KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT

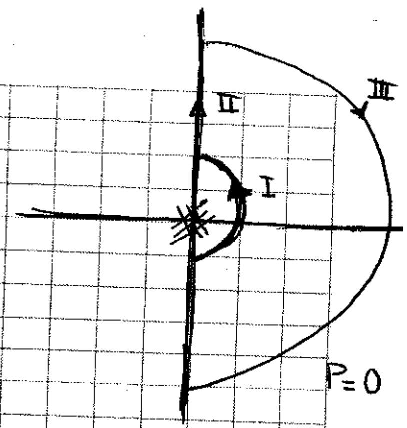
EE380 [081]	**SEC# 3**	**Quiz # 10**
Name:________________	**ID:**________________	**Grade:**________________

Use Nyquist stability and polar plot to determine the range of values for K for which the closed loop system is stable.

$$G(s)H(s) = \frac{K(s+1)^2}{s^3}$$

Verify your result by using Routh-Hurwitz stability.

Along I : $s = \epsilon e^{j\theta} \Rightarrow G(s)H(s) \approx \frac{K}{s^3} = \frac{K}{\epsilon^3} \angle -3\theta$



Along II : $s = j\omega \Rightarrow G(s)H(s) = \frac{K(1+\omega^2)}{\omega^3} \angle 2\tan^{-1}\omega - 270^\circ$

$\angle GH = -180 = -270 + 2\tan^{-1}\omega$

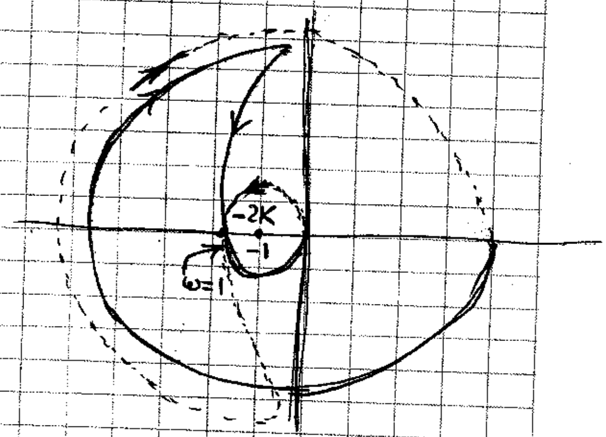
$\therefore 2\tan^{-1}\omega = 90 \Rightarrow \tan^{-1}\omega = 45 \Rightarrow \boxed{\omega = 1}$

$|GH|_{\omega=1} = \frac{2K}{1} = 2K$

if $-1 > -2K \Rightarrow N = 0$

$1 < 2K \Rightarrow Z = N + P = 0$

$\therefore$ Stable if $\boxed{K > 1/2}$



Routh: $s^3 + Ks^2 + 2Ks + K = 0$

s^3	1	2K
s^2	K	K
s^1	2K-1	0
s^0	K	

$\therefore K > 0 \quad 2K - 1 > 0 \Rightarrow \boxed{K > 1/2}$

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EE380 [081]	SEC# 4	Quiz # 10
Name:______________	ID:______________	Grade:__________

Use Nyquist stability and polar plot to determine the range of values for K for which the closed loop system is stable.

$$G(s)H(s) = \frac{K(s+1)^3}{s^3}$$

Verify your result by using Routh-Hurwitz stability.

Along the small arc ①: $s = \epsilon e^{j\theta}$, $\epsilon \to 0$, $\theta \in [0, 90]$

$G(s)H(s) \approx \frac{K}{s^3} = \frac{K}{\epsilon^3} \angle -3\theta$

Along II : $s = j\omega$

$G(s)H(s) = K \frac{(1+j\omega)^3}{(j\omega)^3} = K \frac{(1+\omega^2)^{3/2}}{\omega^3} \angle 3\tan^{-1}\omega - 270°$

$\angle GH = -180 = 3\tan^{-1}\omega - 270 \Rightarrow 3\tan^{-1}\omega = 90°$

$\therefore \tan^{-1}\omega = 30 \Rightarrow \omega_1 = \tan 30 = \frac{1}{\sqrt{3}}$

$|GH|_{\omega=\frac{1}{\sqrt{3}}} = K\left[\frac{1+1/3}{1/3}\right]^{3/2} = K[4]^{3/2} = 8K$

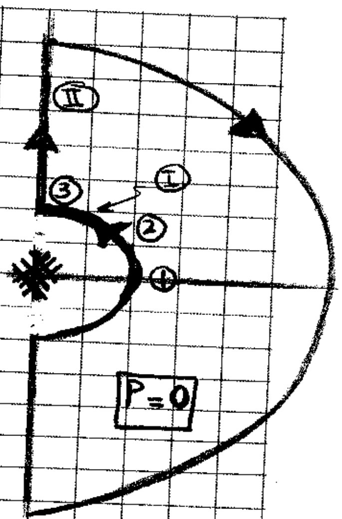
$\angle GH = -90 \Rightarrow \tan^{-1}\omega_2 = 60° \Rightarrow \omega_2 = \sqrt{3}$

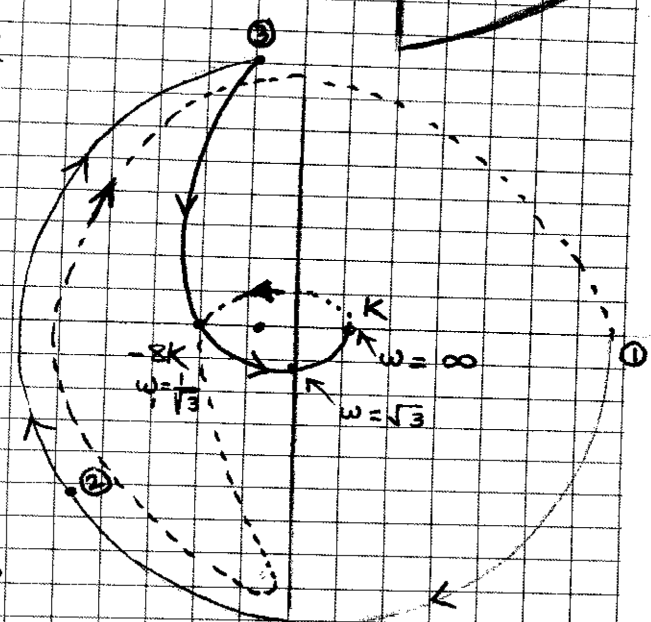
$|GH|_{\omega=\sqrt{3}} = K\frac{8}{\sqrt{27}}$ (not needed)

If $-1 > -8K \Rightarrow$ $\boxed{K > \frac{1}{8}}$

$N = 0$

$\Rightarrow Z = N + P = 0 \Rightarrow$ stable





By Routh:

$(K+1)s^3 + 3Ks^2 + 3Ks + K = 0$

s^3	$K+1$	$3K$
s^2	$3K$	K
s^1	$\frac{8K-1}{3}$	0
s^0	K	

$\Rightarrow K > -1$, $K > 0$, $K > \frac{1}{8}$ $\Rightarrow$ $\boxed{K > \frac{1}{8}}$