# EE 380: Home Work #3

**E5.2** (a) The closed-loop transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{100}{(s+2)(s+5)+100} = \frac{100}{s^2+2\zeta\omega_n s+\omega_n^2} .$$

The steady-state error is given by

$$e_{ss} = \frac{A}{1+K_p} ,$$

where $R(s) = A/s$ and

$$K_p = \lim_{s\to 0} G(s) = \frac{100}{10} = 10 .$$

Therefore,

$$e_{ss} = \frac{A}{11} .$$

(b) The closed-loop system is a second-order system with natural frequency

$$\omega_n = \sqrt{110} ,$$

and damping ratio

$$\zeta = \frac{7}{2\sqrt{110}} = 0.334 .$$

Since the steady-state value of the output is 0.909, we must modify the percent overshoot formula which implicitly assumes that the steady-state value is 1. This requires that we scale the formula by 0.909. The percent overshoot is thus computed to be

$$P.O. = 0.909(100e^{-\pi\zeta/\sqrt{1-\zeta^2}}) = 29\% .$$

**E5.9** The second-order closed-loop transfer function is given by

$$T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} .$$

From the percent overshoot specification, we determine that

$$P.O. \leq 5\% \quad \text{implies} \quad \zeta \geq 0.69 .$$

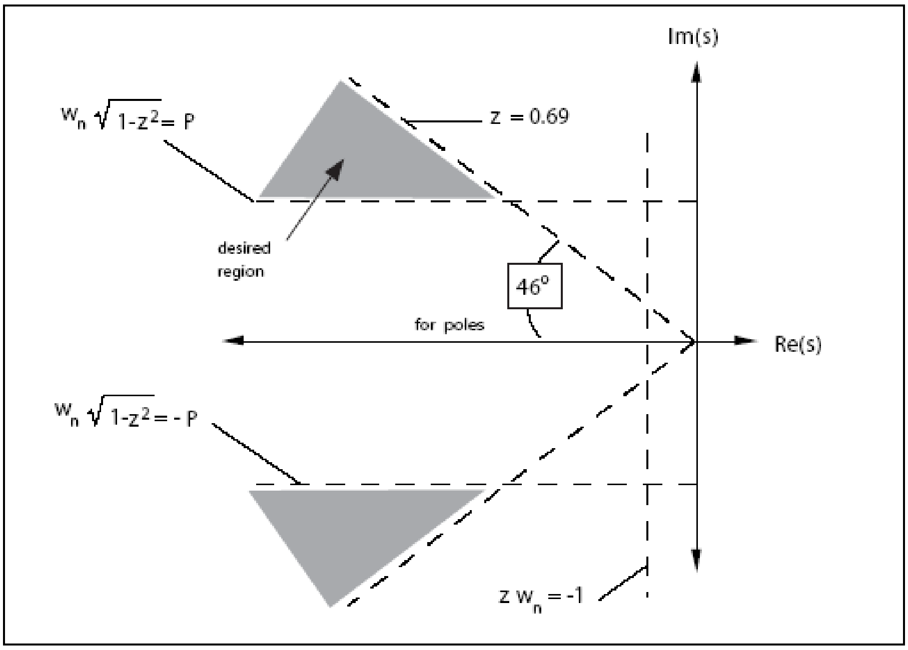
From the settling time specification, we find that

$$T_s < 4 \quad \text{implies} \quad \omega_n\zeta > 1 .$$

And finally, from the peak time specification we have

$$T_p < 1 \quad \text{implies} \quad \omega_n\sqrt{1-\zeta^2} > \pi \ .$$

The constraints imposed on $\zeta$ and $\omega_n$ by the performance specifications define the permissible area for the poles of $T(s)$, as shown in Figure E5.9.

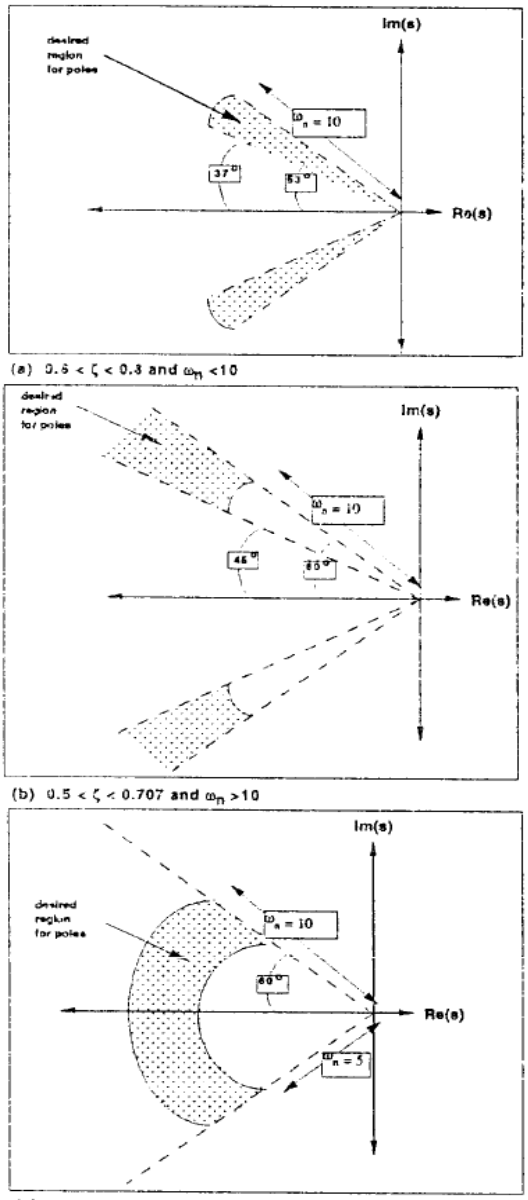



**E5.10** The system is a **type 1**. The error constants are

$$K_p = \infty \quad \text{and} \quad K_v = 1.0 \ .$$

Therefore, the steady-state error to a step input is 0; the steady-state error to a ramp input is $1.0A_0$, where $A_0$ is the magnitude (slope) of the ramp input.

**E5.16** The desired pole locations for the 5 different cases are shown in Figure E5.16.

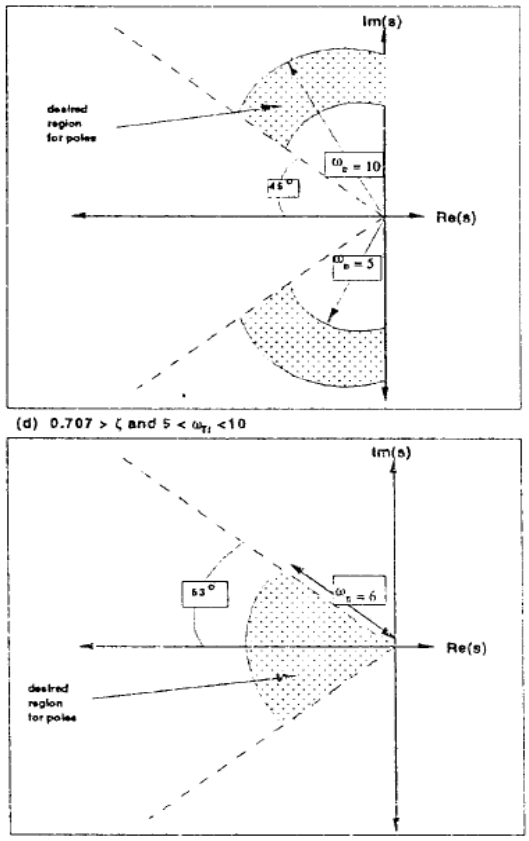



(a) 0.6 < ζ < 0.8 and $\omega_n$ < 10

(b) 0.5 < ζ < 0.707 and $\omega_n$ > 10

(c) 0.5 < ζ and 5 < $\omega_n$ < 10

FIGURE E5.16
Desired pole locations.




(d) 0.707 > ζ and 5 < $\omega_n$ < 10

(e) 0.6 < ζ and $\omega_n$ < 6

FIGURE E5.16
CONTINUED: Desired pole locations.

**P5.19** The steady-state error is

$$e_{ss} = \lim_{s\to 0} \frac{(s+10)(s+12)+K(1-K_1)}{(s+10)(s+12)+K} = \frac{120+K(1-K_1)}{120+K} \ .$$

To achieve a zero steady-state tracking error, select $K_1$ as follows:

$$K_1 = 1 + \frac{120}{K} \ .$$

**P5.20** The closed-loop transfer function is

$$T(s) = \frac{s+a}{s^2+(2k+a)s+2ak+1} \ .$$

(a) If $R(s) = 1/s$, we have the tracking error

$$E(s) = R(s) - Y(s) = [1 - T(s)]R(s)$$

or

$$E(s) = \frac{s^2+(2k+a-1)s+2ak+1-a}{s^2+(2k+a)s+2ak+1} \cdot \frac{1}{s} \ .$$

From the final value theorem we obtain

$$e_{ss} = \lim_{s\to 0} sE(s) = \frac{2ak+1-a}{2ak+1} \ .$$

Selecting $k = (a-1)/(2a)$ leads to a zero steady-state error due to a unit step input.

(b) To meet the percent overshoot specification we desire $\zeta \geq 0.69$. From $T(s)$ we find $\omega_n^2 = 2ak+1$ and $2\zeta\omega_n = 2k+a$. Therefore, solving for $a$ and $k$ yields

$$a = 1.5978 \quad \text{and} \quad \mathrm{k} = 0.1871$$

when we select $\zeta = 0.78$. We select $\zeta > 0.69$ to account for the zero in the closed-loop transfer function which will impact the percent overshoot. With $a$ and $k$, as chosen, we have

$$T(s) = \frac{s+1.598}{s^2+1.972s+1.598}$$

and the step response yields $P.O. \approx 4\%$.

**AP5.6** (a) The closed-loop transfer function is

$$T(s) = \frac{KK_m}{KK_m + s(s + K_m K_b + 0.01)} .$$

The steady-state tracking error for a ramp input $R(s) = 1/s^2$ is

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} s(1 - T(s))R(s) \\ &= \lim_{s\to 0} \frac{s + K_m K_b + 0.01}{KK_m + s(s + K_m K_b + 0.01)} \\ &= \frac{K_m K_b + 0.01}{KK_m} . \end{aligned}$$

(b) With

$$K_m = 10$$

and

$$K_b = 0.05 .$$

we have

$$\frac{K_m K_b + 0.01}{KK_m} = \frac{10(0.05) + 0.01}{10K} = 1 .$$

Solving for $K$ yields

$$K = 0.051 .$$

(c) The plot of the step and ramp responses are shown in Figure AP5.6. The responses are acceptable.

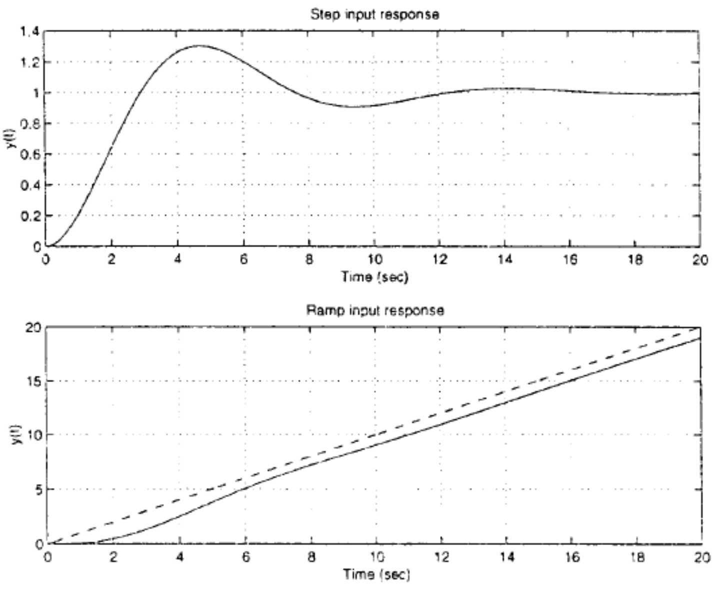



FIGURE AP5.6
Closed-loop system step and ramp responses.