes a system in which k information bits are encoded into a codeword of length n through an (n, k) error-correcting code. This process continues until s codewords are constructed. The transmitter then divides each of the s codewords into L subblocks for transmission over L channels

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such that the ith subblock of all codewords are transmitted as one block over the ith channel, $1 \leqslant i \leqslant L$. On receipt of the outputs for the L channels the received bits are reformed so as to have the same structure as the s codewords, and error decoding is then carried out. Benelli has shown that the system is superior to equal-gain diversity.

In this paper a system of transmission is proposed which employs adaptive forward error correction. The new system, which is referred to as a multichannel adaptive forward error correction (MC–AFEC) system assumes the availability of number of independent parallel channels with the information to be communicated being divided between and sent over the set of available channels. Although the system uses CQE information similar to that needed for MRC, it operates in a fundamentally different manner. The information gained as to channel quality is used at the transmitter to adaptively encode the information that is to be sent over each of the channels in the set.

2 Multichannel adaptive forward error correction (MC–AFEC) system

2.1 System description

The system proposed relates to a transmitter and a receiver that are linked together by a number of independent parallel channels, and it is assumed that the channels are of equal transmission rate. The system is illustrated schematically in Fig. 1*a*.

Consider a block of K information bits that are to be transmitted over L independent channels. Unlike a conventional frequency diversity system, in which the block of K bits would be sent in replica over the parallel channels, in the present system the block of K bits is split into L segments, and each segment is then encoded into one codeword of length n, where the added parity check bits are used purely for the purpose of error correction (see Fig. 1*b*). The number of information bits contained in each of the L segments is based on information obtained from the CQE process, and it is this that gives rise to the adaptive nature of the system.

It will be appreciated that the CQE unit (or channel monitor unit as it is sometimes called) is an essential part of the system. Such units are employed in a number of existing practical communication systems operating over fading channels. Examples are to be found in selective diversity systems in which the channel monitor provides the receiver with an estimation of the channel gains for the various branches so that the branch with the largest gain (that is, that with the highest SNR) is selected. In maximal ratio combining (MRC) diversity systems, both the gain and the phase must be tracked on each diversity channel so that the signal can be brought into phase coincidence, weighted and summed before detection. Other systems based on channel quality estimation, in terms of fade level or SNR, can be found in References 2 to 6 and 26 to 28.

The required circuitry and relevant analysis of channel quality estimation can be found in References 21 and 26. Two approaches are presented. The first is to track the channel attenuation by using information gained from past receptions. The second is to use channel sounding

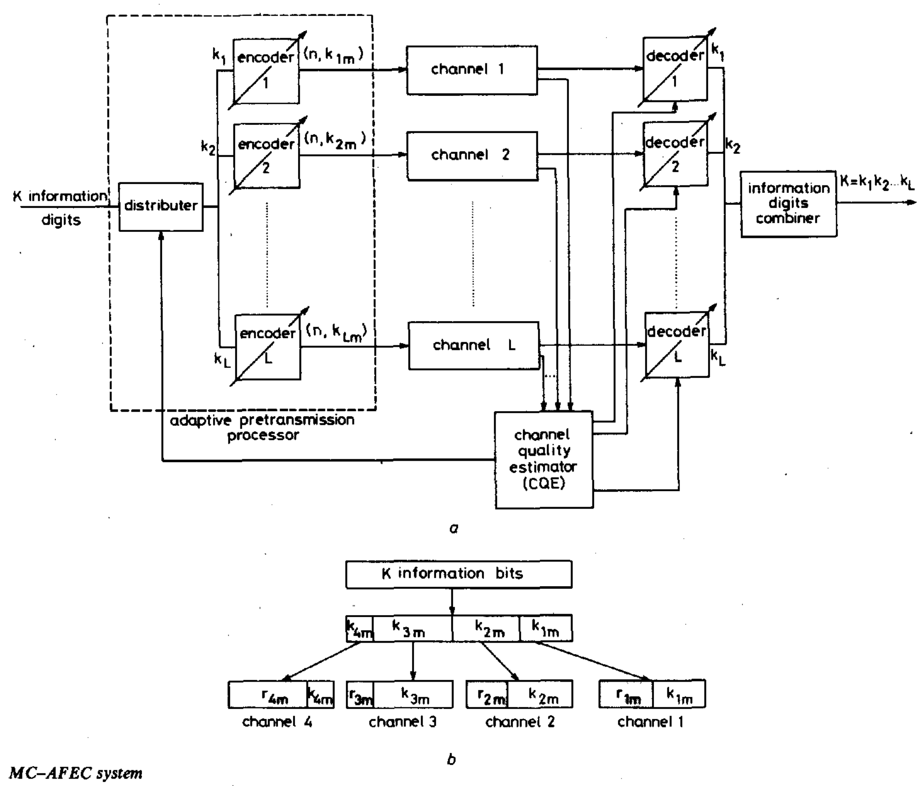

Fig. 1 *MC–AFEC system*
a block diagram
b pretransmission process of K information bits ($L = 4$)

techniques where a pilot signal is transmitted over the fading channel for the purpose of measuring the channel gain and phase shift.

It may be that in the course of error correction an error count is carried out and that it might be used for purposes of CQE, but this is not a matter for consideration in this paper.

The operation of the system can be summarised as follows. Based on the current information available about the quality of each of the L channels, the receiver directs the transmitter on how to control the number of information bits, sand hence the number of check bits, contained in the n-bit codewords that are to be transmitted, over each channel of the system. This is done subject to the constraint that a total of K information bits are transmitted through the transmission of the L codewords. Those channels having a poor quality are allocated fewer information bits and more check bits, whereas the better quality channels are allocated an increased number of information bits and a reduced number of check bits. The code rates are adjusted to match the prevailing channel conditions subject to the contraint of constant throughput. The throughput, R, is defined as the ratio of the number of information bits to the total number of bits, transmitted per unit time.

2.2 System analysis

Let it be assumed that the estimates of the channel qualities are updated periodically, and let τ denote this period. The L estimates of channel quality will be taken to be based on the received energy-per-bit-to-noise ratio (SNR) measurement obtained for each channel during the previous adaptation period. Let the set of L quality estimates be denoted by the vector $V = \gamma_{1m}, \ldots, \gamma_{Lm}$, where γ_{im} is the SNR per bit on the ith channel, measured over the mth adaptation period. Let this be expressed as

$$\gamma_{im} = \alpha_{im}^2 E_b/N_0 \tag{1}$$

where α_{im} is the attenuation factor on channel i during the mth period, E_b is the transmitted energy per bit, and N_0 is the power spectral density of the noise which is assumed to be AWGN. It will be assumed that the channels are slowly varying so that V remains unchanged during an adaptation period, but that it can change from one period to the next. That is,

$$V = \text{constant} \quad m\tau \leqslant t \leqslant (m+1)\tau \tag{2}$$

Based on V, let r_{im} bits be added and used as check bits in respect of the k_{im} information bits that are allocated to the ith channel. The set of information bits k_{im} and check bits r_{im} constitute an (n, k_{im}) error correcting code, $n = k_{im} + r_{im}$; $i = 1, \ldots, L$; capable of correcting all error patterns of weight $\leqslant t_{im}$. The aim is to minimise the system bit error probability subject to the constraint that $K = k_{1m} + \cdots + k_{Lm}$ information bits are to be sent during the course of transmitting the L codewords to the receiver. Generally a number of sets of L codewords are transmitted during each adaptation period τ. In the following analysis it will be assumed that the code rate is changed at the beginning of each adaptation period and that the codes so chosen are retained for the remainder of the adaptation period. The system throughput, R, is held constant from one adaptation period to the next, and is given by

$$R = \sum_{i=1}^{L} k_{im}/nL \tag{3}$$

If p_{im} denotes post-decoding per-bit error probability on the ith channel in respect of the mth period, then, with respect to the mth adaptation interval, the average bit error probability at the system output is

$$P_{e,m} = \sum_{i=1}^{L} k_{im} p_{im}/K \tag{4}$$

The aim of the adaptation process is, during the mth adaptation period, to select the set $\{k_{im}\}$ which minimises the error probability $P_{e,m}$. Expressed alternatively, the problem is to determine the optimum set $\{k_{im}\}$ as a function of the set $\{\gamma_{im}\}$, the elements of the vector V, while maintaining the throughput constant at the rate R.

To evaluate expression 4 it is necessary that p_{im}, $i = 1, \ldots, L$ be known. In general, for a t-error correcting code of block length n operating over a random error channel, the probability, P_C, of correctly decoding a codeword is given by

$$P_C = \sum_{i=0}^{t} \binom{n}{i} \varepsilon^i (1-\varepsilon)^{n-i} \tag{5}$$

where ε is the BER on the channel. The BER is a function of the per bit SNR, and modulation technique used. The probability, P_E, that the codeword is in error is

$$P_E = 1 - P_C \tag{6}$$

Given the codeword error probability, P_E, it is not a simple matter to determine the corresponding bit error probability, p. In general the relation between P_E and p depends on the structure of the code used. In practice a number of approximations have been made [21, 29] for the bit error probability in terms of the codeword error probability. If the number of the information is reasonably large then most approximations reduce to $p \simeq \frac{1}{2}P_E$, as a worst case. The approximation $p = \frac{1}{2}P_E$ will be used throughout the remainder of this paper.

For a given class of codes, the set $\{k_{im}\}$ determines the set $\{t_{im}\}$ and for a given $\{\gamma_{im}\}$ the corresponding channel BER, $\{\varepsilon_{im}\}$, is determined by the modulation technique used. The set $\{t_{im}\}$ and the set $\{\varepsilon_{im}\}$ determine the bit error probability, $\{p_{im}\}$, of the system during the mth period.

Using I, T and E to denote the vectors whose elements are k_{im}, t_{im} and ε_{im}; $i = 1, \ldots, L$, respectively, then the system bit error probability $P_{e,m}$ given in expression 4 can be rewritten as

$$P_{e,m}[I, T, E] = \frac{1}{K} \sum_{i=1}^{L} \frac{k_{im}}{2} \left\{ 1 - \sum_{j=0}^{t_{im}} \binom{n}{j} \varepsilon_{im}^j (1-\varepsilon_{im})^{n-j} \right\} \tag{7}$$

Since, for a given code, the vector T is related to and fully determined by the vector I, one of them can be substituted in the argument of $P_{e,m}$ in eqn. 7. Also since the vector E is mathematically related to the vector V it follows that the analysis of the bit error probability in eqn. 7 can be carried out in terms of I and V, which is more convenient. The problem then becomes one of finding the information vector, I, as a function of the BER vector, E, so that the average bit error probability is minimised subject to the constraint that the throughput is kept equal to R.

The L channels will be considered to be slowly-varying independent Rayleigh fading channels that are subject to AWGN. Within the context of this paper slowly varying will be taken to mean that the signal fade level, and hence the received signal energy, remains constant for at least one adaptation period, which can be as small as the duration of a codeword, and that this level is free to vary from one adaptation period to the next in accordance with a Rayleigh distribution. Then, γ_{im} will have a probability density function $p_{\gamma_{im}}(\gamma_{im})$ of the form

$$p_{\gamma_{im}}(\gamma_{im}) = \frac{1}{\bar{\gamma}_i} \exp\left(-\frac{\gamma_{im}}{\bar{\gamma}_i}\right) \quad \text{for } \gamma_{im} \geqslant 0 \tag{8}$$

where $\bar{\gamma}_i$ is the per bit signal-to-noise ratio on the ith channel averaged over all time. If it is assumed that the channels are identical on average, that is, $\bar{\gamma}_1 = \bar{\gamma}_2 = \cdots = \bar{\gamma}_L = \bar{\gamma}$, then the probability density function $p_V(V)$ associated with the vector V is

$$P_V(V) = \prod_{i=1}^{L} \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma_{im}}{\bar{\gamma}}\right) \quad \text{for } \gamma_{im} \geqslant 0 \tag{9}$$

It thus follows that the overall average bit error probability of the system $\bar{P}_e$ is

$$\bar{P}_e = \int_0^\infty \int_0^\infty \cdots \int_0^\infty P_{e,m}[I, V] p_V(V)\, dV \tag{10}$$

and the task is to minimise expression 10 subject to the constraint

$$R = \sum_{i=1}^{L} k_{im}/nL$$

In general, the problem of minimising a function subject to constraints can be solved by application of the method of Lagrange multiplier. In the case of eqn. 10, however, this method cannot be applied since the variables k_{im} and t_{im} in eqn. 7, and hence in eqn. 10, are discrete. More seriously, the variable t_{im} appears as the upper limit of the second summation and the expression cannot be differentiated with respect to t_{im}. Even if true minimisation could be carried out, it may well yield a combination of n, k and t that is not realisable with known codes.

An alternative approach is adopted in this paper. The two issues of the constraint on the throughput, R, and the minimisation of $\bar{P}_e$ are dealt with separately as follows. First, choose a set, S, of error-correcting codes which have the same codeword length, n, but which are of different rates. Let the set S include the two trivial codes (n, n) and $(n, 0)$. The (n, n) code represents the case when all n bits are information bits, while the $(n, 0)$ code stands for transmitting n dummy bits which carry no information. For a given throughput R and the L parallel channels, form the set G of all groups of L codes (denoted as L-code groups) so that each member of G satisfies the rate constraint in expression 3. Each L-code group corresponds to an information vector I and an error-correction capability vector T which is different from the I and T of any other L-code group. Assume M such groups are obtained, that is, G contains M members.

Secondly, for a given estimate of V obtained from the channel quality monitor, let the transmitter select that L-code group of G which yields the minimum $P_{e,m}[I, V]$, and which thereby minimises the integrand of eqn. 9. The selected L-code group is used for the duration of the adaptation period τ. On obtaining the next estimate of V, which applies to the next adaptation period, the optimum L-code group is again selected. Minimisation is thereby achieved by selecting between the M different L-code groups.

Clearly, in order for the system to operate properly, both the transmitter and the receiver must know the group of codes in use. This implies either that the feedback channel is noise-free, or, more realistically, that the information sent back via the feedback channel can be adequately protected by coding. Since the information that has to be fed back is small (simply the code group number) such protection, and hence the reliability of the system, can be made as high as necessary, and, for all practical purposes, the feedback channel can be rendered error free. In the unlikely event of the transmitter not using the code group directed by the receiver, the two techniques referred to in Reference 21, and mentioned earlier in Section 2.2, can be used to avoid error propagation, since they do not employ any error counting, nor do they employ any error detection.

In this paper the class of BCH codes is used, but, clearly, the application of the ideas is not restricted to this class of codes. BCH codes do, however, constitute a powerful class of multiple error-correcting codes, especially at moderate lengths, and more importantly from the point of view of the present work, they possess the flexibility that for a given codeword length, n, codes having a large range of information bits, k, can be constructed. For example, in the case of $n = 63$, codes having all k values between 1 and 57 exist.

As an illustration of the process of L-code grouping take S to be the set of all BCH codes of length 63 together with the trivial (63, 63) and the 63, 0) codes and consider the complete set of available values of k. This set of available codes can be found in Peterson and Weldon [30]. Suppose further for purpose of illustration that $R = 95/189 \simeq 0.5$ and $L = 3$. When this rate constraint is applied 23 triplets of codes of distinct error-correction capability vectors can be formed, eight of which are shown below in the form (n, k, t).

1	(63, 63, 00)	(63, 32, 05)	(63, 00, —)
2	(63, 63, 00)	(63, 28, 07)	(63, 04, 15)
3	(63, 57, 01)	(63, 36, 05)	(63, 02, 20)
4	(63, 51, 02)	(63, 39, 04)	(63, 05, 15)
5	(63, 51, 02)	(63, 36, 05)	(63, 08, 13)
6	(63, 46, 03)	(63, 39, 04)	(63, 10, 11)
7	(63, 46, 03)	(63, 28, 07)	(63, 21, 08)
8	(63, 39, 04)	(63, 28, 07)	(63, 28, 07)

Note that the total number of information bits in each row is equal to 95 as is required by the rate constraint.

The process of error minimisation referred to earlier involves (in effect) first selecting the appropriate L-code group and then, having selected the L-code group, allocating the L codes to the appropriate channels. This amounts to allocating the higher-rate code to the best channel and continuing down to the point where the lowest rate code is assigned to the poorest channel. In practice this could be implemented quite simply by ordering the codes in an L-code group in descending order of their rates, and assigning them in a simple direct associative manner to the L channels that have been arranged in descending order of quality.

Given a CQE vector V, the question of which L-code group should be selected can also be implemented in a relatively simple manner. One way of doing this would be to divide the L-dimensional space associated with the vector V into cells, and then to associate each cell with the appropriate (best performance) L-code group. This might be done by quantising the elements of V, and then using the quantised vector to address a ROM, in which a number associated with the optimum L-code group has been stored.

3 Results and discussion

The performance of MC–AFEC system has been evaluated numerically. This has been done by considering the way in which the performance varies as a function of the average SNR, $\bar{\gamma}$ (the ratio of the average value of the fading signal to the power of the AWGN), and throughput rate, R, and the way in which the number of channels, L, affects the performance of the system.

In the evaluation BCH codes of length 63 were used and the system was compared with a system that was fixed in the sense that the same code was used on all of the L channels, and the code was kept fixed throughout the operation of the system. In addition, the performance of the MC–AFEC system has also been compared with an L-channel maximal-ratio combining (MRC) diversity system.

Figs. 2–4 show how the performance varies, as function of $\bar{\gamma}$, for $L = 2$, 3 and 4, respectively. As mentioned

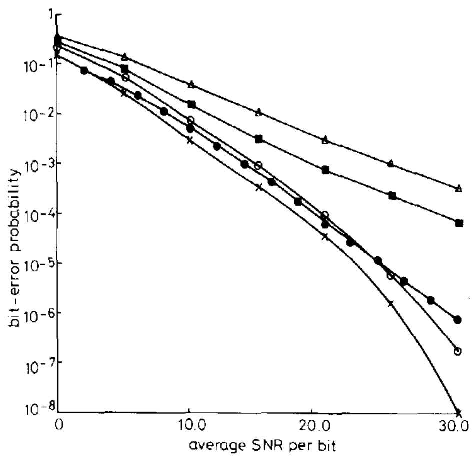

Fig. 2 *Performance of the MC–AFEC system for different throughput values with $L = 2$*

×—× $R = 0.33$
○—○ $R = 0.50$
■—■ $R = 0.60$
△—△ $R = 0.75$
●—● $R = MRC$ (corresponds to $R = 0.5$)

above, $\bar{\gamma}$ is the ratio of the average value of the signal to the noise power. It must be born in mind that each point of the horizontal axis of Figs. 2, 3, 4 etc. relates to a Rayleigh fading channel having a particular mean signal level. A number of observations can be made from those Figures.

As would be expected, the Figures show that a reduction in rate results in a reduction in BER. However, the way in which the BER changes with reduction in rate is not straightforward. For example a rate-reduction of 0.1 (from 0.6 to 0.5) causes a significant improvement for $L = 2$ and a much smaller improvement when $L = 3$. When $L = 4$, the improvement is between that obtained when $L = 2$ and 3. On the other hand, a throughput reduction from 0.75 to 0.6 has a significant effect on performance when $L = 3$, but a much smaller effect when $L = 2$ or 4. From the Figures it will be seen that there tends to be a banding effect with certain groups of BER-vs-rate clustering together. In the case of $L = 2$, the BER curves for rates at or below 0.5 tend to cluster and be distinct from those associated with rates above 0.5. In

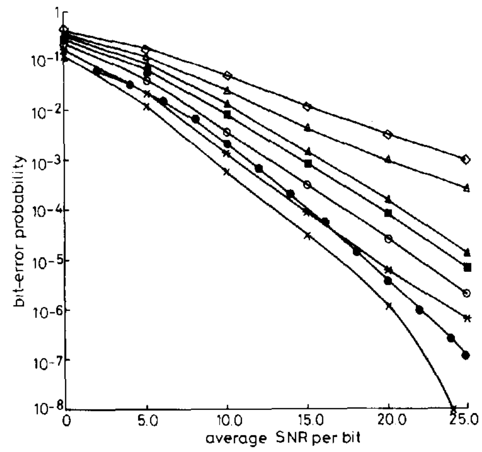

Fig. 3 *Performance of the MC–AFEC system for different throughput values with $L = 3$*

×—× $R = 0.33$
∗—∗ $R = 0.40$
○—○ $R = 0.50$
■—■ $R = 0.60$
▲—▲ $R = 0.66$
△—△ $R = 0.75$
◇—◇ $R = 0.85$
●—● $R = MRC$ (corresponds to $R = 0.33$)

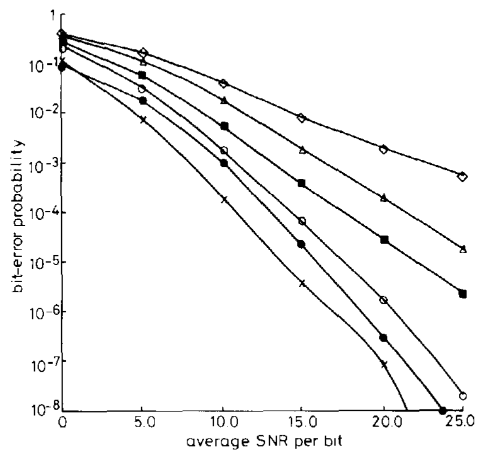

Fig. 4 *Performance of the MC–AFEC system for different throughput values with $L = 4$*

×—× $R = 0.33$
○—○ $R = 0.50$
■—■ $R = 0.60$
△—△ $R = 0.75$
◇—◇ $R = 0.85$
●—● $R = MRC$ (corresponds to $R = 0.25$)

the case $L = 3$ the same effect can be seen with clusters at or below the rates of 0.33 and 0.66. In general, the banding effect depends on the relative values of R and L in the way that for a given L, the clustering tends to be noticeable at rates at and below i/L, $i = 1, \ldots, L$. The effect can be explained as follows. The improvement in performance (the reduction in BER) resulting from decreasing R (or from increasing L, as will be seen shortly) depends on whether the change makes it possible to use a highly redundant code on the worst channel(s) without violating the throughput constraint; if this is possible then, following the change, the improvement will be significant, otherwise, the improvement is minimal.

For purposes of comparison the corresponding performance of an MRC diversity system is also shown on Figs. 2–4. The reason for comparing the MC–AFEC system with the MRC system is that MRC has been examined extensively in the literature and has come to provide a well used standard of comparison. Also, it is, of course, the optimum diversity system.

At first sight it does not appear that a significant improvement has been obtained with the MC–AFEC system. For example, in the case $L = 2$ the performance of MC–AFEC at $R = 0.5$ is virtually identical to that of the corresponding two-channel MRC diversity system (also of rate $R = 0.5$); and improvement with the MC–AFEC system can only be achieved at the expense of throughput reduction. However, when $L > 2$ the situation changes significantly. When $L = 3$ the MC–AFEC will support a throughput rate of approximately 0.4 at the same BER as a three-channel diversity system (operating at $R = 0.33$), and when L is increased to 4, at $\bar{\gamma}$ below 15 dB (see Fig. 4), the MC–AFEC will support a throughput rate of 0.5 whilst maintaining a BER substantially the same as a four-channel diversity system operating at the much reduced rate of 0.25, and although this advantage in BER falls a little with increasing $\bar{\gamma}$, the advantage of MC–AFEC is clear.

Another important advantage of the MC–AFEC system is that it is better able to take account of those periods when the channels are good than is a diversity system, whose rate is fixed at $1/L$. This advantage has an important practical consequence that can be seen, for example, in Fig. 4. If in a practical situation, such as that in the case of a speech communication system, it was decided that a BER of, say, 10^{-4} was acceptable and the channel had $\bar{\gamma} \simeq 20$ dB, then MC–AFEC would be able to operate at a throughput close to 0.6, whereas the fixed diversity system would overperform BER-wise, but would only be able to operate at a throughput rate 0.25. In short, with the MC–AFEC system it is possible for a given acceptable performance level to maximises the throughput rate by appropriate code selection, whereas with a diversity system the rate is fixed.

In Fig. 5 some of the results contained in Figs. 2–4 are re-presented in such a way that, for a given rate R, the effect of varying the number of channels can be seen directly, and compared with a fixed-rate system in which the same code and hence code-rate R is used on each of the channels. In order for any comparisons between the performance curves in Fig. 5 to be valid, it is essential that the total transmission bandwidth and the total transmitted power used should be the same, irrespective to the number of channels, L. By keeping the total transmitted power fixed at P_T and transmitting at a power of P_T/L over each channel, and assuming that the bandwidth per channel is B_T/L, where B_T is the total bandwidth, this amounts to keeping the effective energy/bit divided by the average noise power (E_b/N_0) fixed. This is what is required for a valid comparison, and it was used when evaluating the performance of the system for the various cases.

Fig. 5 shows, irrespective of the rate R, that an improved BER performance results from increasing the number of channels L. The following specific points are worth noting

(i) The MC–AFEC system always outperforms the fixed system. In fact, the performance of the fixed system is unacceptable (BER $\geq 10^{-3}$) even at $\bar{\gamma}$ as high as 30 dB

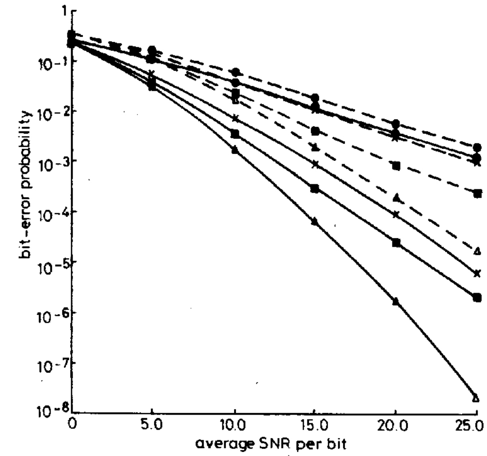

Fig. 5 *Performance of the MC–AFEC system for different L*

—— $R = 0.5$
- - - - $R = 0.75$
×—× $L = 2$
■—■ $L = 3$
Δ—Δ $L = 4$
●—● fixed rate

even though the throughput rate is as low as 1/3. The advantage of the adaptive system over the fixed system arises mainly from the fact that when one or more of the channels are highly noisy, a high redundant code is used on them.

(ii) As the throughput rate increases, the improvement of the adaptive system over the fixed system decreases. This is due to the fact that a higher throughput rate means less freedom for adaptation.

(iii) Increasing the number of channels enhances the performance of the adaptive system over the fixed system but, again, not in a straightforward manner. A large improvement is achieved if the required throughput is small enough to allow the transmitter the freedom to use one or more of the channels in a highly redundant manner, or for it to reject the use of a particularly noisy channel. These points can be illustrated by comparing the curves in which ($L = 4$, $R = 0.75$) and ($L = 2$, $R = 0.50$) and also the curves when ($L = 3$, $R = 0.75$) and ($L = 2$, $R = 0.75$). In the first two cases the combination of the number of channels and rate is such as to allow the system to make little or no use of a channel during those periods when it is particularly noisy. The second two cases do not allow this freedom as even the highly-noisy channel must be used to transmit a significant number of information bits in order to achieve the required throughput. Hence the improvement is smaller.

(iv) The performance of the fixed system is substantially independent of R and it should be noted that it is independent of L, when the total bandwidth and the average transmitted power are kept constant.

Fig. 6 shows a result that has important implications in so far as the implementation of the MC–AFEC is concerned. The figure shows the optimum BER performance obtained for the case $L = 3$, $R = 0.5$ (corresponding to $K = 95$). The optimum performance is obtained by selecting the appropriate triplet (3-code group) out of the complete set of 23 triplets formed based on $R = 0.5$. The other curve represents the BER result obtained when the triplet {(63, 63, 0), (63, 32, 5), (63, 0, —)} alone is used.

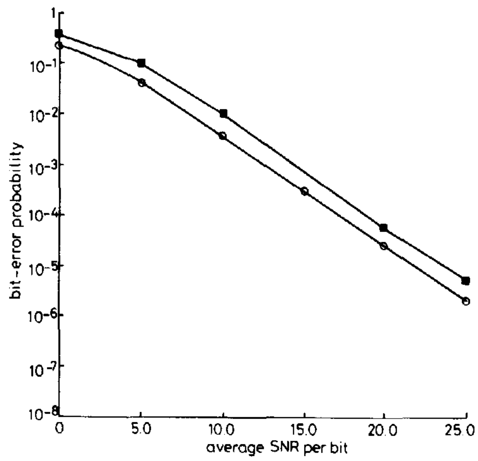

Fig. 6 *Effect of using one triplet only*
$L = 3$ $R = 0.5$
○—○ selection from 23 triplets
■—■ one triplet only

The allocation of the three codes to the three channels can be made very simple. The three channel numbers are first ordered according to their CQE figure, then 63 information bits are transmitted on the best channel without redundancy, the remaining 32 information bits are transmitted on the second ranked channel using a 32/63 rate code, and 63 dummy bits are sent on the worst channel (even though those bits carry no information it is still necessary to send them for the process of channel quality estimation). It is clear from the Figure that only a marginal loss in performance results from using the single triplet.

The results shown in Fig. 6 are general in that they are not restricted to the particular L and R covered by the figure. However, they are not general for any triplet. If a single L-code group is to be selected to operate alone over all times, it should be chosen such that one of its members is as redundant as possible, its next member is again as redundant as possible, and so on. This choice implies that the worst channel is utilised in as redundant a manner as possible, at the cost of transmitting an increased number of information bits on the better channels, with low or no protection against errors being provided. The degradation below the optimum performance would then be minimal because the main cause of the severe deterioration in performance is caused by those intervals when one (or more) of the channels is extremely noisy. The L-code group selected as explained above takes care of these periods very effectively, thus providing a remedy to the most severe cases. During other intervals, when none of the channels are badly affected by noise, the performance of the system is acceptable, and the gain obtained by employing the single L-code group, selected as was just explained, is slightly inferior to that of the optimum L-code group. As a result, the overall performance when this single L-code group is used alone is very close to optimum.

The implication of the result of Fig. 6 is that in practice it will only be necessary to use a small set of triplets (probably one) selected carefully, and that such a system would be relatively easy to implement.

Another important point worth noting is that the reduction of the average BER is not the only gain achieved from reducing the throughput or increasing the number of channels. By doing so, the system becomes more reliable in respect of reducing the percentage of time during which the BER exceeds a given abscissa. Fig. 7 shows the cumulative distribution of BER at $R = 0.33$,

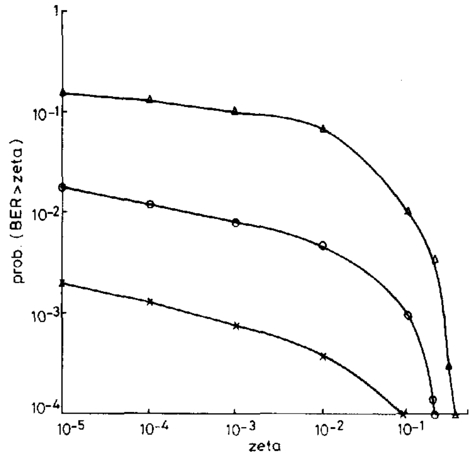

Fig. 7 *BER distribution for the MC–AFEC system for different throughput rates when $L = 3$*
×—× $R = 0.33$
○—○ $R = 0.50$
△—△ $R = 0.60$

0.5, 0.75 when $\bar{\gamma} = 15$ dB, and $L = 3$. From the figure it is clear that the BER exceeds 10^{-3} for 10% of the time when $R = 0.75$, 0.8% of the time when $R = 0.5$, and only 0.08% of the time when $R = 0.33$. Interpreted in practical terms this means that if 10^{-3} is considered to be the worst acceptable bit error rate for the system under consideration then the values given above represent the percentage of time in which the performance of the system is not acceptable.

4 Conclusion

A new multi-channel adaptive forward error-correction (MC–AFEC) system has been proposed and it has been shown that by using a simple adaptation strategy the overall system performance measured in terms of bit error rate is considerably improved as compared with that of either a fixed-rate forward error-correction system or maximal-ratio combining (MRC) diversity system.

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