9. STABILITY IN THE FREQUENCY DOMAIN

INTRODUCTION

A frequency domain stability criterion was developed by H. Nyquist in 1932 and remains a fundamental approach to the investigation of the stability of linear control system. Nyquist's work led to a procedure that can determine whether a system is stable by using the frequency response of the open-loop transfer function $G(s)H(s)$. Thus, knowledge of the open-loop system's frequency response yields information about the stability of the closed-loop system. [This concept is similar to the root locus where we began with information about the open-loop system, its poles and zeros, and developed stability information about the closed-loop system].

The Nyquist procedure involves two parts.

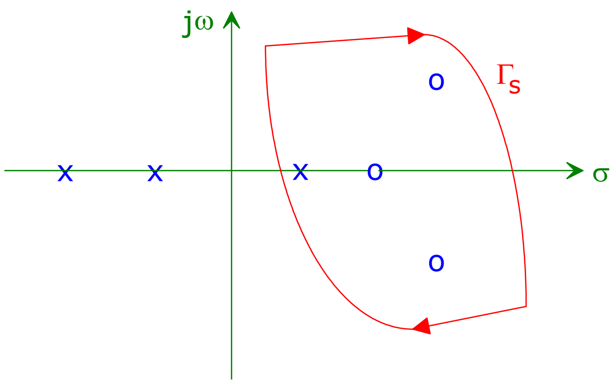
- Generating a Nyquist (polar) plot [See Chapter 8]
- Evaluating the system stability by interpreting the Nyquist plot.

Background

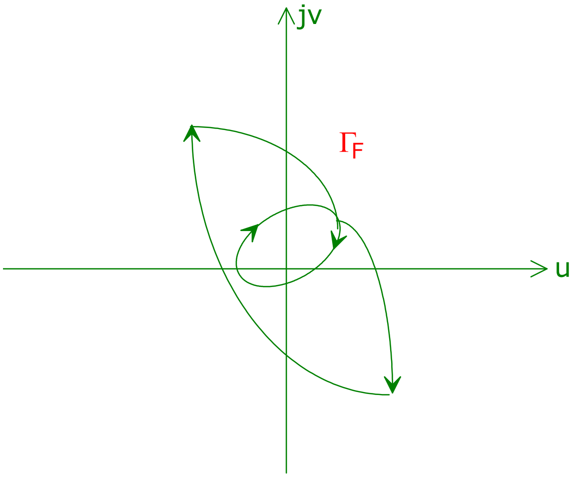
The *Nyquist stability criterion* is based on a theorem in the theory of the function of a complex variable due to Cauchy. Cauchy's theorem, commonly known as the principle of argument, states:

> **If a contour Γ_s in the s-plane encircles Z zeros and P poles of $F(s)$ and does not pass through any poles or zeros of $F(s)$ (traversal is in a clockwise direction along the contour), the corresponding contour Γ_F in the $F(s)$-plane encircles the origin of the $F(s)$-plane $N = Z - P$ times in the clockwise direction.**

As an example of the Cauchy's theorem, consider the pole-zero pattern shown with the contour Γ_s to be considered.

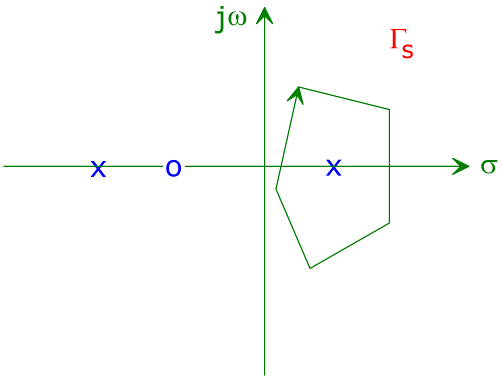


- The contour Γ_s encircles three zeros and one pole.
- Therefore we obtain $N = Z - P = 3 - 1 = +2$

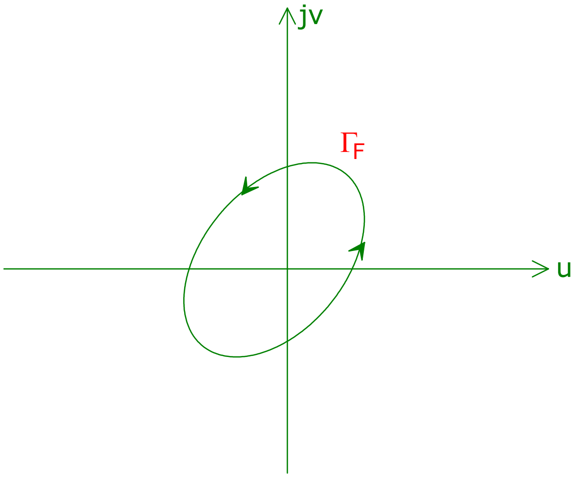


- The contour Γ_F completes two clockwise encirclements of the origin in the $F(s)$-plane.

For the pole and zero pattern shown and the contour Γ_s as shown,



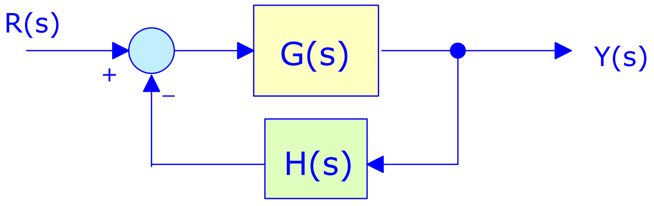
- one pole is encircled and no zeros are encircled.
- Therefore we obtain $N = Z - P = 0 - 1 = -1$



- We expect one encirclement of the origin by the contour Γ_F in the $F(s)$-plane. However since the sign of N is negative, we find the encirclement moves in the counterclockwise direction, as shown.

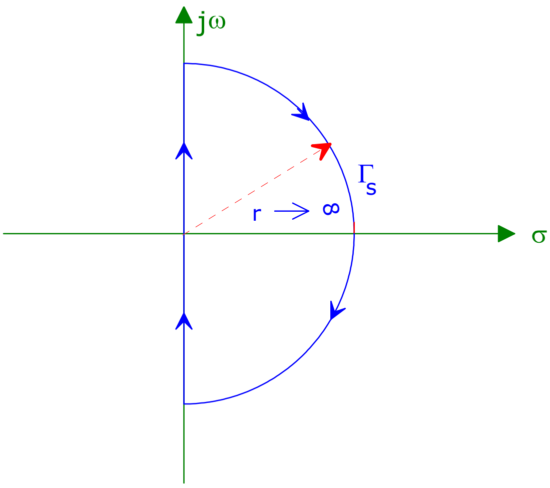
Now that we have illustrated the Cauchy's Theorem, we are ready to consider the stability criterion proposed by Nyquist.

THE NYQUIST STABILITY CRITERION



- For a system to be stable, all the zeros of the characteristic equation $F(s) = 1 + G(s)H(s) = 0$ must lie in the left-hand s-plane. [i.e. To the left of the $j\omega$axis in the s-plane].

Therefore, we choose a contour Γ_s in the s-plane that encloses the entire right-hand s-plane as shown , and we determine whether any zeros of $F(s)$ lie within$\Gamma_s$by utilizing Cauchy's theorem. That is we plot $\Gamma_F = F(s)$ in the $F(s)$-plane and determine the number of encirclements of the origin N.



Then the number of zeros of $F(s)$ within the contour Γ_s [and therefore unstable zeros of $F(s)$] is

$$Z = N + P$$

- Note that P is the number of poles of $F(s)$ in the right-hand s-plane, which is the same as the number of poles of $G(s)H(s)$ in the right-hand s-plane.
- The procedure is greatly simplified if we plot $G(s)H(s)$ instead of $F(s)$ and look for the number of clockwise encirclements of the -1point instead of the number of clockwise encirclements of the origin.

Therefore the Nyquist criterion can be stated as follows:

Number of zeros [roots] of the C.E. in right-hand s-plane	=	Number of CW encirclements of the -1 point on the GH plane	+	Number of right-hand s-plane poles of $G(s)H(s)$

When the number of poles of $G(s)H(s)$ in the right-hand s-plane is zero, (i.e. $P = 0$), the Nyquist criterion is stated as follows:

Number of zeros [roots] of the C.E. in right-hand s-plane	=	Number of CW encirclements of the -1 point on the GH plane

- When the number of poles of $G(s)H(s)$ in the right-hand s-plane is other than zero, (i.e. $P \neq 0$), a system is stable if and only if the -1 point is encircled ACW P times .
- When the number of poles of $G(s)H(s)$ in the right-hand s-plane is zero, (i.e. $P = 0$), a system is stable if and only if the -1 point is not encircled.