7. THE ROOT LOCUS METHOD [CONT.]

THE ROOT LOCUS PROCEDURE

An orderly procedure that facilitates the rapid sketching of the root locus will be developed. The procedure comprises twelve steps.

STEP 1:

Write the characteristic equation as
$1+F(s)=0$
and rearrange the equation, if necessary, so that the parameter of interest, K, appears as the multiplying factor in the form
$1+KP(s)=0$

STEP 2:

Factor $P(s)$, if necessary, and write the polynomial in the form of poles and zeros as follows:

$$1+K\frac{\prod_{i=1}^{M}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}=0 \quad \Rightarrow \quad \prod_{j=1}^{n}(s+p_j)+K\prod_{i=1}^{M}(s+z_i)=0$$

STEP 3:

Locate the open-loop poles and zeros on the s-plane. We are usually interested in determining the locus of roots for $0<K<\infty$.

- Root locus **BEGINS** at the open-loop poles $(K=0)$
- Root locus **ENDS** at the open-loop zeros $(K=\infty)$

STEP 4:

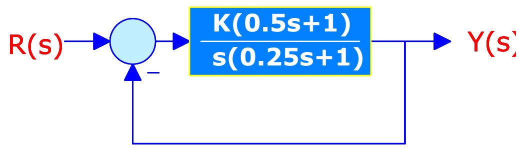
Locate the segments of the real axis that are root loci.

- Root locus on the real axis always lies in a section of the real axis to the left of an odd number of poles and zeros.

These four useful steps will now be illustrated by a suitable examples.

Example

Step 1:



$$1+K\frac{(\frac{1}{2}s+1)}{s(\frac{1}{4}s+1)}=0$$

Step 2:

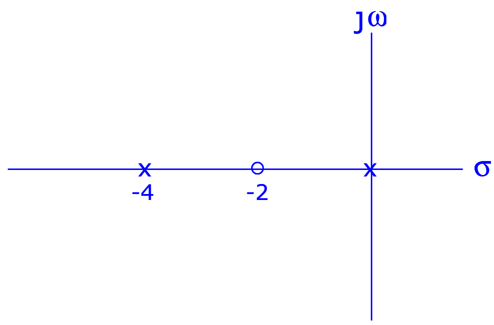
$$1+2K\frac{(s+2)}{s(s+4)}=0$$

Step 3:

We have

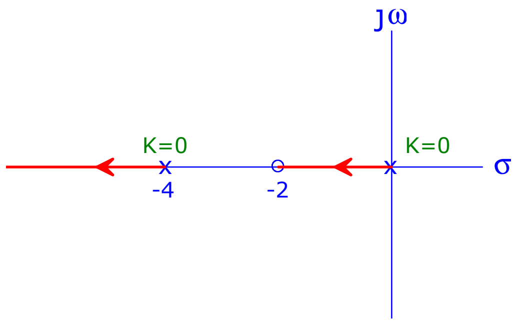
- 2 open-loop poles at $s=0$ & $s=-4$
- 1 open-loop zero at $s=-2$

We locate the poles and zeros as shown.



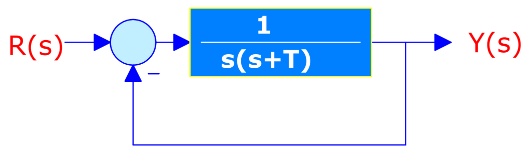
Step 4:

Locate the root locus segments that lie on the real axis



Example

Step 1:



$$1+\frac{1}{s(s+T)}=0 \rightarrow s^2+sT+1=0$$

Step 2:

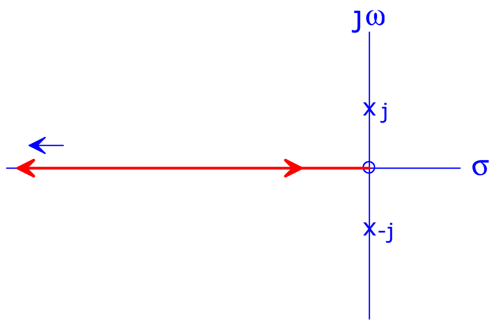
$$sT+(1+s^2)=0 \rightarrow 1+\frac{sT}{s^2+1}=0 \rightarrow 1+\frac{sT}{(s+j1)(s-j1)}$$

Step 3:

We have

- 2 open-loop poles at $s = j$ & $s = -j$
- one open-loop zero at $s = 0$

We locate the poles and zeros as shown.



Step 4:

Locate the root locus segments that lie on the real axis

We now return to developing the remaining steps.

STEP 5:

- The number of separate loci is equal to the number of open-loop poles.
- The number of loci going to ∞ is equal to the number of open-loop poles (n_p)- the number of open-loop zeros (n_z)

STEP 6:

The root loci must be symmetrical with respect to the horizontal real axis