Adaptive Binary Coding for Diversity Communication Systems

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ABSTRACT

Coding and diversity are two powerful techniques to combat fading effects on communication channels. In this paper the available diversity channels are utilized by forward error correction coding in an adaptive fashion to improve the reliability of the system. Based on the quality of the diversity channels, the code rate over each channel is determined using discrete optimization of the overall error probability, subject to the constraint of fixed overall throughput rate. It is found that the proposed system provides noticeable gain over the classical diversity system when binary BCH codes with hard-decision decoding are used. Moreover, the proposed system offers flexibility in choosing the throughput of the system, which the diversity system lacks. The advantages of the proposed system are obtained at only slight increase in implementation complexity.

INTRODUCTION

Fading is a severe problem that many mobile communication systems face. Coding and diversity are two powerful techniques for improving digital transmission over fading channels. Diversity schemes are effective over independently fading channels because it is very improbable that all the received copies of the signal are affected by deep fade. Coding, on the other hand, gives the system the capability of combating the bit errors that are caused by channel noise. For time-varying channels adaptive coding, in which the error correction capability is matched to the prevailing channel conditions, improves the performance significantly.

Coding and diversity have been combined in a number of ways to further enhance the performance of digital communication systems over fading channels. A straightforward way of employing both techniques is first to encode the information sequence for the purpose of error correction (Forward Error Correction Systems, FEC) and then send the resulting codeword over several diversity channels. Upon receiving the several copies of the transmitted signal, the receiver may either select the best one [1-3], or use them all to obtain an estimate of the transmitted codeword that is more reliable than that obtainable from any of the diversity channels [4]. In both cases decoding is performed afterwards. In a similar way. ARQ schemes have been combined with diversity techniques [5].

In [6] coding is proposed as an alternative basis of implementing diversity selection and it has been shown to be more attractive than the conventional diversity selection based on power measurement. Benelli [7] has suggested using a number of independent channels in a manner different from that of classical diversity systems. He proposed a system in which each the codewords to be transmitted are divided into L sub-blocks for transmission over L channels such that the i^{th} sub-block of all codewords are transmitted as one block over the i^{th} channel, $1 \leq i \leq L$.

In this paper we propose an adaptive forward error correction scheme where the distribution of the information bits, and hence the encoding process, are dependent on the relative channel qualities. Both correlated and uncorrelated channels will be considered. The performance criterion of the system is to communicate at as small an error as possible whilst maintaining a fixed throughput rate. The system, in implementation terms, reduces to that of selecting the best group of codes for a given channel estimation. Therefore, the system proposed here may be viewed as a wider-range selective diversity system.

SYSTEM DESCRIPTION

It is assumed that L fading channels are available for transmission. The L channels are utilized equally in terms of transmission rate, i.e. an equal number of bps is transmitted over each channel.

Consider a block of K information bits. To utilize the L channels, the block of K bits will be split into L segments according to the relative qualities of the channels. Each segment is then encoded into a codeword of length n to be transmitted over the available channels. This procedure requires obtaining estimates of the channel quality through a Channel Quality Estimator (CQE) circuit. This circuit is required by many adaptive systems, such as the ones in [6], [8] and [9]. Details of CQE circuits can be found in [8] and [10].

The operation of the system can be summarized as follows. Based on the current information about the quality of each of the L channels, the number of information bits, and hence the number of check bits that are to be transmitted over each channel is determined. This is done subject to the constraint that a total of K information bits are transmitted through the transmission of the L codewords. Those channels having a poor quality are allocated fewer information bits and more check bits,

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whereas the better quality channels are allocated an increased number of information bits and a reduced number of check bits. In other words, the code rates are adjusted to match the prevailing channel conditions, subject to the constraint of constant average rate. This process is illustrated in Figure 1 for L=3.

PERFORMANCE ANALYSIS

The L channels are assumed to be slowly-fading Rayleigh channels with AWGN. During an adaptation period, the i^{th} channel is attenuated by $|\alpha_i|$, where α_i is a complex Gaussian random variable with zero mean and a variance of one-half for both the real and imaginary parts, and $|\bullet|$ denotes the envelope, which is Rayleigh distributed in this case. All the channels are assumed to be identical on the average; i.e., $E[|\alpha_i|^2] = 1$. The channels, however, are not assumed uncorrelated. They are described by the joint probability density function of the α_i's. Let $\underline{\alpha} = \{\alpha_1, \alpha_2, \ldots, \alpha_L\}$. Then, the probability density function of $\underline{\alpha}$ is expressed as [10] :

$$f(\underline{\alpha}) = \frac{1}{\pi^L \det K_{\underline{\alpha}}} \exp(-\underline{\alpha} K_{\underline{\alpha}}^{-1} \underline{\alpha}^*) \qquad (1)$$

where $(\bullet)^*$ denotes the Hermitian transpose, and $K_{\underline{\alpha}}$ is an $L \times L$ covariance matrix with entries $(K_{\underline{\alpha}})_{i,j} = E[\alpha_i \alpha_j^*]$. If the α_i's are uncorrelated then $K_{\underline{\alpha}}$ will simply reduce to the identity matrix .

Define E_b to be the transmitted energy per information bits and N_o to be the one-sided power spectral density of the AWGN. Let the L estimates of channel qualities be denoted by the vector $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_L\}$, where γ_i is the SNR per (information) bit on the i^{th} channel, defined as $(E_b / N_o)|\alpha_i|^2$. Based on Γ, a block of K information bits will be partitioned into L segments of lengths $\{k_1, k_2, \ldots, k_L\}$. Each segment i, $1 \le i \le L$, will be encoded into n bits using an (n,k_i) code before transmitting it over the i^{th} channel. The throughput of the system, R, is given by:

$$R = \frac{\sum_{i=1}^{L} k_i}{n \times L} = \frac{K}{n \times L} \qquad (2)$$

and is kept constant at all times.

Let P_i denote the post-decoding bit error probability on the i^{th} channel. Then the bit error probability averaged over the L channels is:

$$P = \frac{\sum_{i=1}^{L} k_i P_i}{K} \qquad (3)$$

The exact equation relating the post-decoding bit error probability to the channel bit error rate is a function of the code weight structure and the decoding algorithm, and is unknown for most codes. Various close approximations or bounds are usually invoked for analysis purposes. In this study, the bound in [11] is used with slight improvement, to read:

$$P_i < \frac{1}{n} \sum_{j=t_i+1}^{n} \min(j + t_i, n) \cdot \binom{n}{j} \varepsilon^j (1-\varepsilon)^{n-j} \qquad (4)$$

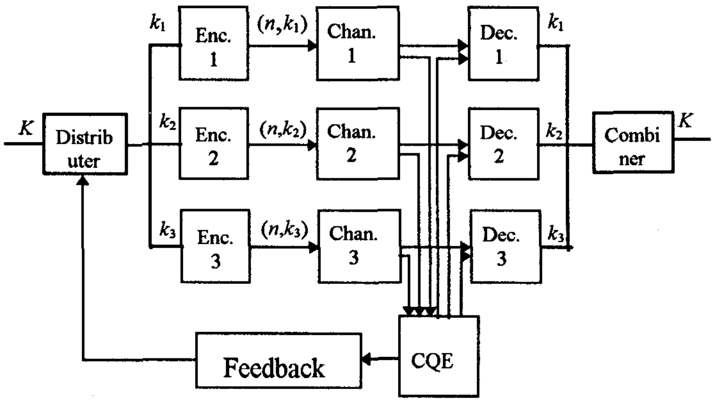

Figure 1.a: Block diagram of the proposed system for L=3

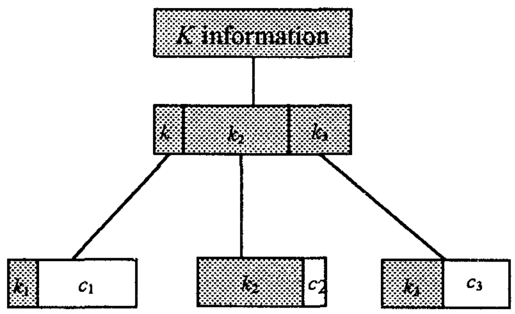

Figure 1.b: Distributing K inf. bits for transmission over three channels

where t_i is the error correction capability of the (n,k_i) code and ε_i is the bit error rate on the i^{th} channel, which is a function of γ_i for a given modulation scheme. Non-coherent BFSK is assumed.

Let $f(\Gamma)$ denote the joint pdf of Γ. The overall average post-decoding bit error probability of the system is obtained by averaging Equation (3) over $f(\Gamma)$, that is

$$\overline{P} = \iint \cdots \int P \times f(\Gamma) d\Gamma \qquad (5)$$

The objective of the adaptation process is to minimize Equation (5) subject to the constraint in Equation (2). It should be noted that P is a function of k_i and ε_i, and both of them are functions of γ_i's, the elements of the vector Γ. What is hoped out of the minimization process is to obtain a set of relations between k_i's minimization process is to obtain a set of relations between k_i's and γ_i's. Unfortunately, such an analytical solution is an extremely tedious task due the complexity of the problem. An alternative discrete optimization is carried out.

The two issues of the constraint on the throughput, R, and the minimization of the average bit error probability, $\overline{P}$, are dealt with separately as follows:

A. Decide on a starting list S of error-correcting codes which have the same codeword length, n, but which are of different rates. For a given throughput R and the L diversity channels form the group G of the sets of L codes (denoted as L-code sets) so that each member of G satisfies the rate constraint in (2). Assume M such L-code sets are obtained. Each L-code set corresponds to some distribution of the total number of information bits K among the L code in the set.

B. For a given estimate of γ, let the transmitter select that L-code set of G which yields the minimum P, according to (4), and which thereby minimizes the integrand of (5). The selected L-code set is used until a new estimate of Γ is available. Based on the recent estimate of Γ, the best L-code set is applied, and so on.

The class of BCH codes of length 63 is used as an example for the application of the ideas developed above. To illustrate the process of L-code grouping, consider the list S of BCH codes of length 63. Now consider a second-order diversity system (L=2) to operate at $R \cong 0.5$. Five pairs satisfying the rate of $R \cong 0.5$ are shown in Table (1). The (63,63) code represents the case when all 63 bits are information bits, while the code (63,0) stands for transmitting 63 dummy bits which carry no information. Note that the first set (pair) corresponds to the selective diversity system.

Set	Code 1	Code 2
1	(63,63,00)	(63,00,--)
2	(63,57,01)	(63,06,15)
3	(63,51,02)	(63,12,11)
4	(63,46,03)	(63,17,10)
5	(63,36,05)	(63,27,07)

Table 1 Five sets (pairs) satisfying the rate $R \cong 0.5$.

The process of error minimization referred to earlier involves (in effect) first selecting the appropriate L-code set and then allocating its L codes to the appropriate channels. This amounts to allocating the higher-rate code to the best channel and continuing down to the point where the lowest-rate code is assigned to the worst channel. Given a vector Γ, the question of which L-code set yields the best performance can be determined in a relatively simple manner. As will be shown later, the L-dimensional space of Γ can be neatly partitioned into regions, where in each of these regions a particular L-code set outperforms other sets. Every new estimate of Γ represents a point in the space; the region in which this point lies determines the best set to use.

RESULTS AND DISCUSSION

The proposed system was studied for different number of channels and different rates. For rate = $1/L$ the system was compared with the selective diversity system. The effect of correlation was also studied and demonstrated for the case L=2.

Figure 2 shows a comparison between adaptive coded systems of 1/L-rate with selective diversity systems for L=2, 3 and 4. The results show the superiority of the adaptive systems. It is also noted that the gains increase with increasing the diversity order. At $\overline{P} = 10^{-5}$, gains of 0.4, 0.8 and 1.3 dB are achieved for L=2, 3 and 4, respectively.

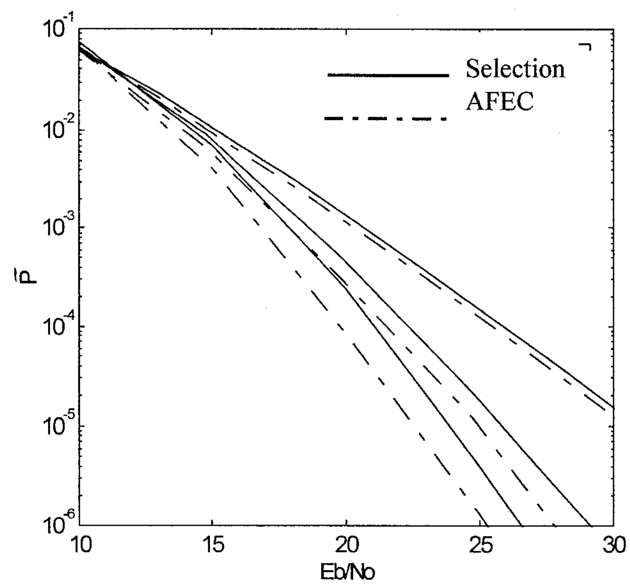

Figure 2: Comparison between AFEC and selective diversity for $L = 2,3,4$.

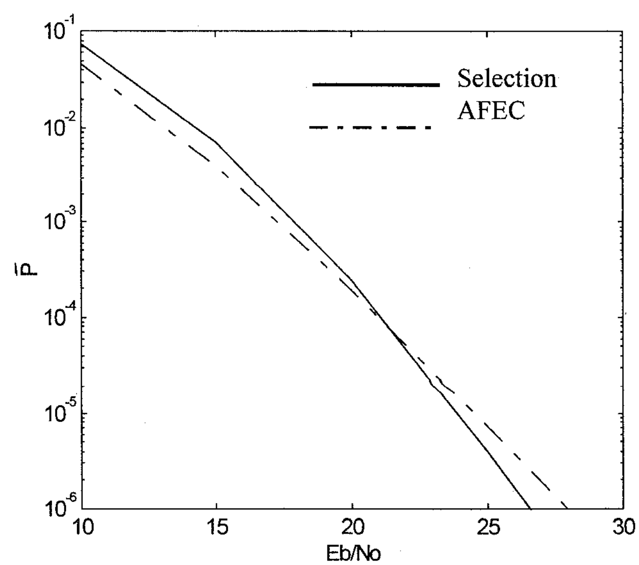

Figure 3: Comparison between selective diversity (R=0.25) and AFEC (R=0.5). In both cases $L = 4$.

Adaptive coding offers another advantage over selective diversity systems; while the latter operates at fixed rates of 1/L, the former can operate at different rates. Figure 3 illustrates this flexibility for $L = 4$, where the adaptive system has a rate of 0.5 (twice the rate of the selective diversity system), yet it performs close to the other system.

The effect of channel correlation is shown in Figure 4, for $L = 2$. The two channels are assumed to be correlated with the correlation matrix $K_{\underline{\alpha}}$

$$K_{\underline{\alpha}} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \quad (6)$$

where ρ here represents the correlation coefficient between the two channels. It is clearly seen that values of ρ as high as 0.5 result in only slight degradation of system performance. This is in agreement with the known results of selective diversity systems.

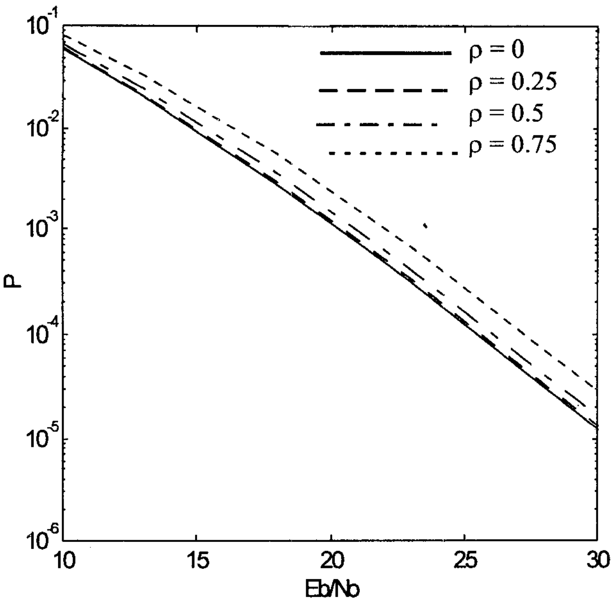

Figure 4: The effect of correlation on the performance of the adaptive system for $L = 2$.

Figure 5 shows the effect of increasing the diversity order for systems with the same rate. Results for codes with rate ½ and L=2, 3, and 4 are shown. It is seen that for L=2 and 3, close performance is achieved, whereas for L=4 a large improvement is obtained. This can be explained as follows. Most of the improvement in the performance is due to the system's freedom to avoid using a channel when it is in deep fade. From that aspect, the adaptive system with rate ½ and L=2 or 3 have the same such freedom, whereas for L=4 the system has more freedom to avoid using two channels when they are in deep fade.

The process of selecting the best set for a given estimation vector can be implemented in a straightforward manner. The space of operation is neatly divided into regions in each of which a set is optimum.

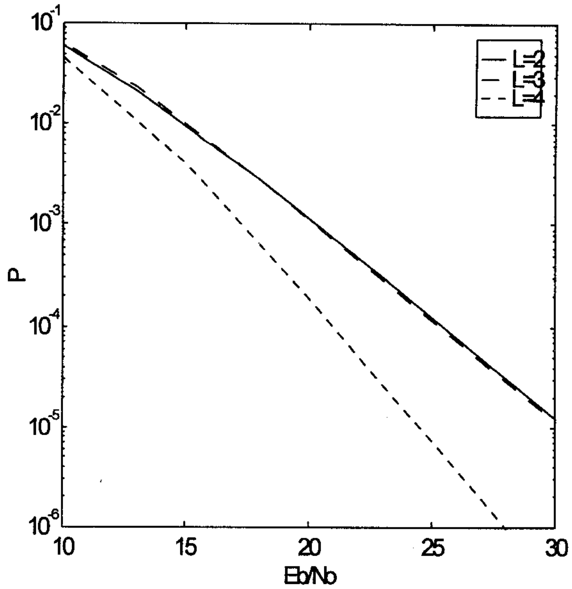

Figure 5: The effect of increasing the diversity order on the adaptive system for R =0.5.

Figure 6 illustrate this partitioning for the case L=2 and the five pairs given in Table 1. The outside region (labeled 1) corresponds to the case where one channel is in deep fade compared to the other channel, and utilizing the latter channel only is the best decision to take (i.e. pair # 1), whereas the most inner region corresponds to the case where the two channels have approximately the same quality, and shall be utilized equally (i.e. pair 5). The other three regions correspond to cases between these two extremes, and the best pair to use is the one given by the label.

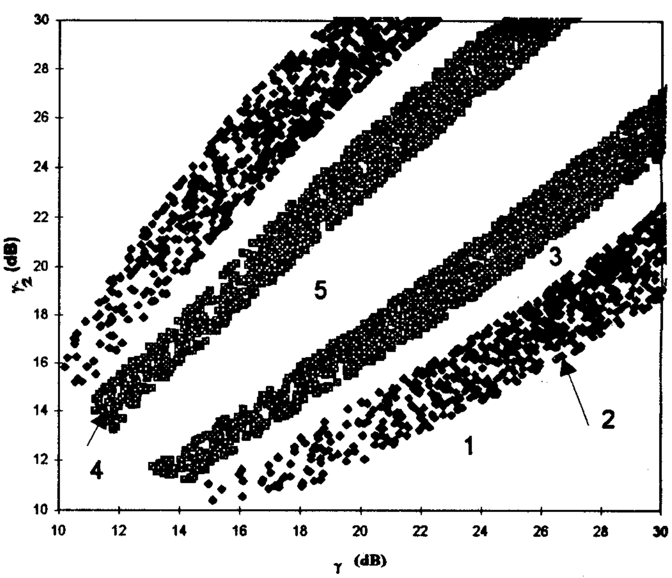

Figure 6: **Regions of operation of each of the five pairs in Table 1**

CONCLUSIONS

An adaptive forward error correction scheme based on binary codes and operating over diversity channels has been presented and analyzed in this paper. Based on the quality of the diversity channels, the code rate over each channel is determined using discrete optimization of the overall error probability, subject to the constraint of fixed overall throughput rate. Using binary BCH codes with hard-decision decoding the proposed system offers a gain of 0.4-1.3 dB over classical diversity systems for diversity orders of 2-4. Moreover, the proposed system can operate at any desirable throughput rate which is an added advantage compared to selective diversity systems. The effect of channel correlation on system performance is similar to that observed in selection diversity.

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