KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

Department of Electrical Engineering

EE 380 - Exam II

(091)

December 28, 2009

1 Hour Exam

Student Name:

Student ID#: Key Solution

Section #:

	Maximum score	Score
Problem 1	25	
Problem 2	25	
Problem 3	25	
Problem 4	25	
Problem 5	25	
Total	125	

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Problem 1

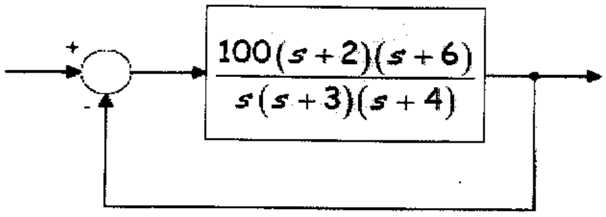
For the system shown below,

a) determine the system type (5 points)

b) find the steady-state error for

i) a unit step input (10 points)

ii) a unit ramp input (10 points)



a) System type = 1

b)

(i) $K_p = \lim_{s \to 0} G(s) = \infty$

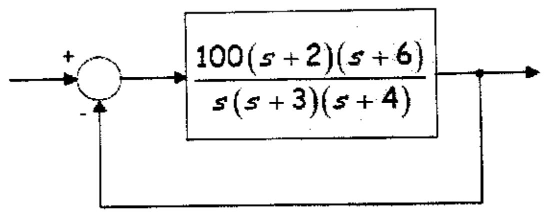
$\therefore \; e_{ss} = \frac{1.0}{1+K_p} = 0$

(ii) $K_v = \lim_{s \to 0} s\,G(s) = 100 \times \frac{2 \times 6}{3 \times 4} = 100$

$\therefore \; e_{ss} = \frac{1.0}{K_v} = \frac{1}{100} = 1\ \%$

Problem 2

For the system below, use the Routh-Hurwitz criterion to determine if the system is stable. (25 points) Note: This method applies to the closed-loop transfer function.



Ch. Eqn : $s(s+3)(s+4) + 100(s+2)(s+6) = 0$

$\therefore \quad s(s^2+7s+12) + 100(s^2+8s+12) = 0$

$s^3 + 7s^2 + 12s + 100s^2 + 800s + 1200 = 0$

$s^3 + 107s^2 + 812s + 1200 = 0$

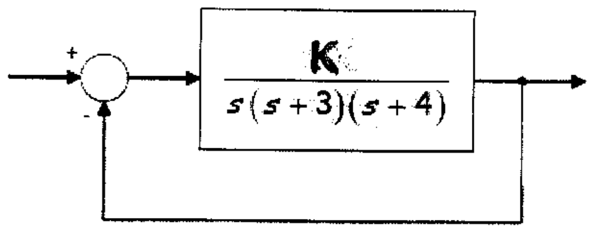
Using Routh - Hurwitz

s^3	1	812
s^2	107	1200
s^1	800.78	0
s^0	1200	0

$\therefore$ System is <u>stable</u> (No sign change)

Problem 3

Given the system below, sketch the root locus of the system.
(25 points)



$\sigma = -\frac{7}{3}$, $\theta = [\pm 60, 180]$

Ch Eqn: $1 + \frac{K}{s(s+3)(s+4)} = 0$

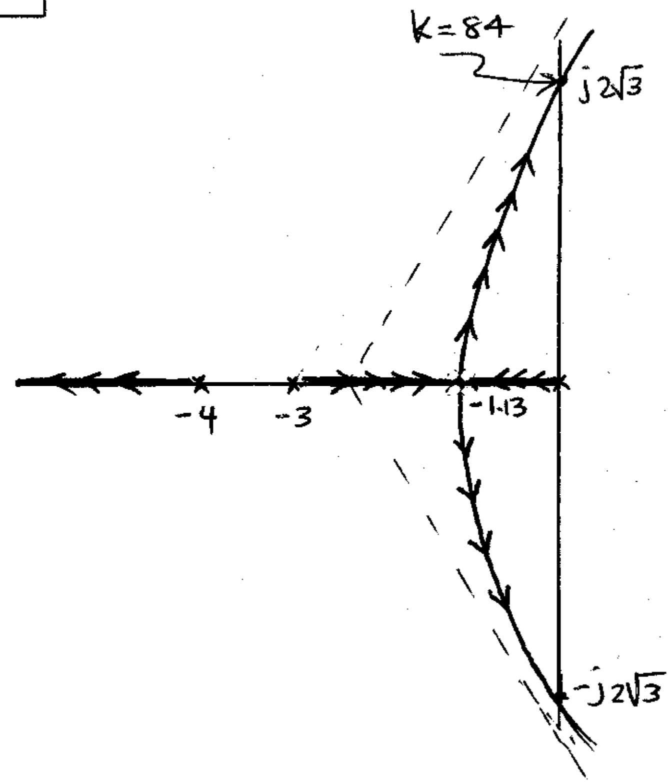
$\frac{dK}{ds} = \frac{d}{ds}\left[s^3 + 7s^2 + 12s\right] = 0$

$3s^2 + 14s + 12 = 0 \Rightarrow s = \frac{-7 \pm \sqrt{13}}{3}$

$\therefore\ s = \underline{-1.13}, -3.53$

Ch. Eqn: $s^3 + 7s^2 + 12s + K = 0$

s^3	1	12
s^2	7	K
s^1	$\frac{84-K}{7}$	0
s^0	K	0



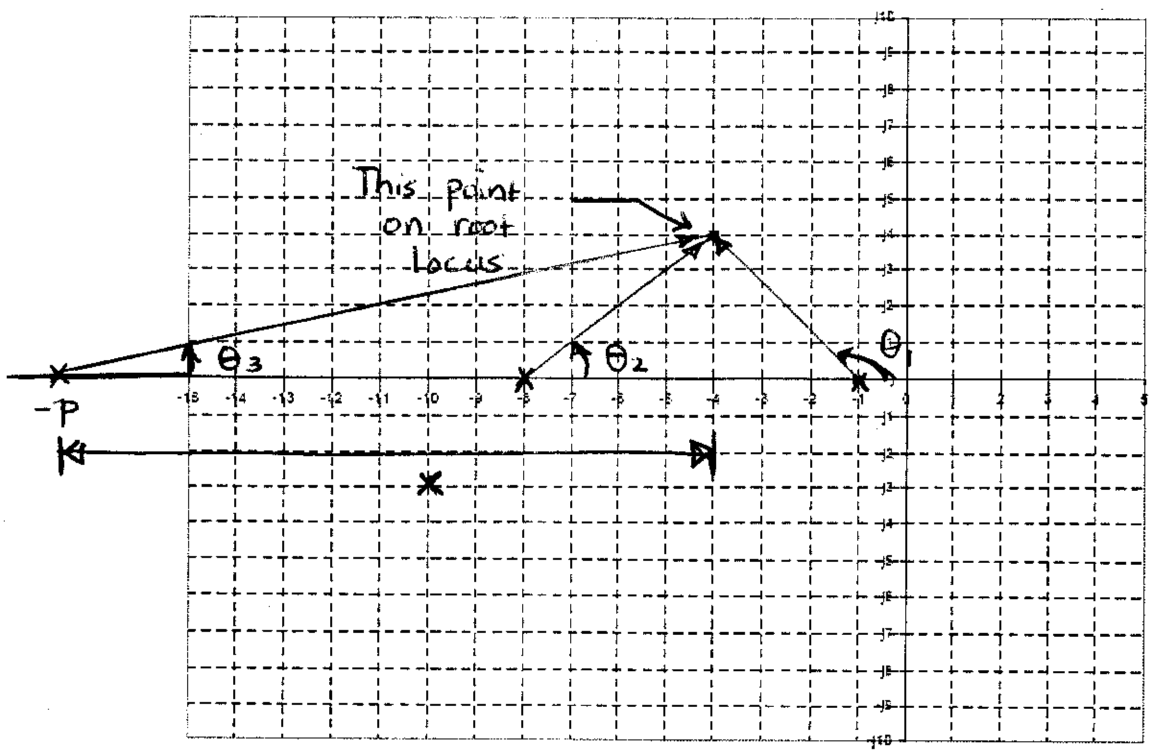
For stability $0 < K < 84$

$A(s) = 7s^2 + 84 = 0$ Auxilary Eqn:

$s^2 + 12 = 0 \Rightarrow s = \pm j\sqrt{12} = \pm j\, 2\sqrt{3}$

Problem 4

In the root locus below, the point s = -4 +j4 is desired. If the first two poles are at -1, and -8, determine the location of the third pole in the system. (25 points)



Assume the third pole at $s = -p$

Using the angle condition $\Rightarrow -[\theta_1 + \theta_2 + \theta_3] = -180$

$\theta_1 = 180 - \tan^{-1} 4/3$

$\theta_2 = 45$

$$\Rightarrow \theta_3 = 180 - \theta_1 - \theta_2$$
$$= 180 - 180 + \tan^{-1} 4/3 - 45$$
$$= \tan^{-1}(4/3) - 45^\circ = 8.13^\circ$$

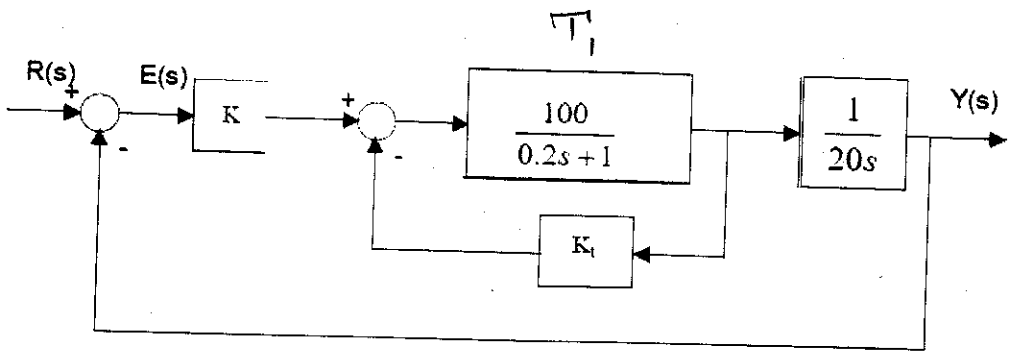
$$\tan\theta_3 = \frac{4}{X} = \frac{1}{7} \Rightarrow X = 28$$

$$\therefore p = X + 4 = 28 + 4 = 32$$

$\therefore$ third poles @ $s = -32$ ◀

Problem 5

For the control system shown, find the values of K and K_t so that the maximum overshoot of the output is approximately 4.3% and the rise time t_r is approximately 0.2 sec.



$$T_1(s) = \frac{100}{0.2s + 1 + 100K_t}$$

$$G(s) = \frac{100K}{20s\left[0.2s + 1 + 100K_t\right]} = \frac{5K}{s\left[\frac{1}{5}s + 1 + 100K_t\right]}$$

$$= \frac{25K}{s\left[s + 5 + 500K_t\right]}$$

C.E : $\quad 1 + G(s) = 0$

$$\therefore \quad s^2 + (5 + 500K_t)s + 25K = 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$O.S = 0.043 = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \Rightarrow \zeta = \frac{1}{\sqrt{2}} = 0.707$$

From C.E. $\quad 2\zeta\omega_n = 5 + 500K_t$

$$\sqrt{2}\,\omega_n = 5(1 + 100K_t), \quad \omega_n = 5\sqrt{K}$$

$$\therefore \quad \boxed{\sqrt{2K} = 1 + 100K_t}$$

$$T_r = \frac{2.16\xi + 0.6}{\omega_n} = \frac{1}{5}$$

$$\therefore \quad \omega_n = 3 + 10.8\xi = 3 + 10.8 * \frac{\sqrt{2}}{2}$$

$$\omega_n = 3 + 5.4 * \sqrt{2} = 10.64 \text{ rad/s}$$

$$K = \frac{\omega_n^2}{25} = \frac{(10.64)^2}{25} = 4.52$$

$$K_t = \frac{\sqrt{2K} - 1}{100} = 0.02006$$