

## ON-LINE IDENTIFICATION OF A SYNCHRONOUS MACHINE USING A RADIAL BASIS FUNCTION NETWORK

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### Abstract

On-line identification of a synchronous machine using a radial basis function network (RBFN) is presented in this paper. The capability of the proposed identifier to capture the nonlinear operating characteristics of synchronous machines is illustrated. A recursive learning algorithm has been developed to update the network parameters. The results of the proposed identifier performance due to random variations in machine inputs are compared to that obtained by time-domain simulations. Correlation-based model validity tests have been carried out to examine the validity of the proposed identifier. The results demonstrate the adequacy and validity of the proposed RBFN identifier.

### 1 Introduction

In power systems, synchronous machines are widely used and models of such machines play important roles in power system studies such as stability and control analysis. Many synchronous machine models have been developed [1]. In general, simple machine models are good for analysis purposes but not accurate enough for predicting machine performance for control purposes. However, higher order models are too cumbersome for on-line applications. Generally, the synchronous machine is a very complex nonlinear system with dynamics and nonlinearities which cannot be modeled in precise mathematical terms.

Recently, it has been shown that feedforward neural networks with one hidden layer can uniformly approximate any continuous function to a chosen degree of accuracy [2]. Feedforward neural networks trained with backpropagation algorithm, referred as backpropagation neural networks (BPNNs), have been applied to identification and control of dynamical systems [3]. Although BPNNs are widely used, there are several problems associated with these networks such as getting stuck in local minima [4] and slow convergence rate [5].

Similar to BPNN, radial basis function network (RBFN) has the universal approximation ability [5-6]. Unlike the former, RBFN has the best approximation property [7]. It has recently been acknowledged that approximation accuracy properties of RBFN are advantageous as compared to the other methods, including BPNNs [5-9]. Even more important for many applications, the RBFNs provide linear approximation in the network weights. This feature makes

powerful tools of the linear system theory applicable to the RBFN identification of nonlinear systems [9]. The "linear in parameters" of RBFN guarantees the convergence of the parameters to the global minimum. Moreover, the local tunability of RBFN makes only a part of the nodes to be affected by any given input [5], and only a portion of the model parameters may need to be adjusted, thus reducing the training time and computational overhead.

In this paper, a single synchronous machine connected to an infinite bus is on-line identified using RBFN. A recursive learning algorithm has been developed to update the network parameters. The potential of the proposed identifier is investigated using various variations in machine inputs. Correlation-based model validity tests have been carried out to examine the validity of the proposed identifier.

### 2 Radial Basis Function Network

Like most feedforward networks, RBF networks have three layers, namely, an input layer, a hidden layer and an output layer. The nodes within each layer are fully connected to the previous layer nodes. A schematic diagram of an RBFN with  $n$  inputs and  $m$  outputs is given in Fig. 1.

The input variables are each assigned to a node in the input layer and pass directly to the hidden layer without weights. The hidden layer nodes are the RBF units. Each node in this layer contains a parameter vector called a center. The node calculates the euclidean distance between the center and the network input vector, and passes the result through a nonlinear function,  $\Phi(\cdot)$ . The output layer is essentially a set of linear combiners. The overall input-output response of the RBFN is a mapping  $f: R^n \rightarrow R^m$  that is,

$$y_i = w_{0i} + \sum_{j=1}^{n_h} w_{ji} \Phi(\|X - C_j\|, \beta_j) \quad (1)$$

Theoretical investigations and practical results suggest that the choice of the nonlinearity  $\Phi(\cdot)$  is not crucial to the performance of the RBFN. Some typical choices of  $\Phi(\cdot)$  are given in [5]. In this study,  $\Phi(\cdot)$  is chosen to be gaussian activation function, that is,

$$\Phi(z, \beta) = \exp(-z^2 / \beta^2) \quad (2)$$

The width parameter of the  $i$ -th hidden unit,  $\beta_i$ , is chosen to be the root mean square distance between  $C_i$  and a number of the nearest neighboring unit centers,  $n_n$ , as

$$\beta_i = \left[ \frac{1}{n_n} \sum_{j=1}^{n_n} (\|C_i - C_j\|)^2 \right]^{1/2} \quad (3)$$

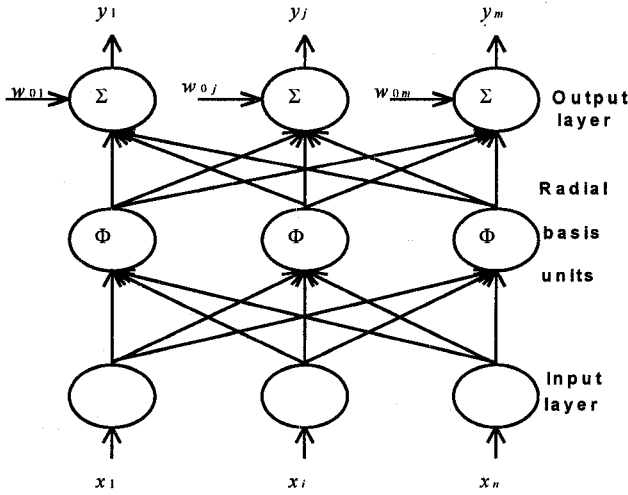


Fig. 1. Schematic diagram of an RBFN

### 3 Recursive Learning Algorithm

For on-line identification using RBFN, a recursive learning algorithm is required to update the network parameters. This algorithm involves a combined form of supervised and unsupervised learning. At each iteration, RBF centers and network weights are updated as follows.

#### 3.1 Updating RBF Centers

Define the input vector of the network at sample  $k$  as

$$V(k) = [Y(k), \dots, Y(k-n_y), U(k), \dots, U(k-n_u)]^T \quad (4)$$

where  $n_y$  and  $n_u$  are the lags of the output and input respectively. Hence the dimension of the input vector and consequently the dimension of the centers of the hidden nodes is given by

$$n_i = m \cdot n_y + n \cdot n_u \quad (5)$$

where  $m$  and  $n$  are the number of outputs and inputs of the network respectively.

Given initial centers  $\{C_j(0), 1 \leq j \leq n_h\}$  which can be generated randomly in the vicinity of the input domain and an initial learning rate  $\alpha(0)$ , the k-means algorithm [10] compute

$$\rho_j(k) = \|V(k) - C_j(k-1)\|, \quad 1 \leq j \leq n_h \quad (6)$$

The updating of a center is based on how far the current input vector is away from the last updated centers. If  $i = \text{argument}\{\min(\rho_j(k)), 1 \leq j \leq n_h\}$  then

$$C_i(k) = C_i(k-1) + \alpha(k)[V(k) - C_i(k-1)] \quad (7)$$

and,

$$C_j(k) = C_j(k-1), \quad \forall j \neq i \quad (8)$$

The learning rate  $\alpha(k) \in (0, 1]$  is given by

$$\alpha(k) = \alpha(k-1) / \sqrt{1 + \text{int}(k/n_h)} \quad (9)$$

where  $\text{int}(x)$  denotes the integer part of  $x$ .

#### 3.2 Updating Network Weights

Once the centers have been updated, the outputs of the hidden layer can be calculated using

$$h_j(k) = \Phi(\|V(k) - C_j(k)\|, \beta_j), \quad 1 \leq j \leq n_h \quad (10)$$

and the input vector to the output layer becomes

$$H(k) = [h_1(k) \dots h_{n_h}(k)]^T \quad (11)$$

Define the  $n_h \times n$  weight matrix at sample  $k$  as

$$W(k) = [w_1(k) \dots w_i(k) \dots w_n(k)] \quad (12)$$

and,

$$w_i(k) = [w_{i1}(k) \dots w_{i n_h}(k)]^T \quad (13)$$

The weighted normal equation can be written as

$$(S_k^T Q_k S_k) W(k) = S_k^T Q_k Y_k \quad (14)$$

where

$$S_k = [H^T(1) \dots H^T(k)]^T \quad (15)$$

$$Y_k = [Y(1) \dots Y(k)]^T \quad (16)$$

and  $Q_k$  is an  $k \times k$  diagonal matrix defined recursively by

$$Q_k = \begin{bmatrix} \mu(k) Q_{k-1} & 0 \\ 0 & 1 \end{bmatrix}, \quad Q_1 = I \quad (17)$$

$\mu(k)$  is the forgetting factor and can be computed as [9]

$$\mu(k) = \mu_0 \mu(k-1) + 1 - \mu_0 \quad (18)$$

$\mu_0$  and  $\mu(0)$  are chosen to be less than but close to 1.

Since the least squares problem in (14) may become ill-conditioned, Givens transformation method [9] has been developed to solve for  $W(k)$  rather than the direct solution of the normal equation because of its numerical advantages.

### 4 Problem Formulation

The seventh order flux-linkage model of a single synchronous machine connected to an infinite bus as shown in Fig. 2 is considered in this study. The system model and parameters are given in the Appendix. The system model can be described by the following nonlinear state space model

$$Z(k+1) = F(Z(k), U(k)) \quad (19)$$

$$Y(k) = G(Z(k)) \quad (20)$$

In the above model,  $F(\cdot)$  and  $G(\cdot)$  are the nonlinear state and output mapping functions respectively. The input vector,  $U(k)$ , represents the mechanical torque,  $T_m$ , and the field voltage,  $V_f$ , that is,  $U(k) = [T_m(k), V_f(k)]^T$ .  $Z(k)$  is the state vector, that is,  $Z(k) = [\delta, \omega, \psi_d, \psi_f, \psi_D, \psi_Q, \psi_Q]^T$  and  $Y(k)$  is the output vector. In this study, we observe two quantities: the

rotor angle,  $\delta$ , because it is an important indicator of generator stability, and the flux linkage in the d-axis,  $\psi_d$ , because it is a representative of the Q-V transient process. Hence, the output vector  $Y(k)=[\delta(k), \psi_d(k)]^T$ .

It has been rigorously proved that a wide class of discrete-time nonlinear systems can be represented by the following difference equation model [5]

$$Y(k+1)=F_s[V(k)] \quad (21)$$

where  $F_s(\cdot)$  is some nonlinear function.

The RBFN expansion  $F_s(V(k))$  is then a one-step-ahead prediction of  $Y(k)$ . The structure of the proposed RBFN identification scheme is shown in Fig. 3 where  $C(k)$  and  $W(k)$  are the updated values of the network centers and weights respectively.

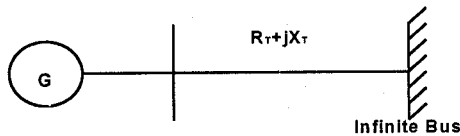


Fig. 2. Synchronous machine connected to an infinite bus

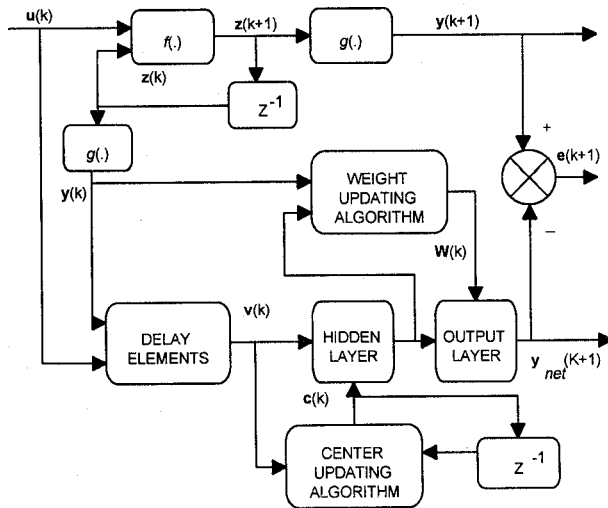


Fig. 3. The structure of the proposed RBFN identifier

### 5 Model Validation

The adequacy of the modeling can be tested using the correlation-based model validity tests. Define the residual vector

$$E(k)=Y(k)-Y_{net}(k) \quad (22)$$

It can be shown [11] that if the identified model is operating correctly then the following correlation tests using residuals and inputs should be satisfied:

$$\begin{aligned} \Gamma_{e_i e_i}(\tau) &= \Gamma_{e_i^2 e_i^2}(\tau) = \lambda(\tau), \forall \tau \\ \Gamma_{u_r e_i}(\tau) &= \Gamma_{u_r^2 e_i^2}(\tau) = 0, \forall \tau \end{aligned} \quad (23)$$

where  $e_i$  is the  $i$ -th residual and  $u_r$  is the  $r$ -th input.  $\Gamma_{xx}$  and  $\Gamma_{yx}$  are the auto-correlation and cross-correlation functions

respectively while  $\lambda(t)$  represents the unit impulse function. In practice, the model will be regarded as adequate if all the tests in (23) fall within the 95% confidence bands at  $\pm 1.96 / \sqrt{N}$  where  $N$  is the number of samples.

### 6 Results and Discussions

In order to investigate the performance of the proposed RBFN identifier, random variations have been applied to the machine inputs,  $T_m$  and  $V_F$ . Initially, the machine was operating at power of 1.0 pu with 0.85 power factor lag and terminal voltage of 1.0 pu. In all cases ten historical values of the inputs and outputs were used to construct the input vector of the network.

#### 6.1 Variations in the Mechanical Torque ( $T_m$ )

The responses of  $\delta$  and  $\psi_d$  due to random variations in the range of 60-140% of the initial value of  $T_m$  are shown in Figs. 4 and 5 respectively. It is worth pointing out that it is difficult to discern a difference between the simulated and identified responses confirming the capability of the proposed identifier to capture the nonlinear operating characteristics of the synchronous machine. The results of the model validation tests, as shown in Fig. 6, fall within the 95% confidence bands and confirming the adequacy of the proposed identifier. The training error is shown in Fig. 7. It is clear that the training error dramatically decreases converging almost to zero in time less than 0.01 sec. This demonstrate the suitability of the proposed identifier for on-line applications. It is found that only 10 neurons in the hidden layer are adequate which makes the structure of the identifier very simple.

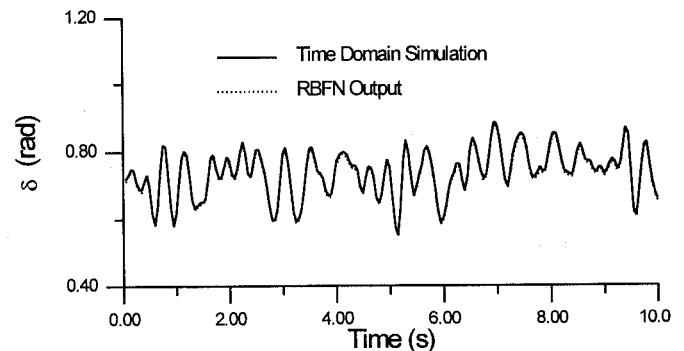


Fig. 4.  $\delta$  response due to 60-140% random change in  $T_m$

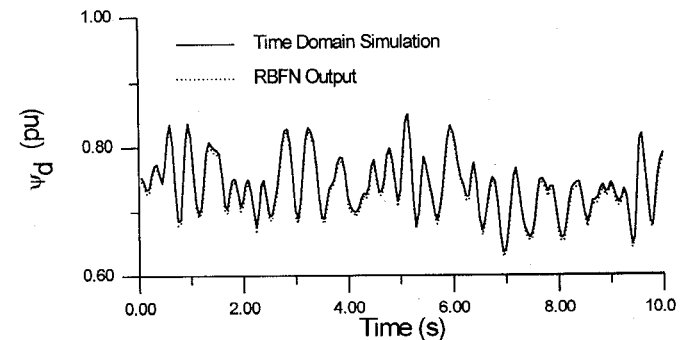


Fig. 5.  $\psi_d$  response due to 60-140% random change in  $T_m$

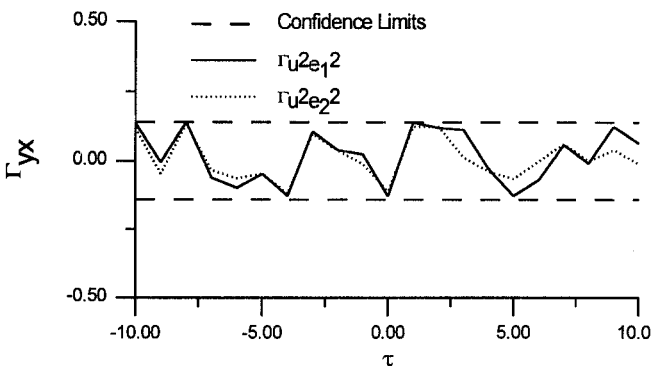
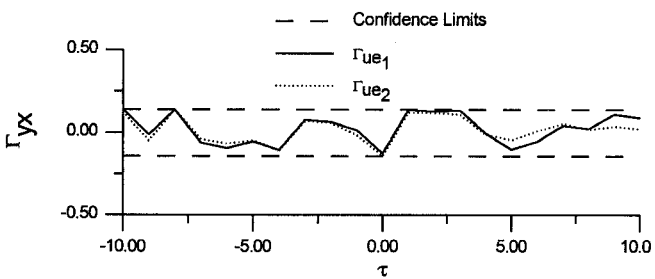
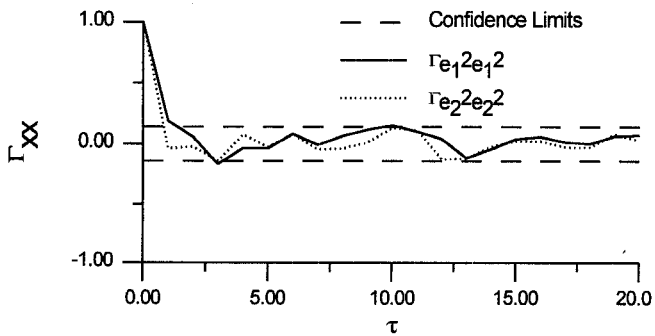
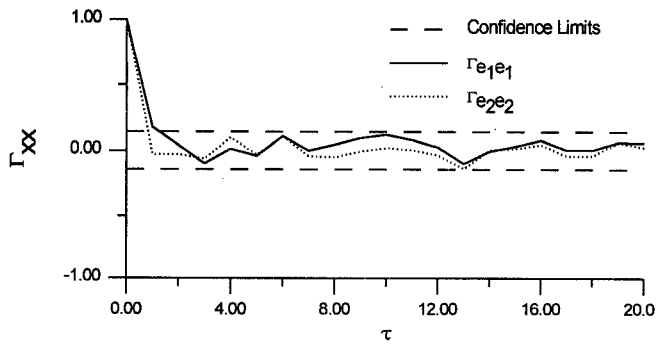


Fig. 6. Correlation tests using residuals and  $T_m$

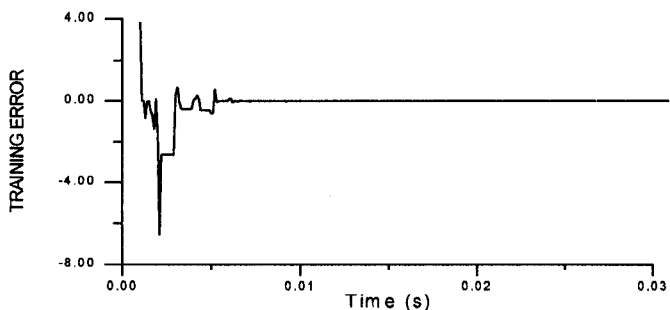


Fig. 7. Training error

## 6.2 Variations in the Field Voltage ( $V_F$ )

The responses of  $\delta$  and  $\psi_d$  due to The random variations in  $V_F$  in the range of 60-140% of the initial value are shown in Figs. 8 and 9 respectively. The results demonstrate the capability of the proposed identifier to learn the underlying characteristics of the synchronous machine. In this case, 11 neurons in the hidden layer are found to be adequate.

## 7 Conclusions

The synchronous machine has been successfully identified using RBFN. The results of this paper show that the nonlinear operating characteristics of the synchronous machine have been accurately captured by the proposed RBFN identifier. The close agreement of the results obtained by simulations and that obtained by the proposed identifier indicates the capability of the proposed identifier for on-line identification of the synchronous machine. The results of the correlation-based model validity tests fall within the 95% confidence bands confirming the adequacy and validity of the proposed identifier. Finally, the results demonstrate the potential of the proposed RBFN identifier that exhibits excellent performance with a relatively simple RBFN architecture.

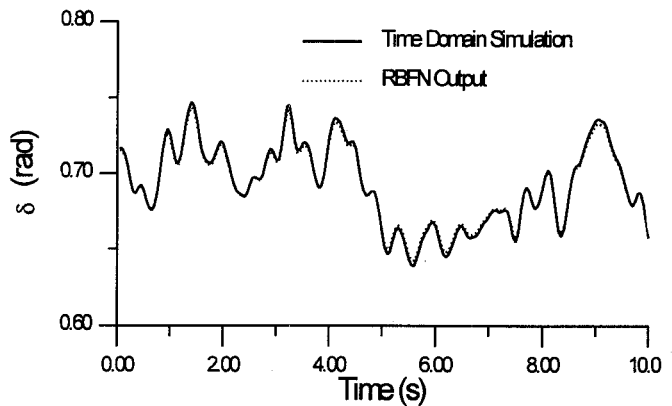


Fig. 11.  $\delta$  response due to 60-140% random change in  $V_F$

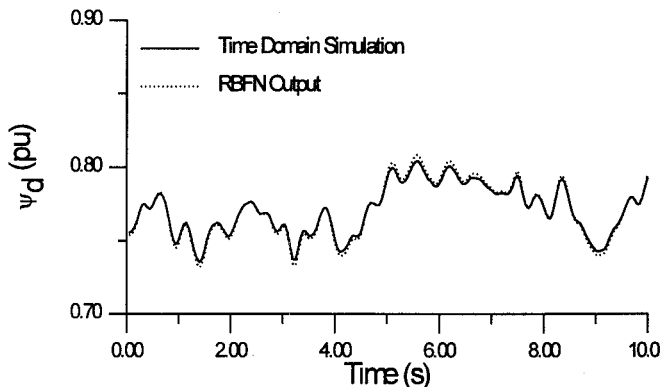


Fig. 12.  $\psi_d$  response due to 60-140% random change in  $V_F$

## 8 Acknowledgement

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## 10 Appendix

### System Model:

$$\rho\delta = \omega_b (\omega - 1) \quad (\text{A.1})$$

$$\rho\omega = [T_m - T_e - K_d (\omega - 1)] / M \quad (\text{A.2})$$

$$\rho\psi_d = \omega_b (v_d + r_a i_d + \omega \psi_q) \quad (\text{A.3})$$

$$\rho\psi_q = \omega_b (v_q + r_a i_q - \omega \psi_d) \quad (\text{A.4})$$

$$\rho\psi_F = \omega_b (V_F - r_F i_F) \quad (\text{A.5})$$

$$\rho\psi_D = \omega_b (-r_D i_D) \quad (\text{A.6})$$

$$\rho\psi_Q = \omega_b (-r_Q i_Q) \quad (\text{A.7})$$

$$T_e = i_q \psi_d + i_d \psi_q \quad (\text{A.8})$$

$$v_d = v_b \sin \delta + R_T i_d - X_T i_q \quad (\text{A.9})$$

$$v_q = v_b \cos \delta + R_T i_q + X_T i_d \quad (\text{A.10})$$

$$v = (v_d^2 + v_q^2)^{1/2} \quad (\text{A.11})$$

$$\begin{bmatrix} \Psi_d \\ \Psi_F \\ \Psi_D \end{bmatrix} = \begin{bmatrix} x_d & x_{md} & x_{md} \\ x_{md} & x_F & x_{md} \\ x_{md} & x_{md} & x_D \end{bmatrix} \begin{bmatrix} -i_d \\ i_F \\ i_D \end{bmatrix} \quad (\text{A.12})$$

$$\begin{bmatrix} \Psi_q \\ \Psi_Q \end{bmatrix} = \begin{bmatrix} x_q & x_{mq} \\ x_{mq} & x_Q \end{bmatrix} \begin{bmatrix} -i_q \\ i_Q \end{bmatrix} \quad (\text{A.13})$$

### System Parameters:

$$\begin{array}{lll} \omega_b = 377 \text{ rad/s}; & K_d = 0.0; & M = 6.5 \text{ s}; \\ x_d = 2.0; & x_q = 1.91; & x_F = 1.97; \\ x_{md} = 1.86; & x_{mq} = 1.77; & x_D = 1.94; \\ x_Q = 1.96; & r_F = 0.0015; & r_a = 0.005; \\ r_D = 0.0078; & r_Q = 0.0084; & R_T = 0.063; \\ X_T = 0.45 & & \end{array}$$

All resistances and reactances are in per-unit.

## 11 Nomenclature

$y_i$	$i$ -th output
$X$	input vector
$w_{oj}$	biasing term
$n_h$	number of hidden units
$w_{ji}$	weight between $j$ -th hidden and $i$ -th output nodes
$C_j$	center of the $j$ -th hidden node
$\  \cdot \ $	euclidean norm
$\beta_j$	$j$ -th hidden unit width
$\Phi(\cdot)$	nonlinear function from $R^+ \rightarrow R$
$\rho$	first derivative w.r.t. time $d/dt$
$\delta$	torque angle
$\omega$	angular speed
$\omega_b$	synchronous speed
$M$	inertia constant
$i_d, i_q$	stator currents in d and q axis circuits respectively
$i_D, i_Q$	damper circuit currents in d and q axes respectively
$\Psi$	flux linkages
$v, v_b$	terminal and infinite bus voltages respectively
$v_F, i_F$	field voltage and current respectively
$R_T, X_T$	line resistance and reactance respectively
$x_d, x_q$	synchronous reactances in d and q axes
$x_{md}, x_{mq}$	mutual reactances in d- and q-axis respectively
$r_F, x_F$	field winding resistance and reactance respectively
$r_a$	stator resistance
$r_D, x_D$	damper winding resistance and reactance in d-axis
$r_Q, x_Q$	damper winding resistance and reactance in q-axis
$T_m$	input mechanical torque
$T_e$	output electric torque
$K_d$	mechanical damping coefficient