7. THE ROOT LOCUS METHOD [CONT.]

MORE EXAMPLES....

Example 2

Plot the root locus for the characteristic equation of a system as $0 < K < \infty$

$$1 + \frac{K}{s^3 + 3s^2 + 2s} = 0$$

Solution

Steps 1 and 2:

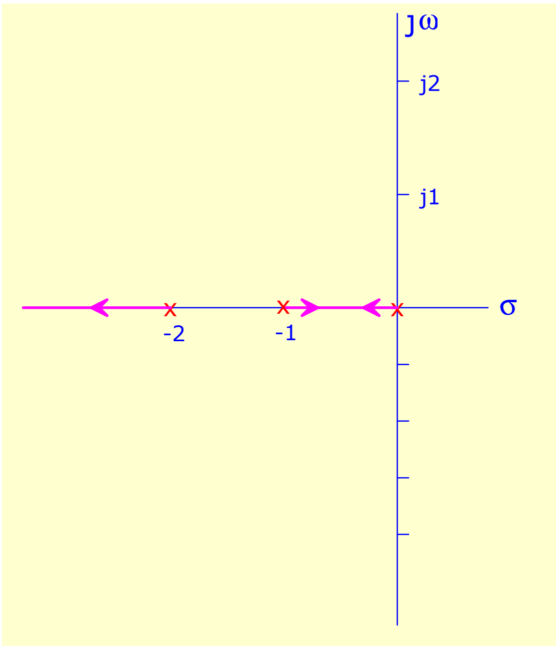
$$1 + K\frac{1}{s(s+1)(s+2)} = 0$$

Step 3:

We have

- 3 open-loop poles at $s = 0$, $s = -1$, $s = -2$
- no open-loop zeros

We locate the poles as shown.



Step 4:

Locate the root locus segments that lie on the real axis as shown .
Segments of the root locus exists on the real axis between $s = 0$ and $s = -1$, and $s = -2$ and $s = -\infty$.

Step 5

The number of separate loci is equal to $n_p = 3$
The number of loci branches proceeding to zeros at infinity is $n_p - n_z = 3$

Step 6

The root loci are symmetrical with respect to the real axis

Step 7:

$n_p = 3; n_z = 0$

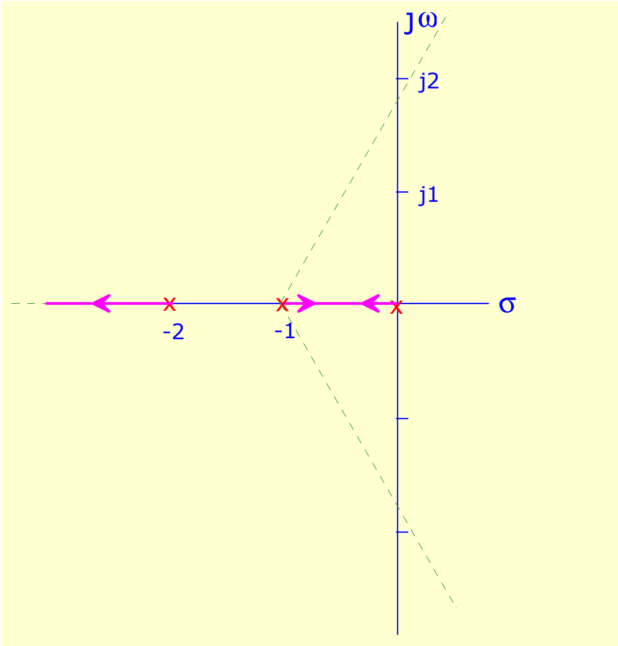
$$\sigma_A = \frac{\sum_{j=1}^{n}(-p_j) - \sum_{i=1}^{M}(-z_i)}{n_p - n_z}$$

$$= \frac{(0-1-2)}{3} = -1$$

$$\phi_A = \frac{(2q+1)}{n_p - n_z} 180^\circ \quad , q = 0, 1, 2$$

$$\phi_A = \pm 60^\circ, 180^\circ$$

Then the 3 asymptotes are drawn as shown.



Step 8:

To determine the imaginary axis crossing, we write the C.E.,

$$s(s+1)(s+2) + K = 0 \quad \Rightarrow \quad s^3 + 3s^2 + 2s + K = 0$$

s^3	1	2	0
s^2	3	K	
s	b_1	0	
s^0	K		

$$b_1 = \frac{6-K}{3}$$

The limiting value of the gain for stability is $K = 6$

To find the points where the locus crosses the imaginary axis, we find the roots of the auxiliary equation,

$$3s^2 + K = 0 \quad \Rightarrow 3s^2 + 6 = 0 \quad \Rightarrow \quad s = \pm j\sqrt{2} \text{ rad/s}$$

Step 9:

To determine the breakaway point [must be between $s = -1$ & $s = 0$], we have

$$K = -(s^3 + 3s^2 + 2s)$$

$$\frac{dK}{ds} = -(3s^2 + 6s + 2) = 0 \quad \rightarrow \quad s^2 + 2s + \frac{2}{3} = 0$$

The roots are $-0.423\ ;-1.577$

Step 10:

To determine the angle of departure at the complex pole

Not Applicable

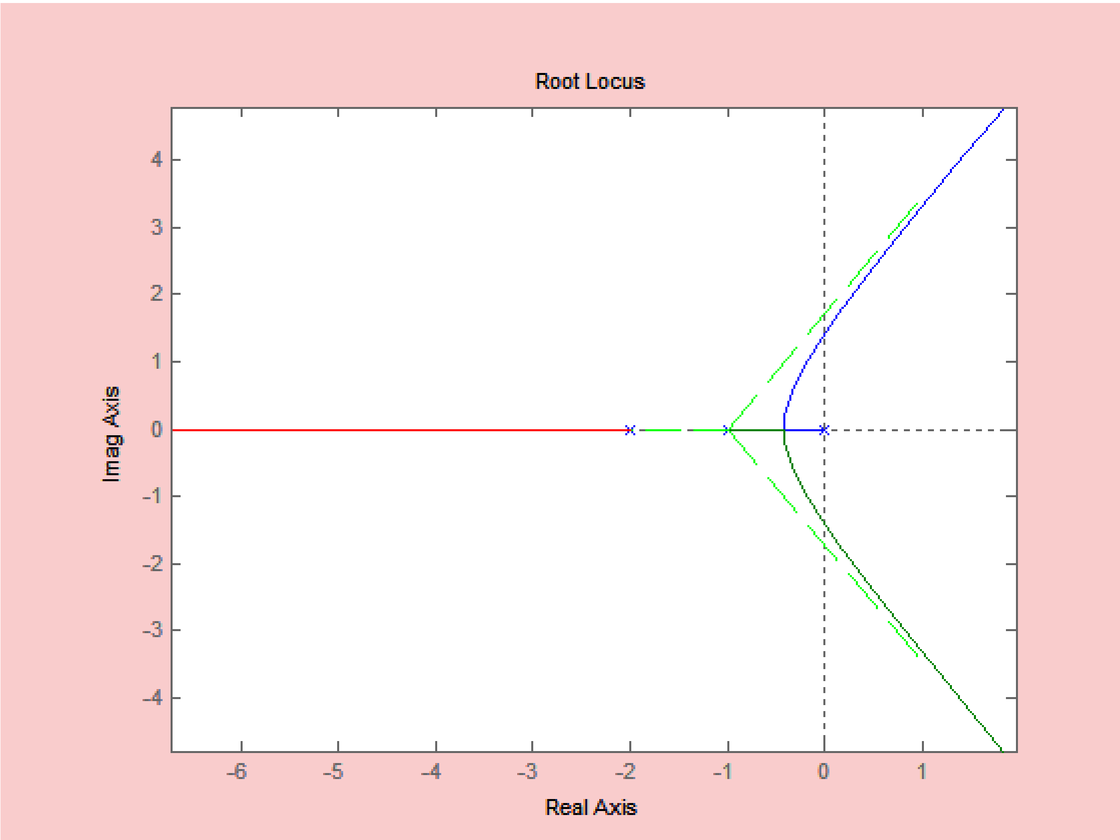
Step 11:

To determine the gain at the breakaway point

$$\left|K\frac{1}{s(s+1)(s+2)}\right|_{s=-0.423} = 1$$

$K_{ba} = 03.85$

The complete root locus plot is shown



Breakaway point: $s = -0.423$

Example 3

Plot the root locus for the characteristic equation of a system as $0 < K < \infty$

$$1 + \frac{K(s+2)}{s^2 + 2s + 3} = 0$$

Solution

Steps 1 and 2:

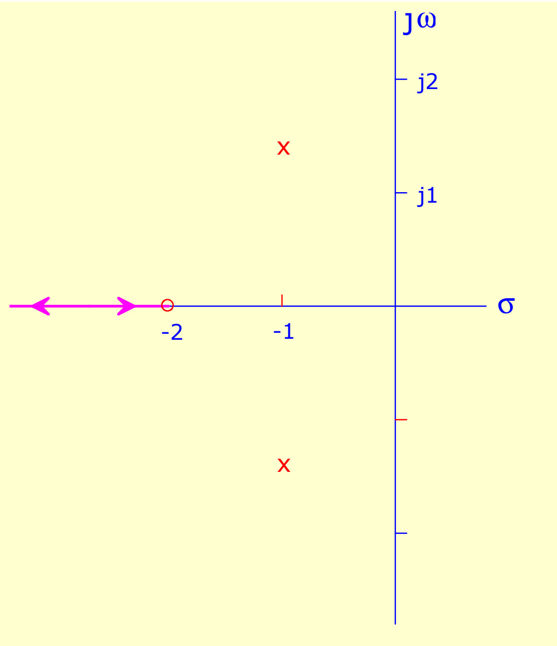
$$1 + K\frac{s+2}{(s+1 \pm j\sqrt{2})} = 0$$

Step 3:

We have

- 2 open-loop poles at $s = -1 + j\sqrt{2}$, $s = -1 - j\sqrt{2}$
- one open-loop zero $s = -2$

We locate the poles as shown.



Step 4:

Locate the root locus segments that lie on the real axis as shown . A segment of the root locus exists on the real axis between $s = -2$ and $s = -\infty$.

Step 5

The number of separate loci is equal to $n_p = 2$

The number of loci branches proceeding to zeros at infinity is $n_p - n_z = 2 - 1 = 1$

Step 6

The root loci are symmetrical with respect to the real axis

Step 7:

$n_p = 2; n_z = 1$

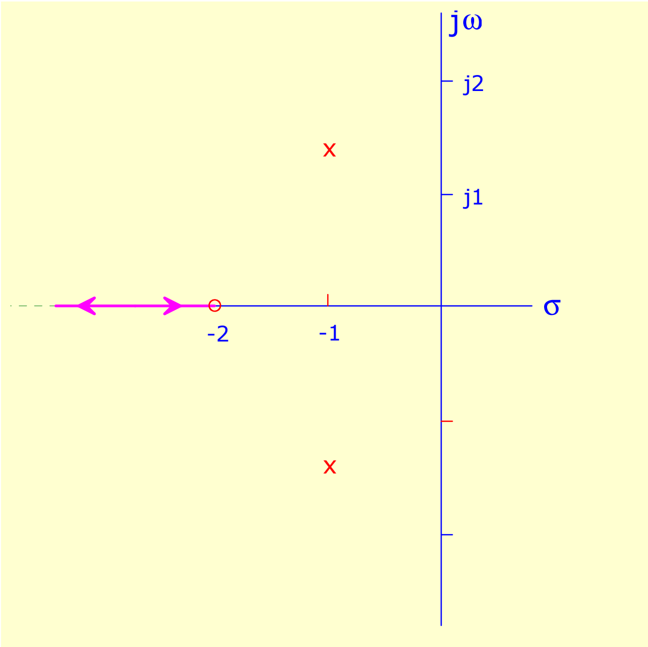
$$\sigma_A = \frac{\sum_{j=1}^{n}(-p_j) - \sum_{i=1}^{M}(-z_i)}{n_p - n_z}$$

$$= \frac{(-1-1)-(-2)}{2-1} = 0$$

$$\phi_A = \frac{(2q+1)}{n_p - n_z}180^\circ \ , q = 0$$

$$\phi_A = 180^\circ$$

Then the asymptote is drawn as shown.



Step 8:

To determine the imaginary axis crossing (if any), we write the C.E.,

$$s^2 + 2s + 3 + K(s+2) = 0 \ \Rightarrow s^2 + (2+K)s + (3+2K) = 0$$

s^2	1	3+2K
s	2+K	0
s^0	3+2K	

It is clear that the root locus does not cross the iamginary axis for $K > 0$.

Step 9:

To determine the breakaway point [must be between $s = -2 \ \& s = -\infty$], we have

$$K = -\frac{(s^2 + 2s + 3)}{s+2}$$

$$\frac{dK}{ds} = (s^2 + 4s + 1) = 0$$

The roots are $-3.73 \ ; -0.27$

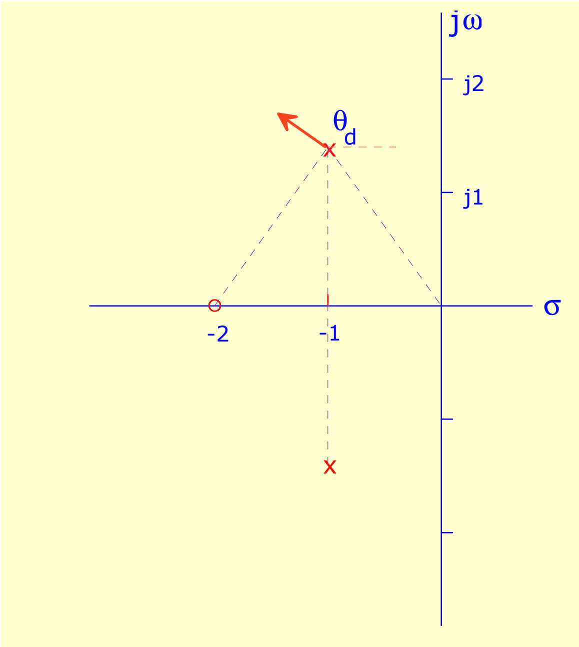
The breakaway point is $s = -3.73$

Step 10:

To determine the angle of departure at the complex pole

$$\tan^{-1}(\frac{\sqrt{2}}{1}) - (\theta_d + 90^\circ) = -180^\circ$$

$$54.7^\circ - (\theta_d + 90^\circ) = -180^\circ$$
$$\theta_d \simeq 145^\circ$$



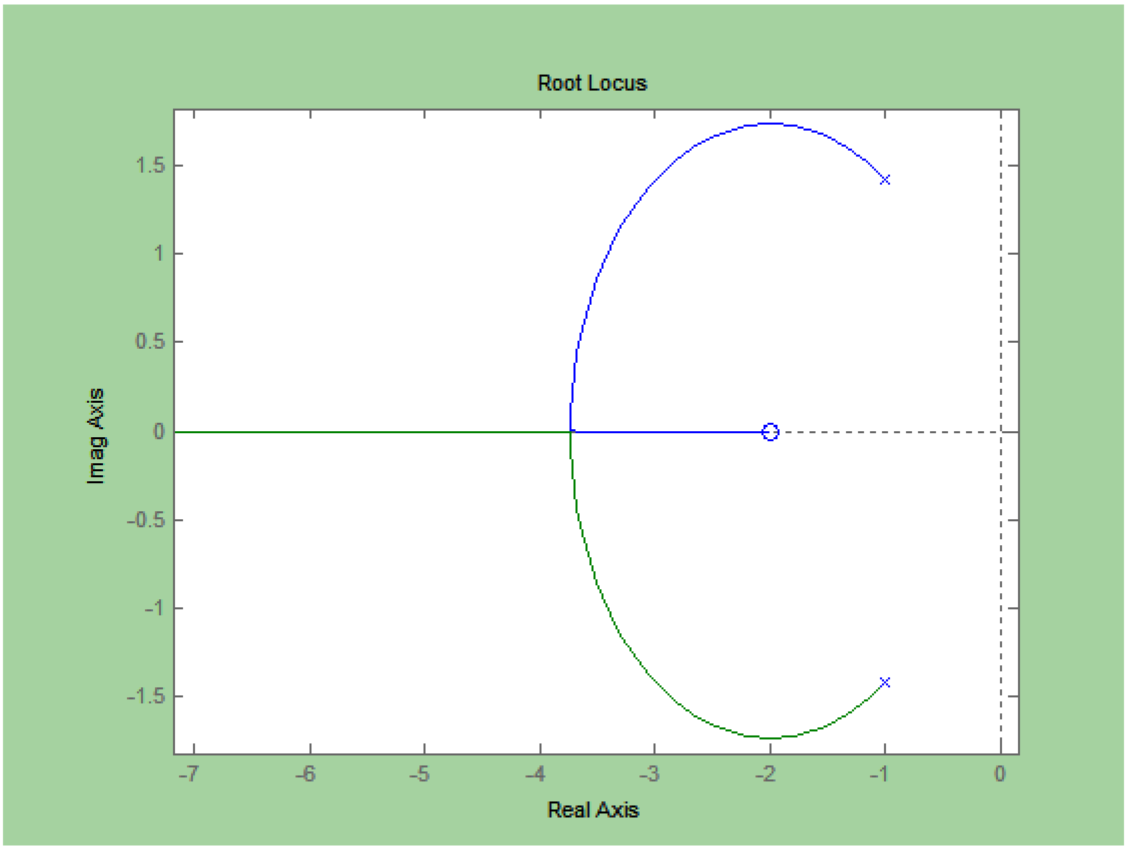
Step 11:

To determine the gain at the breakaway point

$$\left| K \frac{s+2}{(s+1+j\sqrt{2})} \right|_{s=-3.73} = 1$$

$$K_{ba} = 5.46$$

The complete root locus plot is shown



Breakaway point: $s = -3.73$