EE 380: Home Work #4

E6.12 The characteristic equation associated with the system matrix is

$$s^3 + 3s^2 + 5s + 6 = 0 \ .$$

The roots of the characteristic equation are $s_1 = -2$ and $s_{2,3} = -5 \pm j1.66$. The system is stable.

P6.1 (a) Given

$$s^2 + 5s + 2 \ ,$$

we have the Routh array

$$\begin{array}{c|cc} s^2 & 1 & 2 \\ s^1 & 5 & 0 \\ s^o & 2 & \end{array}$$

Each element in the first column is positive, thus the system is stable.

(b) Given

$$s^3 + 4s^2 + 8s + 4 \ ,$$

we have the Routh array

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 4 & 4 \\ s^1 & 7 & 0 \\ s^o & 4 & \end{array}$$

Each element in the first column is positive, thus the system is stable.

(c) Given

$$s^3 + 2s^2 - 4s + 20 \ ,$$

we determine by inspection that the system is unstable, since it is necessary that all coefficients have the same sign. There are two roots in the right half-plane.

(d) Given

$$s^4 + s^3 + 2s^2 + 10s + 8 \ ,$$

we have the Routh array

s^4	1	2	8
s^3	1	10	0
s^2	-8	8	0
s^1	11	0	
s^o	8		

There are two sign changes in the first column, thus the system is unstable with two roots in the right half-plane.

(e) Given

$$s^4 + s^3 + 3s^2 + 2s + K \ ,$$

we have the Routh array

s^4	1	3	K
s^3	1	2	0
s^2	1	K	
s^1	$2-K$	0	
s^o	K		

Examining the first column, we determine that the system is stable for $0 < K < 2$.

(f) Given

$$s^5 + s^4 + 2s^3 + s + 6 \ ,$$

we know the system is unstable since the coefficient of the s^2 term is missing. There are two roots in the right half-plane.

(g) Given

$$s^5 + s^4 + 2s^3 + s^2 + s + K \ ,$$

we have the Routh array

s^5	1	2	1
s^4	1	1	K
s^3	1	$1-K$	
s^2	K	K	
s^1	$-K$	0	
s^o	K		

Examining the first column, we determine that for stability we need $K > 0$ and $K < 0$. Therefore the system is unstable for all K.

P6.4 (a) The closed-loop characteristic equation is

$$1 + GH(s) = 1 + \frac{K(s+40)}{s(s+10)(s+20)} = 0 \ ,$$

or

$$s^3 + 30s^2 + 200s + Ks + 40K = 0 \ .$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 200+K \\ s^2 & 30 & 40K \\ s^1 & 200 - \frac{K}{3} & 0 \\ s^o & 40K & \end{array}$$

Therefore, for stability we require $200 - K/3 > 0$ and $40K > 0$. So, the range of K for stability is

$$0 < K < 600 \ .$$

(b) At $K = 600$, the auxilary equation is

$$30s^2 + 40(600) = 0 \quad \text{or} \quad s^2 + 800 = 0 \ .$$

The roots of the auxiliary equation are

$$s = \pm j28.3 \ .$$

(c) Let $K = 600/2 = 300$. Then, to the shift the axis, first define $s_o = s + 1$. Substituting $s = s_o - 1$ into the characteristic equation yields

$$(s_o-1)^3+30(s_o-1)^2+500(s_o-1)+12000 = s_o^3+27s_o^2+443s_o+11529 \ .$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 443 \\ s^2 & 27 & 11529 \\ s^1 & 16 & 0 \\ s^o & 11529 & \end{array}$$

All the elements of the first column are positive, therefore all the roots lie to left of $s = -1$. We repeat the procedure for $s = s_o - 2$ and obtain

$$s_o^3 + 24s_o^2 + 392s_o + 10992 = 0 \; .$$

The Routh array is

s^3	1	392
s^2	24	10992
s^1	-66	0
s^o	10992	

There are two sign changes in the first column indicating two roots to right of $s = -2$. Combining the results, we determine that there are two roots located between $s = -1$ and $s = -2$. The roots of the characteristic equation are

$$s_1 = -27.6250 \quad \text{and} \quad s_{2,3} = -1.1875 \pm 20.8082j \; .$$

We see that indeed the two roots $s_{2,3} = -1.1875 \pm 20.8082j$ lie between -1 and -2.

AP6.3 (a) The steady-state tracking error to a step input is

$$e_{ss} = \lim_{s \to 0} s(1 - T(s))R(s) = 1 - T(0) = 1 - \alpha \; .$$

We want

$$|1 - \alpha| < 0.05 \; .$$

This yields the bounds for α

$$0.95 < \alpha < 1.05 \; .$$

(b) The Routh array is

s^3	1	α
s^2	$1+\alpha$	1
s^1	b	0
s^o	1	

where

$$b = \frac{\alpha^2 + \alpha - 1}{1 + \alpha} \; .$$

Therefore, using the condition that $b > 0$, we obtain the stability range for α:

$$\alpha > 0.618 \ .$$

(c) Choosing $\alpha = 1$ satisfies both the steady-state tracking requirement and the stability requirement.

AP6.4 The closed-loop transfer function is

$$T(s) = \frac{K}{s^3 + (p+1)s^2 + ps + K} \ .$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & p \\ s^2 & 1+p & K \\ s^1 & b & 0 \\ s^o & K & \end{array}$$

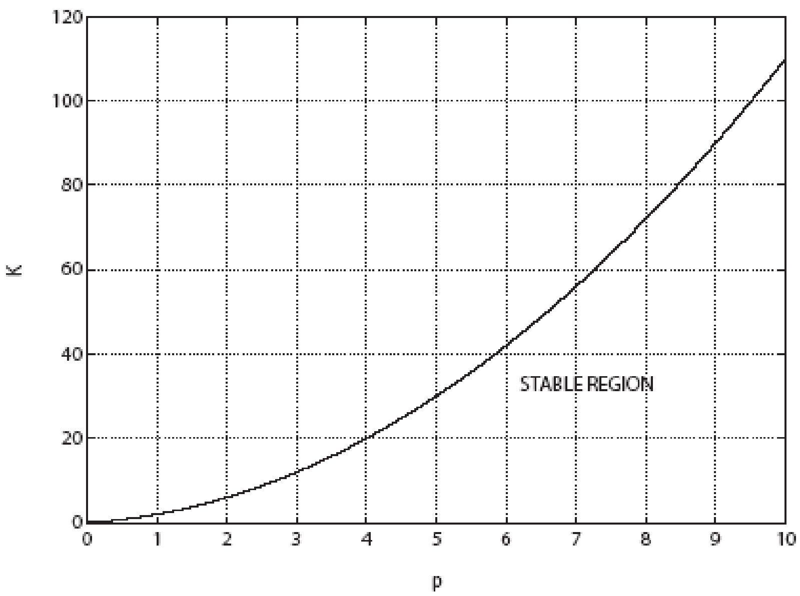
where

$$b = \frac{p^2 + p - K}{1+p} \ .$$

Therefore, using the condition that $b > 0$, we obtain the the relationship

$$K < p^2 + p \ .$$

The plot of K as a function of p is shown in Figure AP6.4.



DP 6.3

Ch Eqn : $s(1+\tau s)(1+2s) + K(s+2) = 0$

$$\Rightarrow \quad 2\tau s^3 + (\tau+2)s^2 + (K+1)s + 2K = 0$$

Routh Array

s^3	2τ	$K+1$
s^2	$\tau+2$	$2K$
s^1	a	0
s^0	$2K$	

$$a = \frac{(\tau+2)(K+1) - 4K\tau}{\tau+2}$$

For a stable system $\tau > 0$, $K > 0$

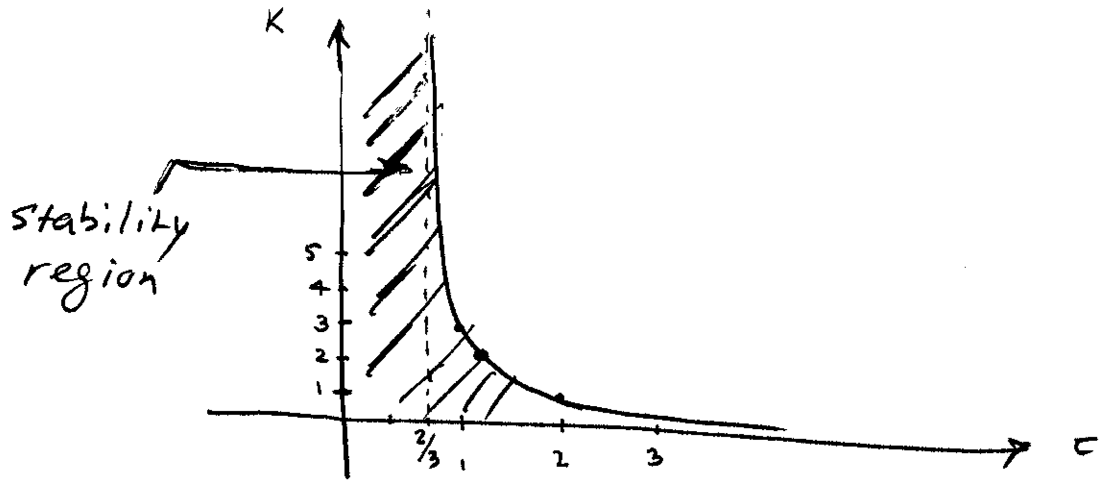
$$a > 0 \Rightarrow (\tau+2)(K+1) > 4K\tau$$

$$\therefore \quad K\tau + 2K + \tau + 2 > 4K\tau$$

$$\tau + 2 > 3K\tau - 2K = (3\tau - 2)K$$

or $K < \dfrac{\tau+2}{3\tau-2}$ $\qquad 3\tau - 2 > 0$, $\tau > 2/3$

$$\therefore \quad \begin{matrix} K \leq \\ K \geq \end{matrix} \begin{cases} \dfrac{\tau+2}{3\tau-2} & \tau > 2/3 \\ 0 & t \leq 2/3 \end{cases}$$



$$e_{ss} = \frac{A}{K_v}, \quad K_v = \lim_{s\to 0} s\,G(s) = \frac{2K}{1}$$

$$e_{ss} = \frac{A}{2K} \leq 0.25\,A \Rightarrow \frac{1}{2}K \leq 1 \Rightarrow \boxed{K \geq 2}$$

$$K = 2 = \frac{\tau + 2}{3\tau - 2} \Rightarrow \tau = 1.2$$

or let $\tau = 0.5 < 2/3 \Rightarrow$ we stay in stable region

b) $K \geq 2, \quad \tau = 0.5$

$$G(s) = \frac{K(s+2)}{s(1+\tau s)(1+2s)} = \frac{K(s+2)}{s(1+s/2)(1+2s)}$$

$$= \frac{2K(s+2)}{s(s+2)(1+2s)} = \frac{K}{s(s+1/2)} = \frac{K}{s(s+2\zeta\omega_n)}$$

$\therefore \quad K = \omega_n^2 = 2 \Rightarrow \omega_n = \sqrt{2}$

$2\zeta\omega_n = \frac{1}{2} \Rightarrow \zeta\omega_n = \frac{1}{4} \Rightarrow \zeta = \frac{1}{4\sqrt{2}}$

$\zeta = \frac{\sqrt{2}}{8} = \frac{1.4}{8} = 0.175$

From fig 5.8 : P.O. $\simeq$ 55 %

MP6.7 The characteristic equation is

$$p(s) = s^3 + 10s^2 + 15s + 10 \ .$$

```
A=[0 1 0;0 0 1;-10 -15 -10]; b=[0;0;10];c=[1 1 0]; d=[0];
sys = ss(A,b,c,d);
%
% Part (a)
%
p=poly(A)
%
% Part (b)
%
roots(p)
%
% Part (c)
%
step(sys)
```

```
p =

   1.0000   10.0000   15.0000   10.0000

r =

  -8.3464
  -0.8268 + 0.7173i
  -0.8268 - 0.7173i
```

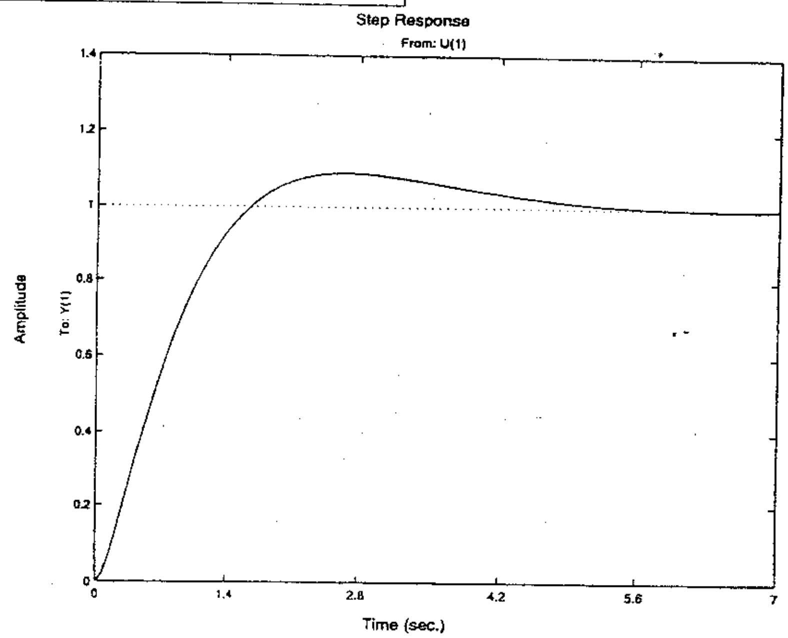


FIGURE MP6.7
Characteristic equation from the state-space representation using the **poly** function.

The roots of the characteristic equation are

$$s_1 = -8.3464 \quad \text{and} \quad s_{2,3} = -0.8268 \pm 0.7173j \ .$$