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Optimal power flow using particle swarm optimization

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Abstract

This paper presents an efficient and reliable evolutionary-based approach to solve the optimal power flow (OPF) problem. The proposed approach employs particle swarm optimization (PSO) algorithm for optimal settings of OPF problem control variables. Incorporation of PSO as a derivative-free optimization technique in solving OPF problem significantly relieves the assumptions imposed on the optimized objective functions. The proposed approach has been examined and tested on the standard IEEE 30-bus test system with different objectives that reflect fuel cost minimization, voltage profile improvement, and voltage stability enhancement. The proposed approach results have been compared to those that reported in the literature recently. The results are promising and show the effectiveness and robustness of the proposed approach.

Keywords: Optimal power flow; Particle swarm optimization; Combinatorial optimization

1. Introduction

In the past two decades, the problem of optimal power flow (OPF) has received much attention. It is of current interest of many utilities and it has been marked as one of the most operational needs. The OPF problem solution aims to optimize a selected objective function such as fuel cost via optimal adjustment of the power system control variables, while at the same time satisfying various equality and inequality constraints. The equality constraints are the power flow equations, while the inequality constraints are the limits on control variables and the operating limits of power system dependent variables. The problem control variables include the generator real powers, the generator bus voltages, the transformer tap settings, and the reactive power of switchable VAR sources, while the problem dependent variables include the load bus voltages, the generator reactive powers, and the line flows. Generally, the OPF problem is a large-scale highly constrained nonlinear nonconvex optimization problem.

A wide variety of optimization techniques have been applied in solving the OPF problems [1–19] such as nonlinear programming [1–6], quadratic programming [7,8], linear programming [9–11], Newton-based techniques [12,13], sequential unconstrained minimization technique [14], and interior point methods [15,16]. Generally, nonlinear programming based procedures have many drawbacks such as insecure convergence properties and algorithmic complexity. Quadratic programming based techniques have some disadvantages associated with the piecewise quadratic cost approximation. Newton-based techniques have a drawback of the convergence characteristics that are sensitive to the initial conditions and they may even fail to converge due to the inappropriate initial conditions. Sequential unconstrained minimization techniques are known to exhibit numerical difficulties when the penalty factors become extremely large. Although linear programming methods are fast and reliable they have some disadvantages associated with the piecewise linear cost approximation. Interior point methods have been reported as computationally efficient, however, if the step size is not chosen properly, the sub-linear problem may have a solution that is infeasible in the original nonlinear domain [15]. In addition, interior point methods, in general, suffer from bad initial, termination, and optimality criteria and, in most cases, are unable to solve nonlinear and quadratic objective functions [16]. For more discussions on these techniques, we direct the reader to consult the comprehensive survey presented in Ref. [17].

Generally, most of these approaches apply sensitivity analysis and gradient-based optimization algorithms by linearizing the objective function and the system constraints around an operating point. Unfortunately, the problem of the OPF is a highly nonlinear and a *multimodal* optimization problem, i.e. there exist more than one local optimum.

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Hence, local optimization techniques, which are well elaborated, are not suitable for such a problem. Moreover, there is no local criterion to decide whether a local solution is also the global solution. Therefore, conventional optimization methods that make use of derivatives and gradients are, in general, not able to locate or identify the global optimum. On the other hand, many mathematical assumptions such as convex, analytic, and differential objective functions have to be given to simplify the problem. However, the OPF problem is an optimization problem with, in general, nonconvex, nonsmooth, and nondifferentiable objective functions. These properties have become more evident and dominant if the effects of the valve-point loading of thermal generators and the nonlinear behavior of electronic-based devices such as FACTS are taking into consideration. Hence, it becomes essential to develop optimization techniques that are efficient to overcome these drawbacks and handle such difficulties.

Heuristic algorithms such as genetic algorithms (GA) [18]and evolutionary programming [19] have been recently proposed for solving the OPF problem. The results reported were promising and encouraging for further research in this direction. Unfortunately, recent research has identified some deficiencies in GA performance [20]. This degradation in efficiency is apparent in applications with highly *epistatic* objective functions, i.e. where the parameters being optimized are highly correlated. In addition, the premature convergence of GA degrades its performance and reduces its search capability.

Recently, a new evolutionary computation technique, called particle swarm optimization (PSO), has been proposed and introduced [21–24]. This technique combines social psychology principles in socio-cognition human agents and evolutionary computations. PSO has been motivated by the behavior of organisms such as fish schooling and bird flocking. Generally, PSO is characterized as simple in concept, easy to implement, and computationally efficient. Unlike the other heuristic techniques, PSO has a flexible and well-balanced mechanism to enhance and adapt to the global and local exploration abilities.

In this paper, a novel PSO based approach is proposed to solve the OPF problem. The problem is formulated as an optimization problem with mild constraints. In this study, different objective function has been considered to minimize the fuel cost, to improve the voltage profile, and to enhance power system voltage stability. The proposed approach has been examined and tested on IEEE 30-bus standard system. The potential and effectiveness of the proposed approach are demonstrated. Additionally, the results are compared to those reported in the literature.

2. Problem formulation

The OPF problem is to optimize the steady state performance of a power system in terms of an objective function while satisfying several equality and inequality constraints. Mathematically, the OPF problem can be formulated as follows.

$$\text{Min } J(\mathbf{x}, \mathbf{u}) \tag{1}$$

$$\text{Subject to : } g(\mathbf{x}, \mathbf{u}) = 0 \tag{2}$$

$$h(\mathbf{x}, \mathbf{u}) \le 0 \tag{3}$$

where $\mathbf{x}$ is the vector of dependent variables consisting of slack bus power P_{G_1}, load bus voltages V_L, generator reactive power outputs Q_G, and transmission line loadings S_l. Hence, $\mathbf{x}$ can be expressed as

$$\mathbf{x}^T = [P_{G_1}, V_{L_1} \cdots V_{L_{NL}}, Q_{G_1} \cdots Q_{G_{NG}}, S_{l_1} \cdots S_{l_{nl}}] \tag{4}$$

where NL, NG, and nl are number of load buses, number of generators, and number of transmission lines, respectively.

$\mathbf{u}$ is the vector of independent variables consisting of generator voltages V_G, generator real power outputs P_G except at the slack bus P_{G_1}, transformer tap settings T, and shunt VAR compensations Q_c. Hence, $\mathbf{u}$ can be expressed as

$$\mathbf{u}^T = [V_{G_1} \cdots V_{G_{NG}}, P_{G_2} \cdots P_{G_{NG}}, T_1 \cdots T_{NT}, Q_{c1} \cdots Q_{c_{NC}}] \tag{5}$$

where NT and NC are the number of the regulating transformers and shunt compensators, respectively. J is the objective function to be minimized. g is the equality constraints represent typical load flow equations. h is the system operating constraints that include

(a) *Generation constraints*: Generator voltages, real power outputs, and reactive power outputs are restricted by their lower and upper limits as follows:

$$V_{G_i}^{\min} \le V_{G_i} \le V_{G_i}^{\max}, \quad i = 1, \ldots, NG \tag{6}$$

$$P_{G_i}^{\min} \le P_{G_i} \le P_{G_i}^{\max}, \quad i = 1, \ldots, NG \tag{7}$$

$$Q_{G_i}^{\min} \le Q_{G_i} \le Q_{G_i}^{\max}, \quad i = 1, \ldots, NG \tag{8}$$

(b) *Transformer constraints*: Transformer tap settings are bounded as follows:

$$T_i^{\min} \le T_i \le T_i^{\max}, \quad i = 1, \ldots, NT \tag{9}$$

(c) *Shunt VAR constraints*: Shunt VAR compensations are restricted by their limits as follows:

$$Q_{ci}^{\min} \le Q_{ci} \le Q_{ci}^{\max}, \quad i = 1, \ldots, NC \tag{10}$$

(d) *Security constraints*: These include the constraints of voltages at load buses and transmission line loadings as follows:

$$V_{L_i}^{\min} \le V_{L_i} \le V_{L_i}^{\max}, \quad i = 1, \ldots, NL \tag{11}$$

$$S_{l_i} \le S_{l_i}^{\max}, \quad i = 1, \ldots, \mathrm{nl} \tag{12}$$

It is worth mentioning that the control variables are self-constrained. The hard inequalities of P_{G_1}, V_L, Q_G, and S_l can be incorporated in the objective function as quadratic penalty terms. Therefore, the objective function can be augmented as follows:

$$J_{\mathrm{aug}} = J + \lambda_P (P_{G_1} - P_{G_1}^{\lim})^2 + \lambda_V \sum_{i=1}^{\mathrm{NL}} (V_{L_i} - V_{L_i}^{\lim})^2 + \lambda_Q \sum_{i=1}^{\mathrm{NG}} (Q_{G_i} - Q_{G_i}^{\lim})^2 + \lambda_S \sum_{i=1}^{\mathrm{nl}} (S_{l_i} - S_{l_i}^{\max})^2 \tag{13}$$

where λ_P, λ_V, λ_Q, and λ_S are penalty factors and $x^{\lim}$ is the limit value of the dependent variable x given as

$$x^{\lim} = \begin{cases} x^{\max}; & x > x^{\max} \\ x^{\min}; & x < x^{\min} \end{cases} \tag{14}$$

3. Particle swarm optimization

3.1. Overview

Like evolutionary algorithms, PSO technique conducts search using a population of particles, corresponding to individuals. Each particle represents a candidate solution to the problem at hand. In a PSO system, particles change their positions by flying around in a multi-dimensional search space until a relatively unchanging position has been encountered, or until computational limitations are exceeded. In social science context, a PSO system combines a social-only model and a cognition-only model [21]. The social-only component suggests that individuals ignore their own experience and adjust their behavior according to the successful beliefs of individuals in the neighborhood. On the other hand, the cognition-only component treats individuals as isolated beings. A particle changes its position using these models.

3.2. PSO algorithm

The basic elements of PSO technique are briefly stated and defined as follows:

- *Particle*, $X(t)$: It is a candidate solution represented by an m-dimensional vector, where m is the number of optimized parameters. At time t, the jth particle $X_j(t)$ can be described as $X_j(t) = [x_{j,1}(t), \ldots, x_{j,m}(t)]$, where xs are the optimized parameters and $x_{j,k}(t)$ is the position of the jth particle with respect to the kth dimension, i.e. the value of the kth optimized parameter in the jth candidate solution.
- *Population*, pop(t),: It is a set of n particles at time t, i.e. $\mathrm{pop}(t) = [X_1(t), \ldots, X_n(t)]^T$.
- *Swarm*: It is an apparently disorganized population of moving particles that tend to cluster together while each particle seems to be moving in a random direction [23].
- *Particle velocity*, $V(t)$: It is the velocity of the moving particles represented by an m-dimensional vector. At time t, the jth particle velocity $V_j(t)$ can be described as $V_j(t) = [v_{j,1}(t), \ldots, v_{j,m}(t)]$, where $v_{j,k}(t)$ is the velocity component of the jth particle with respect to the kth dimension.
- *Inertia weight*, $w(t)$: It is a control parameter that is used to control the impact of the previous velocities on the current velocity. Hence, it influences the trade-off between the global and local exploration abilities of the particles [23]. For initial stages of the search process, large inertia weight to enhance the global exploration is recommended while, for last stages, the inertia weight is reduced for better local exploration.
- *Individual best*, $X^*(t)$: As a particle moves through the search space, it compares its fitness value at the current position to the best fitness value it has ever attained at any time up to the current time. The best position that is associated with the best fitness encountered so far is called the individual best, $X^*(t)$. For each particle in the swarm, $X^*(t)$ can be determined and updated during the search. In a minimization problem with objective function J, the individual best of the jth particle $X_j^*(t)$ is determined such that $J(X_j^*(t)) \le J(X_j(\tau)), \quad \tau \le t$. For simplicity, assume that $J_j^* = J(X_j^*(t))$. For the jth particle, individual best can be expressed as $X_j^*(t) = [x_{j,1}^*(t), \ldots, x_{j,m}^*(t)]$.
- *Global best*, $X^{**}(t)$: It is the best position among all individual best positions achieved so far. Hence, the global best can be determined such that $J(X^{**}(t)) \le J(X_j^*(t)), j = 1, \ldots, n$ For simplicity, assume that $J^{**} = J(X^{**}(t))$.
- *Stopping criteria*: these are the conditions under which the search process will terminate. In this study, the search will terminate if one of the following criteria is satisfied: (a) the number of iterations since the last change of the best solution is greater than a prespecified number or (b) the number of iterations reaches the maximum allowable number.

In this study, the basic PSO has been developed as follows:

- An annealing procedure has been incorporated in order to make uniform search in the initial stages and very local search in the later stages. A decrement function for decreasing the inertia weight given as $w(t) = \alpha w(t-1)$, α is a decrement constant smaller than but close to 1, is proposed in this study.
- Feasibility checks procedure of the particle positions has been imposed after the position updating to prevent the particles from flying outside the feasible search space.
- The particle velocity in the kth dimension is limited by some maximum value, $v_k^{\max}$. This limit enhances the local

exploration of the problem space and it realistically simulates the incremental changes of human learning [21]. To ensure uniform velocity through all dimensions, the maximum velocity in the kth dimension is proposed as:

$$v_k^{\max} = (x_k^{\max} - x_k^{\min})/N \tag{15}$$

where N is a chosen number of intervals.

In PSO algorithm, the population has n particles and each particle is an m-dimensional vector, where m is the number of optimized parameters. Incorporating the above modifications, the computational flow of PSO technique can be described in the following steps.

Step 1 (Initialization): Set the time counter $t = 0$ and generate randomly n particles, $\{X_j(0), j = 1, \ldots, n\}$, where $X_j(0) = [x_{j,1}(0), \ldots, x_{j,m}(0)]$. $x_{j,k}(0)$ is generated by randomly selecting a value with uniform probability over the kth optimized parameter search space $[x_k^{\min}, x_k^{\max}]$. Similarly, generate randomly initial velocities of all particles, $\{V_j(0), j = 1, \ldots, n\}$, where $V_j(0) = [v_{j,1}(0), \ldots, v_{j,m}(0)]$. $v_{j,k}(0)$ is generated by randomly selecting a value with uniform probability over the kth dimension $[-v_k^{\max}, v_k^{\max}]$. Each particle in the initial population is evaluated using the objective function, J. For each particle, set $X_j^*(0) = X_j(0)$ and $J_j^* = J_j$, $j = 1, \ldots, n$. Search for the best value of the objective function J_{best}. Set the particle associated with J_{best} as the global best, $X^{**}(0)$, with an objective function of J^{**}. Set the initial value of the inertia weight $w(0)$.
Step 2 (Time updating): Update the time counter $t = t + 1$.
Step 3 (Weight updating): Update the inertia weight $w(t) = \alpha w(t-1)$.
Step 4 (velocity updating): Using the global best and individual best of each particle, the jth particle velocity in the kth dimension is updated according to the following equation:

$$v_{j,k}(t) = w(t)v_{j,k}(t-1) + c_1 r_1 (x_{j,k}^*(t-1) - x_{j,k}(t-1)) + c_2 r_2 (x_{j,k}^{**}(t-1) - x_{j,k}(t-1)) \tag{16}$$

where c_1 and c_2 are positive constants and r_1 and r_2 are uniformly distributed random numbers in [0,1]. It is worth mentioning that the second term represents the cognitive part of PSO where the particle changes its velocity based on its own thinking and memory. The third term represents the social part of PSO where the particle changes its velocity based on the social-psychological adaptation of knowledge. If a particle violates the velocity limits, set its velocity equal to the limit.
Step 5 (Position updating): Based on the updated velocities, each particle changes its position according to the following equation:

$$x_{j,k}(t) = v_{j,k}(t) + x_{j,k}(t-1) \tag{17}$$

If a particle violates the its position limits in any dimension, set its position at the proper limit.
Step 6 (individual best updating): Each particle is evaluated according to its updated position. If $J_j < J_j^*$, $j = 1, \ldots, n$, then update individual best as $X_j^*(t) = X_j(t)$ and $J_j^* = J_j$ and go to step 7; else go to step 7.
Step 7 (Global best updating): Search for the minimum value $J_{\min}$ among J_j^*, where min is the index of the particle with minimum objective function, i.e. $\min \in \{j; j = 1, \ldots, n\}$. If $J_{\min} < J^{**}$, then update global best as $X^{**}(t) = X_{\min}(t)$ and $J^{**} = J_{\min}$ and go to step 8; else go to step 8.
Step 8 (Stopping criteria): If one of the stopping criteria is satisfied then stop; else go to step 2.

3.3. PSO implementation

The proposed PSO based approach was implemented using the FORTRAN language and the developed software program was executed on a 166 MHz Pentium I PC. Initially, several runs have been done with different values of the PSO key parameters such as the initial inertia weight and the maximum allowable velocity. In our implementation, the initial inertia weight $w(0)$ and the number of intervals in each space dimension N are selected as 1.0 and 10 respectively. Other parameters are selected as: number of particles $n = 50$, decrement constant $\alpha = 0.98$, $c_1 = c_2 = 2$, and the search will be terminated if (a) the number of iterations since the last change of the best solution is greater than 50; or (b) the number of iterations reaches 500.

To demonstrate the effectiveness of the proposed approach, different cases with various objectives are considered in this study.

4. Numerical results

The proposed PSO-based approach has been tested on the standard IEEE 30-bus test system shown in Fig. 1. The system line and bus data are given in Refs. [18,25]. The system has six generators at buses 1, 2, 5, 8, 11, and 13 and four transformers with off-nominal tap ratio in lines 6–9, 6–10, 4–12, and 28–27. In addition, buses 10, 12, 15, 17, 20, 21, 23, 24, and 29 have been selected in Ref. [25] as shunt VAR compensation buses. The minimum and maximum limits on control variables along with the initial operating point are given in Table 1.

In order to demonstrate the effectiveness and robustness of the technique, several cases with different objectives to minimize the total fuel cost, to improve the voltage profile, and to enhance the system voltage stability have been considered as follows.

4.1. Case 1: minimization of fuel cost

In this case, the objective function J is considered as the

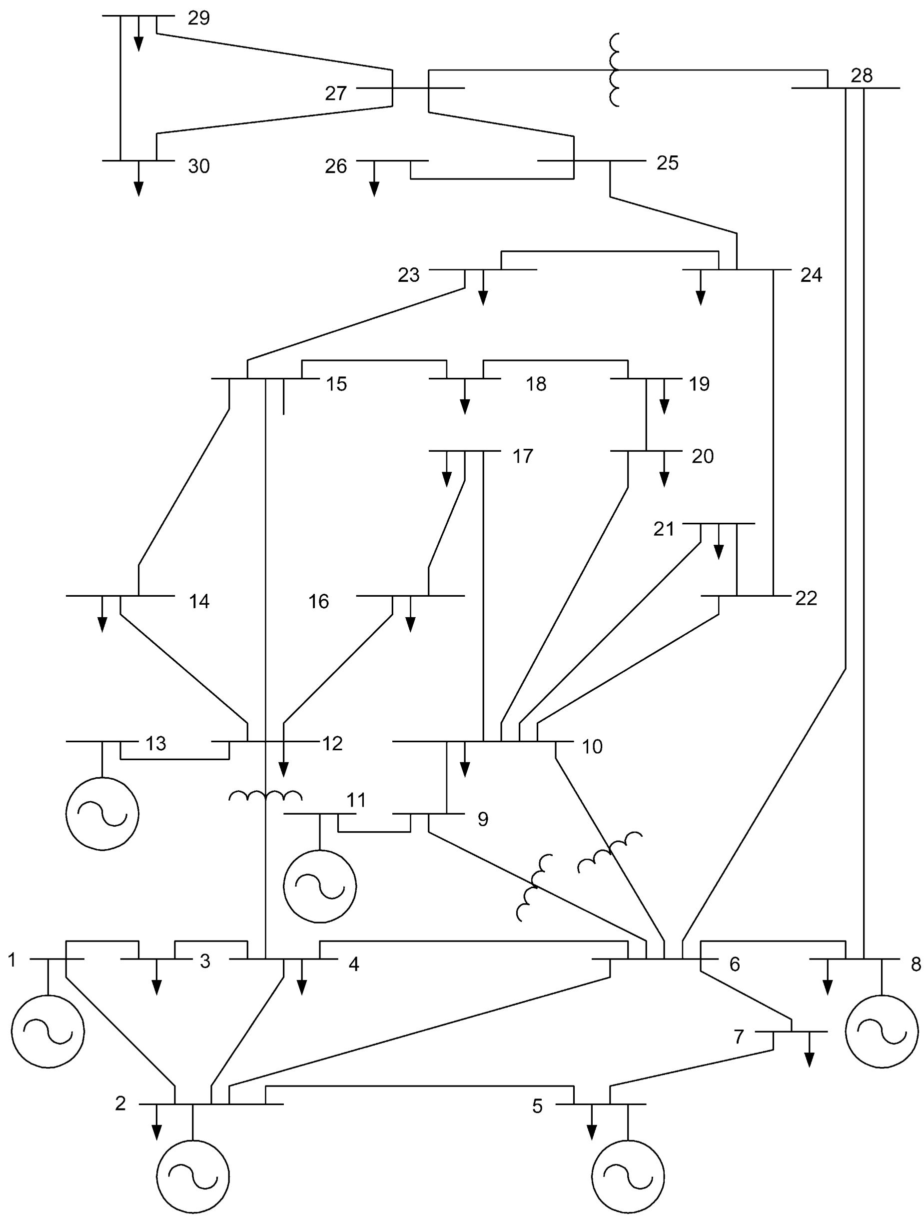

Fig. 1. Single-line diagram of IEEE 30-bus test system.

total fuel cost, i.e.

$$J = \sum_{i=1}^{\mathrm{NG}} f_i \ (\$/\mathrm{h}) \tag{18}$$

where f_i is the fuel cost of the ith generator.

The generator cost curves are represented by quadratic functions as

$$f_i = a_i + b_i P_{\mathrm{G}_i} + c_i P_{\mathrm{G}_i}^2 \ (\$/\mathrm{h}) \tag{19}$$

where a_i, b_i, and c_i are the cost coefficients of the ith generator. The values of these coefficients are given in Table 2.

The variation of the total fuel cost is shown in Fig. 2. The optimal settings of the control variables are given in Table 1. Initially, the total fuel cost was \$901.88. The total cost obtained by the proposed technique is \$800.41. It is clear that the total fuel cost is greatly reduced (11.25% reduction). It is worth mentioning that the initial operating point has voltage violations at buses 19–30. However, all these violations have been alleviated with the proposed optimal control variable settings. With the same control variable limits, initial conditions, and other system data, the problem was solved using gradient-based approach [25] and improved genetic algorithm-based approach [18] with optimal fuel costs of \$804.583 and \$800.805, respectively. It is clear that the proposed PSO based approach outperforms the gradient and GA techniques.

4.2. Case 2: voltage profile improvement

Bus voltage is one of the most important security and

Table 1
Optimal settings of control variables

	Min	Max	Initial	Case 1	Case 2	Case 3	Case 4
P_1	0.50	2.00	0.9921	1.7696	1.7368	1.7553	1.4000
P_2	0.20	0.80	0.8000	0.4898	0.4910	0.4798	0.5500
P_5	0.15	0.50	0.5000	0.2130	0.2181	0.2092	0.2415
P_8	0.10	0.35	0.2000	0.2119	0.2330	0.2450	0.3500
P_{11}	0.10	0.30	0.2000	0.1197	0.1388	0.1151	0.1851
P_{13}	0.12	0.40	0.2000	0.1200	0.1200	0.1200	0.1779
V_1	0.95	1.10	1.0500	1.0855	1.0142	1.0891	1.0500
V_2	0.95	1.10	1.0400	1.0653	1.0022	1.0693	1.0412
V_5	0.95	1.10	1.0100	1.0333	1.0170	1.0464	1.0170
V_8	0.95	1.10	1.0100	1.0386	1.0100	1.0465	1.0282
V_{11}	0.95	1.10	1.0500	1.0848	1.0506	1.0277	1.0910
V_{13}	0.95	1.10	1.0500	1.0512	1.0175	1.0294	1.0876
T_{11}	0.90	1.10	1.0780	1.0233	1.0702	0.9694	1.0192
T_{12}	0.90	1.10	1.0690	0.9557	0.9000	0.9238	0.9573
T_{15}	0.90	1.10	1.0320	0.9724	0.9954	0.9467	1.0120
T_{36}	0.90	1.10	1.0680	0.9728	0.9703	0.9820	0.9505
Q_{c10}	0.00	0.05	0.0	0.0335	0.0403	0.0162	–
Q_{c12}	0.00	0.05	0.0	0.0220	0.0369	0.0424	–
Q_{c15}	0.00	0.05	0.0	0.0198	0.0500	0.0256	–
Q_{c17}	0.00	0.05	0.0	0.0315	0.0000	0.0465	–
Q_{c20}	0.00	0.05	0.0	0.0454	0.0500	0.0348	–
Q_{c21}	0.00	0.05	0.0	0.0381	0.0500	0.0500	–
Q_{c23}	0.00	0.05	0.0	0.0398	0.0500	0.0488	–
Q_{c24}	0.00	0.05	0.0	0.0500	0.0500	0.0500	–
Q_{c29}	0.00	0.05	0.0	0.0251	0.0259	0.0500	–
Fuel cost ($/h)			901.88	800.41	806.38	801.16	647.69
$\sum$ voltage deviations			1.1554	0.8765	0.0891	0.9607	0.7722
L_{max}			0.1681	0.1296	0.1392	0.1246	0.1417

service quality indices. Considering only cost-based objectives in OPF problem may result in a feasible solution that has unattractive voltage profile. In this case, a two-fold objective function is proposed in order to minimize the fuel cost and improve voltage profile by minimizing the load bus voltage deviations from 1.0 per unit. The objective function can be expressed as

$$J = \sum_{i=1}^{NG} f_i + w \sum_{i \in NL} |V_i - 1.0| \tag{20}$$

where w is a weighting factor. The optimal settings of the control variables are given in Table 1. The variation of the total fuel cost is shown in Fig. 3. The system voltage profile of this case is compared to that of case 1 as shown in Fig. 4. It is evident that the voltage profile is greatly improved compared to that of case 1. Specifically, the total sum of voltage deviations is reduced from 0.8765 in case 1 to 0.0891 in case 2 as given in Table 1. This gives a reduction ratio of 90%. The total generation cost in this case, however, is slightly increased by 0.75% of that of case 1.

Table 2
Generator cost coefficients

	G_1	G_2	G_5	G_8	G_{11}	G_{13}
a	0.0	0.0	0.0	0.0	0.0	0.0
b	200	175	100	325	300	300
c	37.5	175	625	83.4	250	250

4.3. Case 3: voltage stability enhancement

The power system ability to maintain constantly acceptable bus voltage at each bus under normal operating conditions, after load increase, following system configuration changes, or when the system is being subjected to a disturbance is a very important characteristic of the system. The nonoptimized control variables may lead to progressive

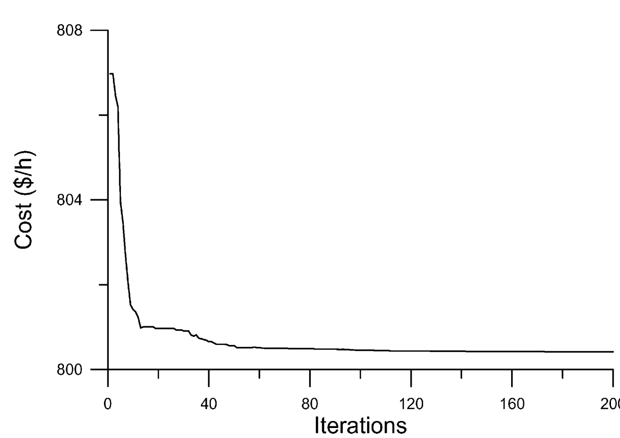

Fig. 2. Fuel cost variation of case 1.

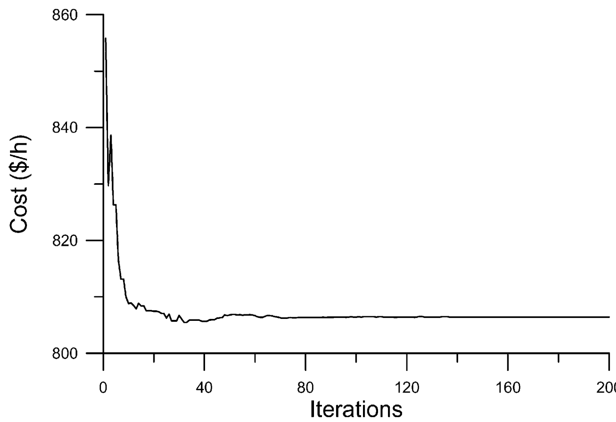

Fig. 3. Fuel cost variation of case 2.

and uncontrollable drop in voltage resulting in an eventual wide spread voltage collapse.

In this case, a two-fold objective function is proposed in order to minimize the fuel cost and enhance the voltage stability profile through out the whole power network. In this study, voltage stability enhancement is achieved through minimizing the voltage stability indicator L-index [26–28] values at every bus of the system and consequently the global power system L-index.

Generally, L-index at any bus varies between zero (no load case) and one (voltage collapse). In order to enhance the voltage stability and move the system far from the voltage collapse point, the following objective function is proposed

$$J = \sum_{i=1}^{NG} f_i + wL_{max} \tag{21}$$

where w is a weighting factor and L_{max} is the maximum value of L-index defined as

$$L_{max} = \max\{L_k, K = 1, \ldots, NL\} \tag{22}$$

The optimal settings of the control variables are given in Table 1. The variation of the total fuel cost is shown in

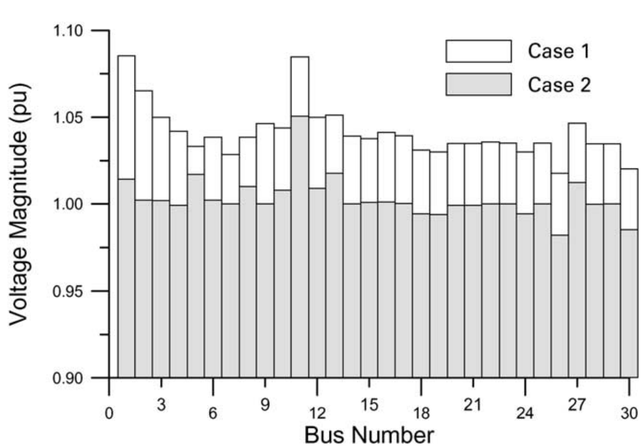

Fig. 4. System voltage profile.

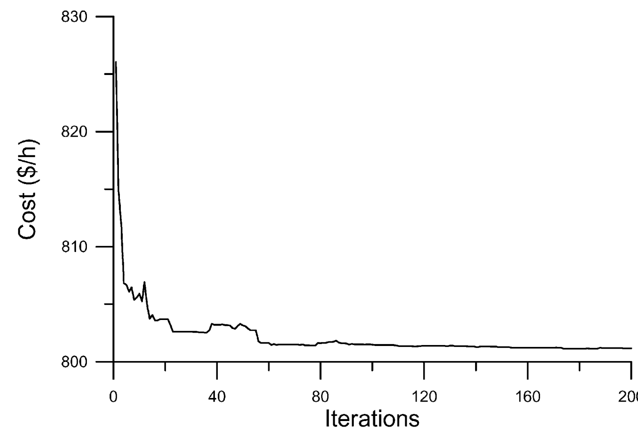

Fig. 5. Fuel cost variation of case 3.

Fig. 5. It can be seen from Table 1 that the value of L_{max} at load buses is significantly reduced in this case. Consequently, the voltage stability distance from collapse has increased.

An additional test has been carried out for different load levels starting from 50% of the base load with a step increase of all loads in the system by 0.05 till voltage collapse. The results show that the maximum load factor with the initial case is 2.65. On the other hand, the load factor increases to 3.2 with the optimized control variables of case 3. This extends the stability margin and gives a gain in power system MVA loading of 21%. The above positive results demonstrate the potential of the proposed approach to improve and enhance the system voltage stability.

4.4. Case 4: piecewise quadratic cost curve

For comparison purposes, the proposed PSO-based OPF algorithm has been applied to the standard IEEE 30-bus test system with the system line and bus data as given in Refs. [2,19]. The upper limit of voltage magnitude at bus 1 is 1.05 pu and there are no shunt VAR compensation buses. In this case, the cost curves of the generators at buses 1 and 2 are represented by piecewise quadratic functions as given in Table 3 [19]. The cost curves of the other generators are the same as case 1. It is obvious that the search space has several local optimal solutions and, therefore, the gradient methods are susceptible to getting trapped on local optimal solution. For this case, the fuel cost objective function of case 1 is considered. The variation of the total fuel cost is shown in

Table 3
Generator cost coefficients for case 4

	From MW	To MW	Cost coefficients		
			a	*b*	*c*
Gen 1	50	140	55.0	0.70	0.0050
	140	200	82.5	1.05	0.0075
Gen 2	20	55	40.0	0.30	0.0100
	55	80	80.0	0.60	0.0200

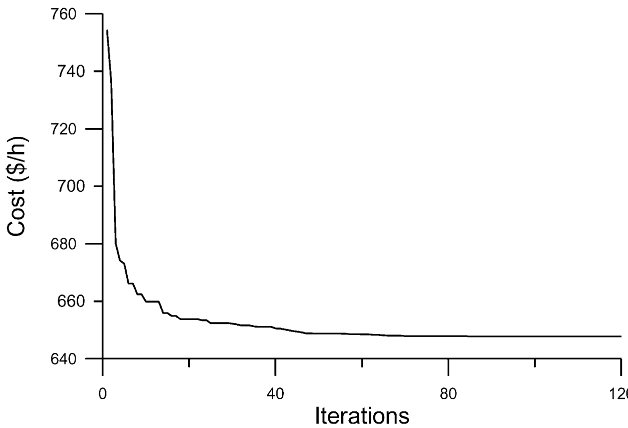

Fig. 6. Fuel cost variation of case 4.

Fig. 6. The optimal settings of control variables are given in Table 1. The total cost obtained by the proposed technique is $647.69 while the minimum cost obtained by evolutionary programming [19] after 100 runs was $647.79. It is clear that the proposed technique outperforms the evolutionary programming.

5. Discussion

Some comments on the proposed approach are now in order:

- Unlike the gradient methods, PSO is a population-based search algorithm, i.e. PSO has implicit parallelism. This property ensures PSO to be less susceptible to getting trapped on local minima.
- PSO uses objective function information to guide the search in the problem space. Therefore, PSO can easily deal with nondifferentiable and nonconvex objective functions. Additionally, this property relieves PSO of assumptions and approximations, which are often required by traditional optimization methods.
- PSO uses probabilistic rules for particle movements, not deterministic rules. Hence, PSO is a kind of stochastic optimization algorithm that can search a complicated and uncertain area. This makes PSO more flexible and robust than conventional methods.
- Unlike GA, PSO has the flexibility to control the balance between the global and local exploration of the search space. This property enhances the search capabilities of PSO technique and avoids the premature convergence of the search process.
- Unlike the traditional methods, the solution quality of the proposed approach does not rely on the initial population. Starting anywhere in the search space, the algorithm ensures the convergence to the optimal solution. With 100 different initializations of case 4, a minimum, maximum, and average value of the fuel cost is compared to those of EP [19]. The results given in Table 4 emphasize that the proposed approach finally leads to the optimal solution regardless the initial one. The results also confirm the robustness and superiority of the proposed approach.
- The candidate solutions in PSO are coded as a set of real numbers. However, most of the control variables such as transformer tap settings and switchable shunt capacitors change in discrete manner. Real coding of these variables represents a limitation of the proposed technique as simple round-off calculations may lead to significant errors.

Table 4
Fuel cost of 100 runs with different initializations

	Min	Max	Ave
Proposed	647.69	647.87	647.73
EP [19]	647.79	652.67	649.67

6. Conclusion

In this paper, a novel particle swarm optimization based approach to OPF problem has been presented. The proposed approach utilizes the global and local exploration capabilities of PSO to search for the optimal settings of the control variables. Different objective functions have been considered to minimize the fuel cost, to improve the voltage profile, and to enhance voltage stability. The proposed approach has been tested and examined with different objectives to demonstrate its effectiveness and robustness. The results using the proposed approach were compared to those reported in the literature. The results confirm the potential of the proposed approach and show its effectiveness and superiority over the classical techniques and genetic algorithms.

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