

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT

EE380 [091]	SEC# _____	Quiz # 9
Name: _____	Key Solution	ID: _____
		Grade: _____

Use Nyquist to find the stability for a unity feedback control system with open loop function given by

$$KG(s)H(s) = \frac{K}{s(s+2)(s+10)}$$

Select your Nyquist contour such that the pole at origin is inside it ($P=1$).

Ch Eqn: $s^3 + 12s^2 + 20s + K = 0$

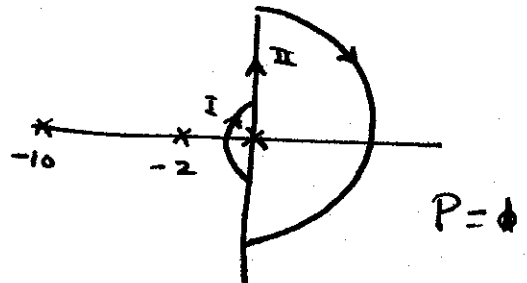
Using R.H.

$$\begin{array}{r} s^3 \quad 1 \quad 20 \\ s^2 \quad 12 \quad K \\ s^1 \quad \frac{240-K}{12} \\ s^0 \quad K \end{array} \Rightarrow 0 < K < 240 \quad (\text{stable})$$

Along I: $s = \rho e^{j\theta}$, $\theta \in [180, 90]$

$$KGH \approx \frac{K}{s} = \frac{K}{\rho} \angle -\theta$$

Along II: $s = j\omega$



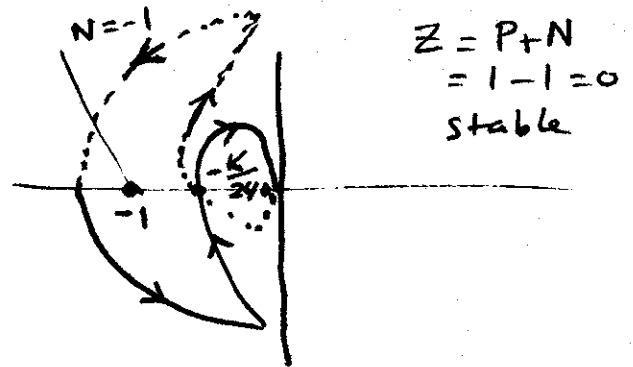
$P=1$

$$|KGH| = \left| \frac{K}{j\omega(2+j\omega)(10+j\omega)} \right| = \frac{K}{\omega \sqrt{\omega^2+4} \sqrt{\omega^2+100}}$$

$$\angle KGH = -90 - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/10)$$

$$\angle = -180 \Rightarrow \omega^2 = 20 \Rightarrow \omega = \sqrt{20}$$

$$\begin{aligned} |KGH|_{\omega=\sqrt{20}} &= \frac{K}{\sqrt{20} \sqrt{24} \sqrt{120}} \\ &= \frac{K}{240} \end{aligned}$$



$$\begin{aligned} Z &= P + N \\ &= 1 - 1 = 0 \\ &\text{stable} \end{aligned}$$

$$N = -1 \Rightarrow \frac{-K}{240} > -1 \Rightarrow \frac{K}{240} < 1 \Rightarrow K < 240$$

\therefore stable if $0 < K < 240$