8. **FREQUENCY RESPONSE METHODS** (CONT.)

EXAMPLE OF THE DRAWING THE BODE DIAGRAM

The bode diagram of a transfer function $G(s)$, which contains several poles and zeros is obtained by adding the plot due to each individual pole and zero. An example will now be given by considering a transfer function which has all the factors considered.

Example

Draw the bode diagram of the following transfer function

$$G(j\omega) = \frac{5(1+j0.1\omega)}{j\omega(1+j0.5\omega)(1+j0.6\frac{\omega}{50}+\left(\frac{j\omega}{50}\right)^2)}$$

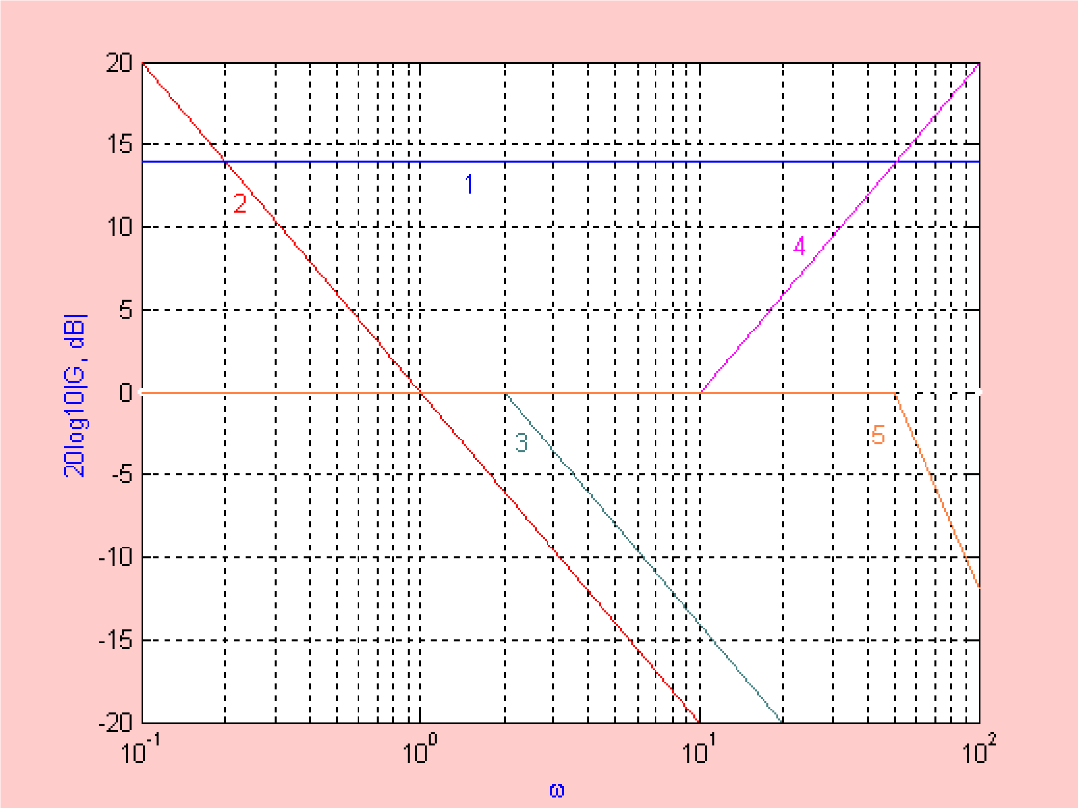
Solution

The factors, in order of their occurrence as frequency increases, are as follows:

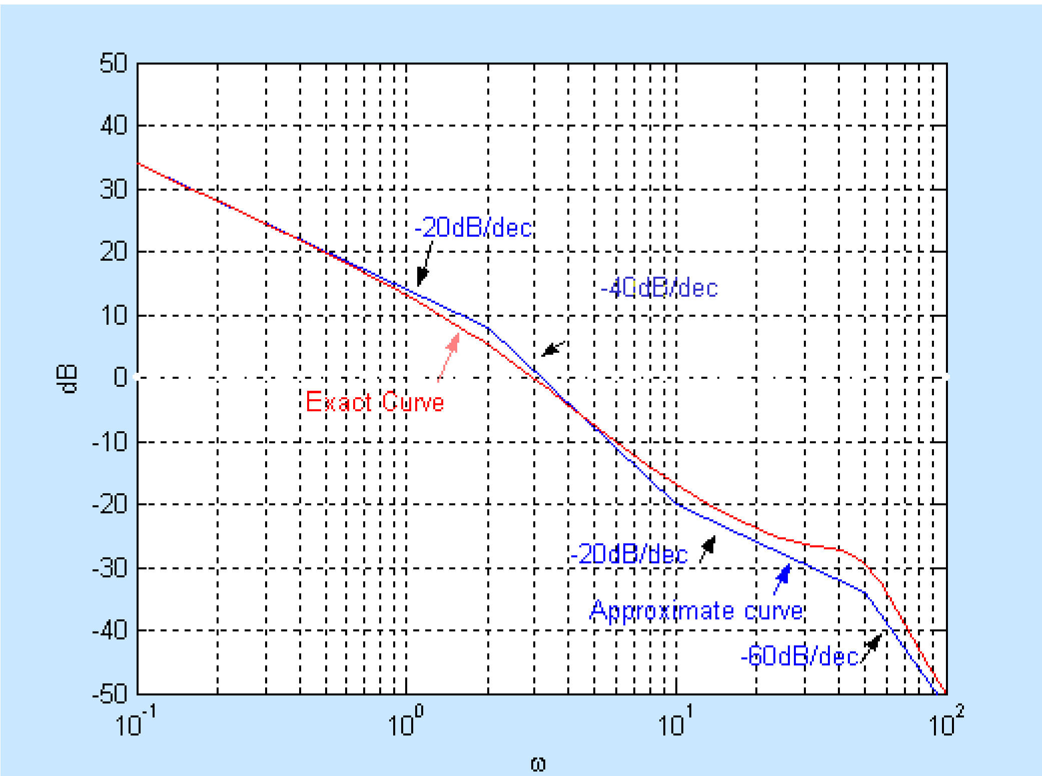
1. A constant gain $K=5$
2. A pole at the origin
3. A pole at $\omega=2$
4. A zero at $\omega=10$
5. A pair of complex poles at $\omega=\omega_n=50$

First we plot the magnitude characteristic for each individual pole and zero factor and the constant gain

1) The constant gain is $20\log_{10}5 = 14\text{dB}$, as shown in the figure.
2) The magnitude of the pole at the origin extends from zero frequency to infinite frequencies and has a slope of $-20\,\text{dB/decade}$ intersecting the $0-\text{dB}$ line at $\omega=1$, as shown in the figure.
3) The asymptotic approximation of the magnitude of the pole at $\omega=2$ has a slope of -20dB/decade beyond the break frequency at $\omega=2$. The asymptotic magnitude below the break frequency is $0\,\text{dB}$, as shown in the figure.
4) The asymptotic approximation for the zero at $\omega=10$ has a slope of $+20\text{dB/decade}$ beyond the break frequency at $\omega=10$. The asymptotic magnitude below the break frequency is $0\,\text{dB}$, as shown in the figure.
5) The asymptotic approximation for the pair of complex poles has a slope of -40dB/decade beyond the break frequency at $\omega=\omega_n=50$. The asymptotic magnitude below the break frequency is $0\,\text{dB}$, as shown in the figure. This approximation must be corrected to the actual magnitude because the damping ratio is $\zeta=0.3$, and the magnitude differs appreciably from the approximation.



The total asymptotic magnitude can be plotted by adding the asymptotes due to each factor, as shown . The exact magnitude curve, obtained using MATLAB, is also shown for comparison.



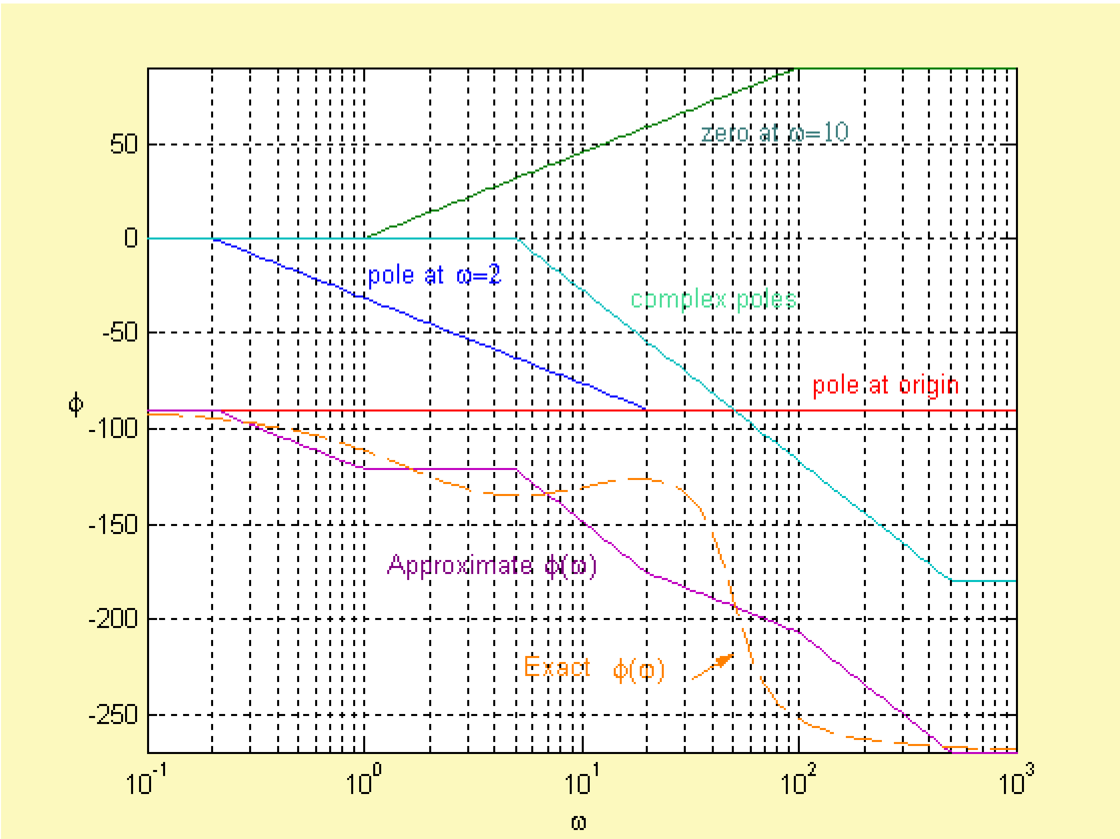
The linear approximations of the phase characteristic of the individual factors are as follows:

1) The phase of the constant gain is 0°.
2) The phase of the pole at the origin is a constant -90°.
3) The linear approximation of the phase of the pole at $\omega = 2$ has a slope of -45°/decade between the frequencies $\omega = \frac{2}{10}$ and $\omega = 10 * 2$.
4) The linear approximation of the phase of the zero at $\omega = 10$ has a slope of $+45^\circ$/decade between the frequencies $\omega = \frac{10}{10}$ and $\omega = 10 * 10$.
5) The linear approximation of the phase of the complex poles at $\omega_n = 50$ has a slope of -90°/decade between the frequencies $\omega = \frac{50}{10}$ and $\omega = 10 * 50$.

The individual linear approximations of the phase characteristics for the poles and zeros are shown in the figure. The approximate total phase characteristic, $\phi_a(\omega)$, is obtained by adding the phase due to each factor. The exact phase characteristic calculated from

$$\phi(\omega) = -90^\circ - \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{2\zeta\frac{\omega}{50}}{1-(\frac{\omega}{50})^2}$$

is also shown for comparison.



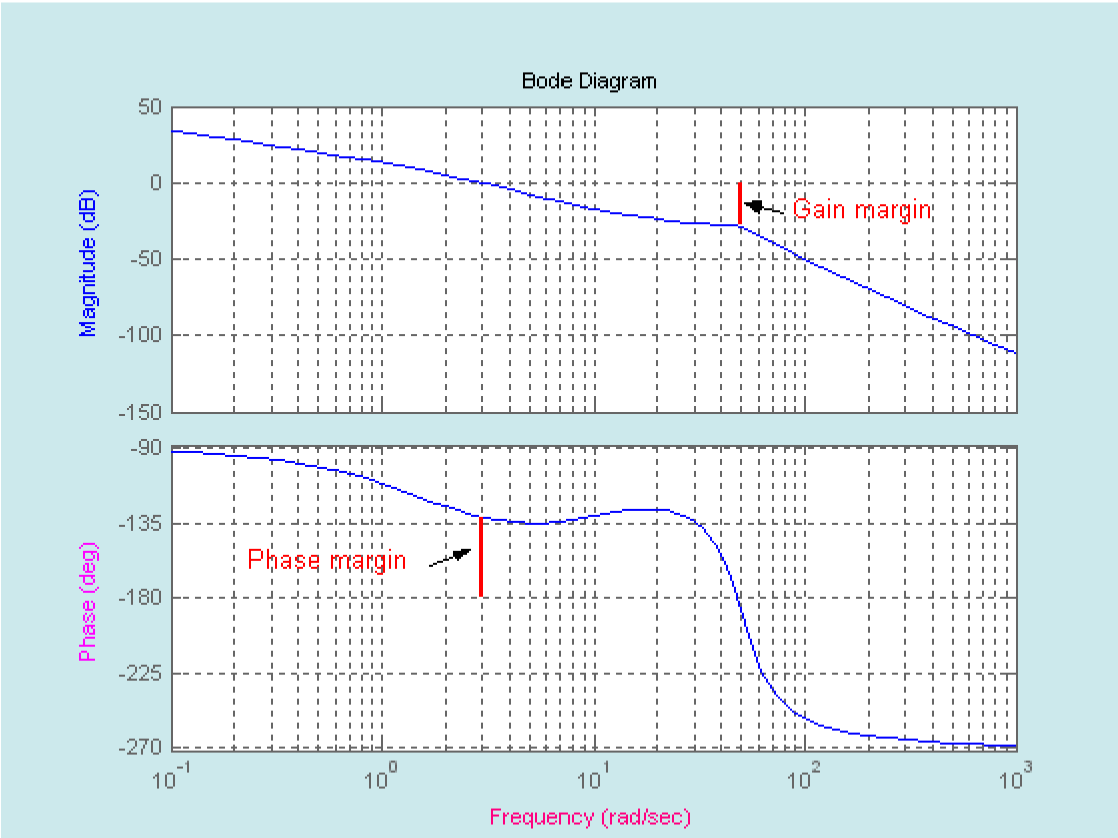
The frequency response of $G(j\omega)$ can be calculated and plotted using MATLAB. To do this , rewrite $G(j\omega)$ as:

$$G(s)=\frac{5*0.1(10+s)}{0.5*\frac{1}{2500}*s(2+s)(2500+30s+s^2)}=\frac{2500(10+s)}{s(2+s)(2500+30s+s^2)}$$

In MATLAB, run the following command:

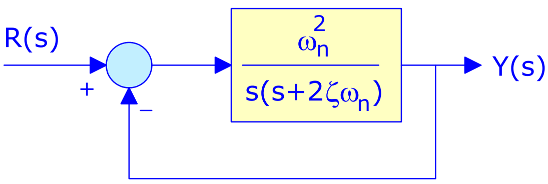
Bode(conv([2500],[1 10]) , conv([1 2 0],[1 30 2500]));

The following plots will be generated



PERFORMANCE SPECIFICATIONS IN THE FREQUENCY DOMAIN

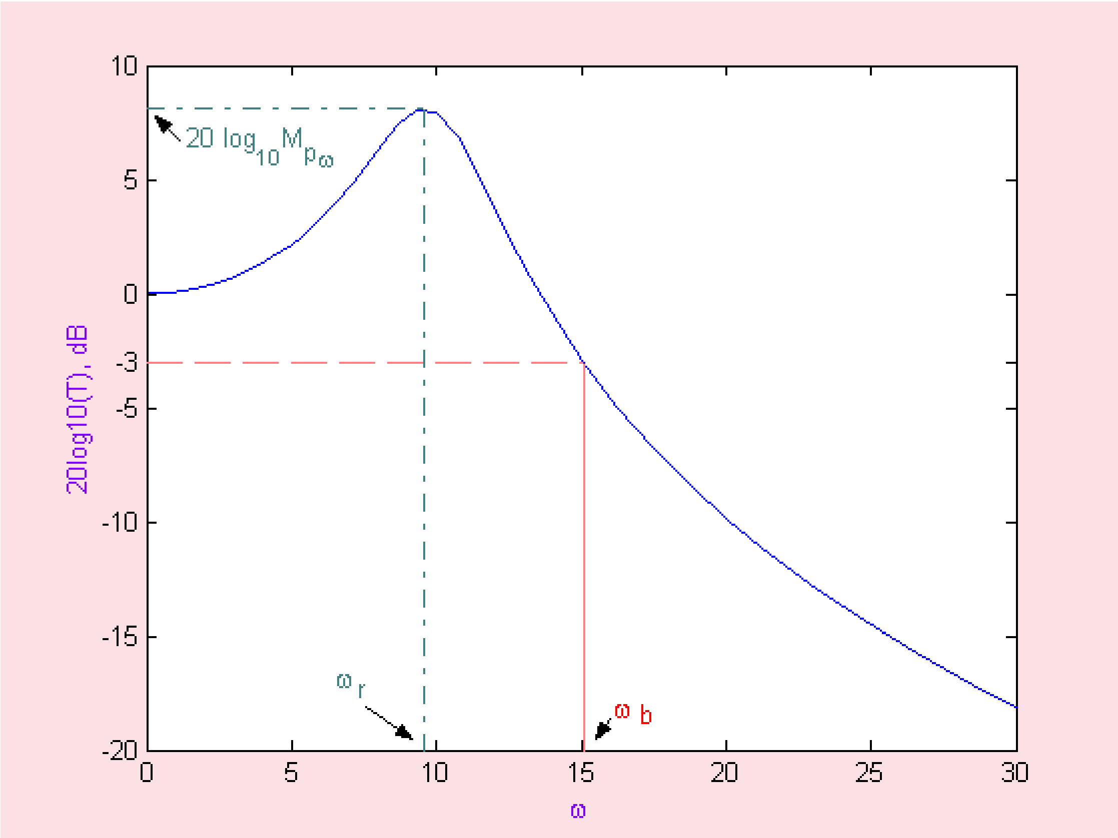
Consider the second-order system shown



The closed-loop transfer function is given by

$$T(s)=\frac{Y(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

The frequency response of this system for $\zeta = 0.2; \omega_n = 10$ will appear as shown



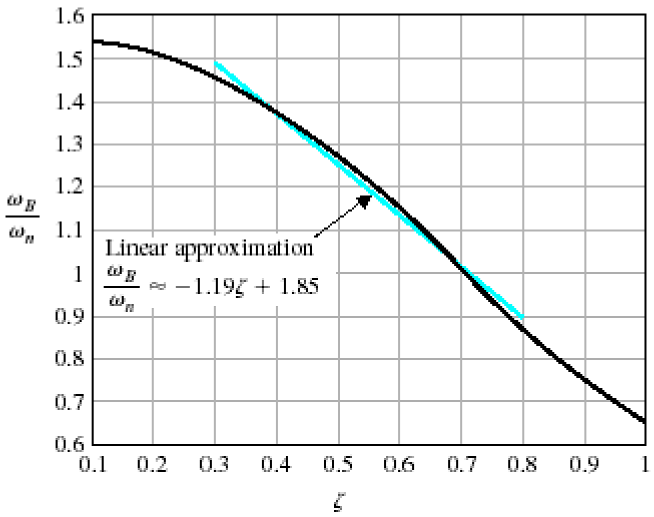
We already know that the frequency response will have a maximum magnitude $M_{p_\omega} = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$ which occurs at the resonant frequency $\omega_r = \omega_n\sqrt{1-2\zeta^2}$.

The bandwidth, ω_b is a measure of the system ability to faithfully reproduce an input signal. It is defined as follows

> THE BANDWIDTH IS THE FREQUENCY, ω_b, AT WHICH THE FREQUENCY RESPONSE HAS DECLINED 3 dB FROM ITS LOW-FREQUENCY VALUE.

HOW DOES THE BANDWIDTH ω_b VARY WITH ζ?

To answer this question, a simulation of the second-order system considered for different values of ζ was performed. For each ζ, a frequency response of the system was obtained. The bandwidth was then estimated from the frequency response plot. The figure shows the normalized bandwidth $\frac{\omega_B}{\omega_n}$ versus ζ



The following observations can be made:

- For a given ω_n, the bandwidth is inversely proportional to ζ . If we recall that the rise time is directly proportional to ζ [$T_r = \frac{2.16\zeta + 0.6}{\omega_n}$], then one can conclude that

> The larger the bandwidth, the faster the system response

Thus desirable frequency-domain specifications are as follows:

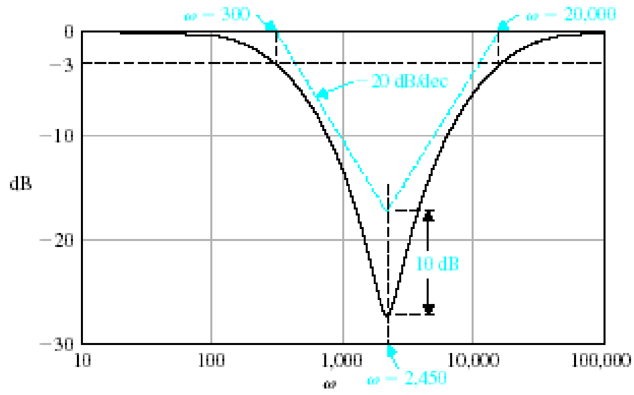
1. Relatively large bandwidth so that the system time constant $\tau = \frac{1}{\zeta\omega_n}$ is sufficiently small.
2. Relatively small resonant peak $M_{p_\omega} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$**: <** 1.5 for example

Frequency response measurements

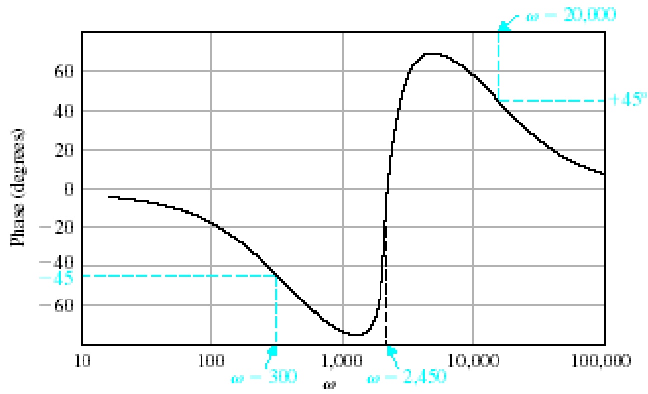
A sine wave can be used to measure the open-loop frequency response of a control system. In practice, a plot of amplitude versus frequency and phase versus frequency will be obtained. From these two plots the open-loop transfer function $G(j\omega)H(j\omega)$ can be deduced. Similarly the closed-loop frequency response of a control system, $T(j\omega)$, may be obtained and the actual transfer function deduced.

Example

Consider the plot shown.



(a)



(b)

- Notice that the magnitude plot declines at about $-20\,\mathrm{dB/decade}$ as ω increases between 100 and 1000, and because the phase is -45° and the magnitude is $-3\,\mathrm{dB}$ at $300\,\mathrm{rad/s}$, one can deduce that one factor is a pole at $p_1 = 300$.
- Because the slope of the magnitude curve changes from $-20\,\mathrm{dB/decade}$ to $+20\,\mathrm{dB/decade}$ at $\omega_n = 2450$, and the phase changes abruptly by nearly 180° passing through 0° at $\omega_n = 2450$, we deduce that a pair of complex zeros with $\zeta = 0.16$[the difference in magnitude from the corner frequency ($\omega_n = 2450$) of the asymptotes to the minimum response is $10\,\mathrm{dB} = M_{p_\omega}$] , and $\omega_n = 2450$ exist.
- Because the slope of the magnitude curve returns to $0\,\mathrm{dBc/decade}$as ω exceeds $50,000$, we determine that there is a second pole at $p_2 = 20000$. This is because the magnitude is $-3\,\mathrm{dB}$ from the asymptote and the phase is 45°at this point.

Therefore the transfer function is

$$T(s) = \frac{\left((\frac{s}{2450})^2 + \frac{0.32}{2450}s + 1 \right)}{(\frac{s}{300} + 1)(\frac{s}{20000} + 1)}$$