KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT

EE380 CONTROL ENGINEERING 081

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[MAJOR EXAM # 2]

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Name:	Key Solution
ID #:	
Section	

PROBLEM #	SCORE	MAXIMUM
1		35
2		22
3		18
4		25
TOTAL		**100**

Problem#1:
The characteristic equation of a feedback system is :

$$s^3 + 19s^2 + (K-20)s + K = 0 \qquad \text{Where} \quad K > 0 \;,$$

Fill in the blank and **sketch the root locus**

1. There are ....1.... Open-loop finite zeros at ....-1
2. There are ...3.... Open-loop finite poles at ...0, 1, -20
3. The number of asymptotes is ..2.. with angles ....$\pm 90^\circ$ and centroid at ...-9
4. There is ...2..... Break-away points at ..0.42 & -8.7
5. There is ....1.... Break-in points at ....-2.7
6. The root locus crosses the imaginary axis at the points $\pm j\sqrt{\frac{10}{9}}$ when K=..$\frac{190}{9}$.. and the third pole is at ...-19
7. The system is stable when .21.11 $\le K \le$ ..∞....

Ch. Eqn: $s^3 + 19s^2 + Ks + K - 20s = s^3 + 19s^2 - 20s + K(s+1) =$

$$\therefore \quad 1 + \frac{K(s+1)}{s(s^2+19s-20)} = 1 + K\,\frac{(s+1)}{s(s+20)(s-1)}$$

$$n = 3, \quad m = 1 \Rightarrow n - m = 2 \Rightarrow \theta = \pm 90^\circ$$

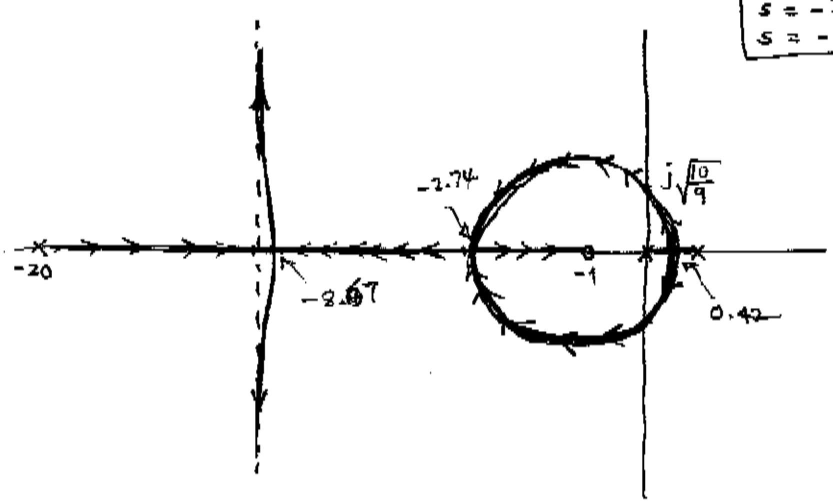
$$\sigma = \frac{(-20+1)-(-1)}{2} = -\frac{18}{2} = -9$$

$$K = -\left[\frac{s^3+19s^2-20s}{(s+1)}\right] \Rightarrow \frac{dK}{ds} = 0 \Rightarrow (s+1)\left[3s^2+38s-20\right] - \left[s^3+19s^2-20s\right] = 0$$

$$\therefore \quad \begin{aligned} &3s^3 + 38s^2 - 20s \\ &\quad + 3s^2 + 38s - 20 \\ &-s^3 - 19s^2 + 20s \quad = 0 \end{aligned} \quad \Rightarrow \quad 2s^3 + 22s^2 + 38s - 20 = 0$$

$$\boxed{s^3 + 11s^2 + 19s - 10 = 0}$$

$$\boxed{\begin{aligned} s &= 0.42 \\ s &= -2.74 \\ s &= -8.67 \end{aligned}}$$



$$s^3 + 19s^2 + (K-20)s + K = 0$$

Using Routh – Hurwitz

s^3	1	$K-20$
s^2	19	K
s^1	$\frac{19(K-20)-K}{19}$	0
s^0	K	

$\therefore$ For stability $K > 0$ & $18K - 380 > 0 \Rightarrow \boxed{K > 21.11}$

$$\therefore \boxed{21.11 < K \leq \infty}$$

$$A(s) = 19s^2 + 21.11 = 0 \Rightarrow s^2 + \frac{10}{9} = 0$$

$$\therefore \boxed{s = \pm j\sqrt{\frac{10}{9}}}$$

Ch Eqn: $s^3 + 19s^2 + (K-20)s + K = 0$

$$K = \frac{380}{18} = \frac{190}{9} \Rightarrow K - 20 = \frac{190}{9} - \frac{180}{9} = \frac{10}{9}$$

$$\therefore s^3 + 19s^2 + \frac{10}{9}s + \frac{190}{9}$$

By long division $\boxed{s + 19}$

$$s^2 + \frac{10}{9} \,\Big)\, s^3 + 19s^2 + \frac{10}{9}s + \frac{190}{9}$$

$$s^3 + \frac{10}{9}s$$

$$19s^2 + \frac{190}{9}$$

$$19s^2 + \frac{190}{9}$$

$$0$$

Problem #2:

The open loop transfer function of a feedback control system is given by

$$G(s)H(s) = \frac{Ks + \frac{T}{s}}{s^3 + s^2 + 5}$$

The parameters K and T may be represented in a plane with K as horizontal axis and T as the vertical axis. Determine the region in which the closed loop system is stable

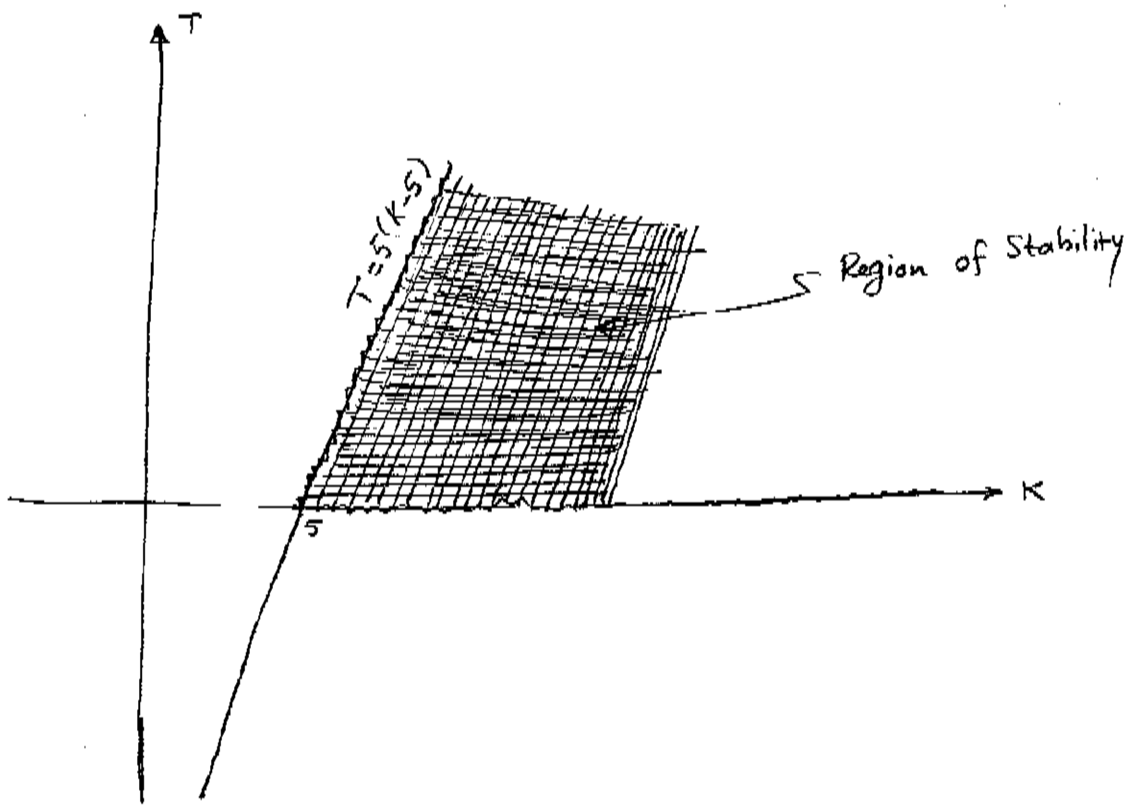
Ch Eqn: $1 + \frac{Ks + T/s}{s^3 + s^2 + 5} = 0$

$s^3 + s^2 + 5 + Ks + \frac{T}{s} = 0 \Rightarrow s^4 + s^3 + Ks^2 + 5s + T = 0$

s^4	1	K	T
s^3	1	5	0
s^2	K-5	T	0
s^1	5(K-5)-T	0	0
s^0	T	0	0

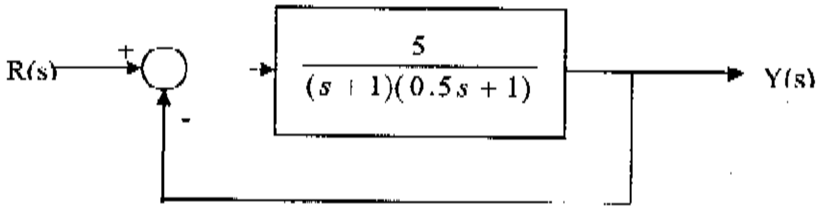
$K > 5$

$5(K-5) > T > 0$



Problem #3

1. Find the steady state error for a step input of 10 units.
2. What should be done in order to reduce the error by 50%?



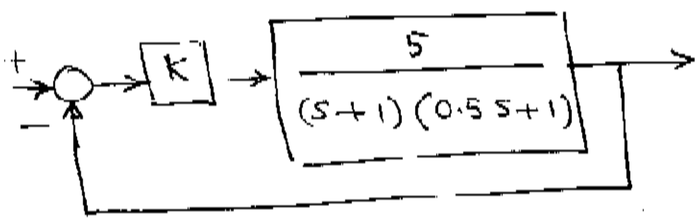
$$K_p = \lim_{s \to 0} G(s) = 5$$

① $$e_{ss} = \frac{A}{1+K_p} = \frac{10}{1+5} = \frac{10}{6} = \frac{5}{3}$$

② $$e_{ss} = \frac{1}{2} \times \frac{5}{3} = \frac{5}{6} = \frac{A}{1+\hat{K}_p} = \frac{10}{1+\hat{K}_p}$$

$$\therefore \quad \hat{K}_p = 11$$

$$\therefore \quad \lim_{s \to 0} K\, G(s) = 5K = 11 \Rightarrow \boxed{K = \frac{11}{5}}$$



Use a proportional controller $K = \frac{11}{5}$

Problem#4:

Draw the asymptotic magnitude and phase Bode plot for the following function

$$G(s) = \frac{20(s+2)}{s(s+10)}$$

$$G(j\omega) = \frac{4(1+j\omega/2)}{j\omega(1+j\omega/10)}$$

$$\angle G(j\omega) = -90 + \tan^{-1}(\omega/2) - \tan^{-1}(\omega/10)$$