Some Useful Math Formulas for Electric Circuits I

Trigonometry

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta), \qquad \cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

Law of sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$

Analytic Geometry

Polar to rectangular conversion

$$x = r\cos\theta, \qquad y = r\sin\theta$$

Rectangular to polar conversion

$$r = \sqrt{x^2 + y^2}, \qquad \theta = \tan^{-1}\frac{y}{x}$$

Straight Lines

$$y = mx + b$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Calculus

$$\frac{d}{dt}(uv) = u\frac{dv}{dt} + v\frac{du}{dt}$$

$$\frac{d}{dt}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dt} - u\frac{dv}{dt}}{v^2}$$

Let $u = \phi(t)$, and $\mathrm{y} = f(u) = f\left[\phi(t)\right] = F(t)$

Then $\frac{dy}{dt} = \frac{dy}{du} \bullet \frac{du}{dt}$, or $F'(t) = f'\left[\phi(t)\right] \bullet \phi'(t) = f'(u) \bullet \phi'(t)$

$$\frac{d}{dt}\left(t^n\right) = nt^{n-1}, \qquad \int \mathrm{t}^{\mathrm{n}} dt = \frac{t^{n+1}}{n+1} + C$$

$$\frac{d}{dt}\left(e^{at}\right) = ae^{at}, \qquad \int e^{at} dt = \frac{1}{a} e^{at} + C$$

$$\frac{d}{dt}\left(\sin(\omega t + \theta)\right) = \omega \cos(\omega t + \theta)$$

$$\frac{d}{dt}\left(\cos(\omega t + \theta)\right) = -\omega \sin(\omega t + \theta)$$

$$\int \sin(\omega t + \theta) dt = -\frac{1}{\omega}\cos(\omega t + \theta) + C$$

$$\int \cos(\omega t + \theta) dt = \frac{1}{\omega}\sin(\omega t + \theta) + C$$

$$\int u dv = uv - \int v du$$

AC Steady State and Complex Numbers

$$v(t) = V_{\max} \cos(\omega t + \theta) = V_{rms} \sqrt{2} \cos(\omega t + \theta)$$

Identitys

$$\sin(\omega t + \phi) = \cos(\omega t + \phi - 90^\circ)$$

$$\sin(\omega t + \phi) = \cos(\omega t + \theta)$$

$$\omega = 2\pi f \qquad \theta = \phi - 90^\circ$$

Example

$$v(t) = 15 \sin(2000\pi t + 30^\circ) \quad V$$

$$v(t) = 15 \cos(2000\pi t + 30^\circ - 90^\circ) V$$

$$v(t) = 15 \cos(2000\pi t - 60^\circ) V$$

$$V_{\max} = 15V$$

$$V_{rms} = \frac{15}{\sqrt{2}} V_{rms} = 10.6066 V_{rms}$$

$$\omega = 2000\pi \, rad / s$$

$$f = 1000 \, Hz$$

$$T = \frac{1}{f} = 1 \, ms = 10^{-3} \, s$$

$$\theta = -60^\circ$$

Complex Numbers and Impedances

$$j = \sqrt{-1}$$

Euler's Identity $e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$

$$Z_L = j\omega L \ , \qquad Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

Complex Identities

$$e^{j90^\circ} = j = 1\angle 90^\circ$$

$$e^{-j90^\circ} = -j = 1\angle -90^\circ$$

$$e^{\pm j180^\circ} = 1\angle \pm 180^\circ = -1 = j^2$$

$$j^3 = j^2 j = -j$$

$$\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$$

$$-j15 = 15\angle -90^\circ$$

Complex Arithmetic

$$z_1 = x_1 + jy_1 \ , \qquad z_2 = x_2 + jy_2$$

$$z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$$

$$z_1 z_2 = (x_1 + jy_1)(x_2 + jy_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + j(x_2 y_1 + x_1 y_2)$$

$$c = \sqrt{x^2 + y^2} \ , \qquad \theta = \tan^{-1} \frac{y}{x}$$

$$z_1 = c_1 e^{j\theta_1} \ , \qquad z_2 = c_2 e^{j\theta_2}$$

$$z_1 z_2 = c_1 e^{j\theta_1} c_2 e^{j\theta_2} = c_1 c_2 e^{j(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \frac{c_1 e^{j\theta_1}}{c_2 e^{j\theta_2}} = \frac{c_1}{c_2} e^{j(\theta_1 - \theta_2)}$$