Continuous Locational Marginal Pricing (CLMP)

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***Abstract*—This paper proposes a solution to eliminate the step change in the curve of Location Marginal Price (LMP) with respect to load variation. The new solution is named Continuous Locational Marginal Pricing (Continuous LMP or CLMP) because it is a continuous function with respect to load. The present LMP methodology leads to a step change when a new constraint, either transmission or generation, becomes binding as load increases. Similarly, there is also a step change of LMP if an existing constraint is no longer binding when load decreases. The proposed CLMP methodology smooths the step changes in the price curve and introduces a fourth component, Future Limit Risk (FLR) Price, in addition to the present three LMP components, Energy Price, Congestion Price, and Loss Price. Also, FLR is an indication of how close the present system state moves to the next constraint. An algorithm is proposed in this paper to give a technically efficient method to calculate CLMP and FLR price. Two case studies are presented to demonstrate the proposed CLMP methodology.**

***Index Terms*—Continuous locational marginal pricing (CLMP), DCOPF, energy markets, locational marginal pricing (LMP), optimal power flow (OPF), power markets.**

I. Introduction

THE Locational Marginal Pricing (LMP) methodology has been a dominant approach in energy market operation and planning to identify the nodal price and manage the transmission congestion. LMP methodology has been implemented or is under consideration at a number of ISOs such as PJM, New York ISO, ISO-New England, California ISO, Midwest ISO, etc. [1]–[4].

LMP may be decomposed into three components including marginal energy price, marginal congestion price, and marginal loss price [1]–[6]. LMP calculation can be formulated with Optimal Power Flow (OPF) in the same way as the old-fashioned generation dispatch under regulated structure. DCOPF-based incremental Linear Programming (LP) model (or a linear approximation of ac model) has been adopted in LMP calculation for power system operation and planning [1]–[3], [7], [8]. Particularly, DCOPF is broadly employed by a number of industrial LMP simulators, such as ABB's GridView™, GE's MAPS™, Siemens' Promod IV®, and PowerWorld for price forecasting and system planning [9], [10]. The popularity of DCOPF lies in its natural fit to the Linear Programming (LP) model for robustness and efficiency. Also, other previous works testify the general acceptability of dc model in most scenarios in power flow study [11] and LMP calculation [8], [12], compared with ac models. In addition, it is straightforward to use DCOPF to decompose LMP into three components. So, the main research work and the following discussion will be based on DCOPF.

It should be noted that a nonlinear generation cost curve may be linearized with a piece-wise-linear curve to fit into LP model. Also, the discussion below assumes that there is only one piece (or block) in the cost curve of each unit for notational simplicity. Certainly, the actual generation cost with multiple blocks may be modeled similarly, and the limits of each block may behave similarly to the limits of each generator discussed below. In addition, the mathematical formulations in this paper assume that each bus has one generator and one load for notational simplicity as well.

In general, DCOPF can be formulated as

$$\text{Min} \sum_{i=1}^{N} c_i \times G_i \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{N} DF_i \times G_i - \sum_{i=1}^{N} DF_i \times D_i + P_{loss} = 0 \tag{2}$$

$$\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i) \leq Limit_k, \text{ for } k \in \{\text{all lines}\} \tag{3}$$

$$G_i^{\min} \leq G_i \leq G_i^{\max} \text{ for } i \in \{\text{all generators}\} \tag{4}$$

where

N	number of buses;
M	number of lines;
c_i	generation cost at Bus i ($/MWh);
G_i	generation dispatch at Bus i (MWh);
D_i	demand at Bus i (MWh);
GSF_{k-i}	generation shift factor to line k from bus i;
$Limit_k$	transmission limit of line k;
DF_i	delivery factor at bus i;
P_{Loss}	system loss to offset the doubled loss using marginal delivery factor.

Then, LMP at Bus i can be written as follows:

$$LMP_i = LMP^{energy} + LMP_i^{cong} + LMP_i^{loss} \tag{5}$$

$$LMP^{energy} = \lambda \tag{6}$$

$$LMP_i^{cong} = \sum_{k=1}^{M} GSF_{k-i} \times \mu_k \tag{7}$$

$$LMP_i^{loss} = \lambda \times (DF_i - 1) \tag{8}$$

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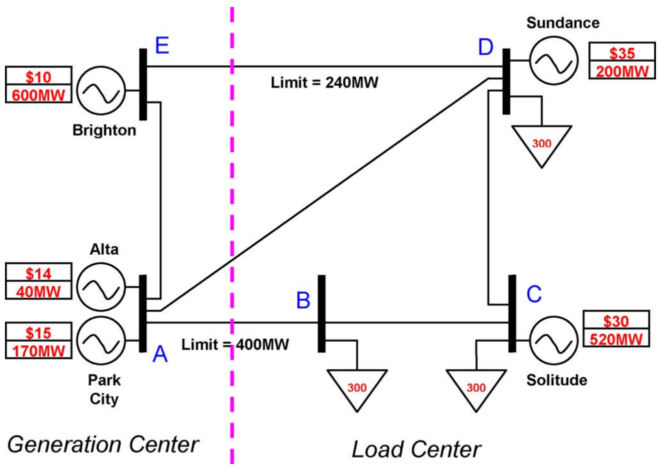

Fig. 1. Base case modified from the PJM five-bus example.

where

λ Lagrangian multiplier of the equality constraint, i.e., system energy balance equation (2);

μ_k Lagrangian multiplier of the kth transmission constraint.

The actual solution of DCOPF-based LMP calculation, especially LMP loss component, remains a challenging task because delivery factors and actual generation dispatches are mutually dependent. [8] proposes an iterative approach to address DCOPF-based LMP calculation. In the discussion in this work, the loss price is ignored to avoid the complicated issue with delivery factors and to emphasize the main point to be presented. Hence, the dispatch model and LMP calculation can be simplified to a lossless DCOPF model with $DF_i = 1$ at all buses and $P_{loss} = 0$ in (2).

This work will report a step change issue in LMP methodology when load increases or decreases. In other words, LMP curve with respect to load is discrete at some load levels. The possible step changes may not give a good price signal to market participants especially when load is close to the load level of the next step change. To address this issue, this paper will present a solution called Continuous Locational Marginal Pricing (Continuous LMP or CLMP) methodology to achieve a continuous price curve.

This paper is organized as follows. Section II presents an actual example of LMP step changes when load varies in the PJM five-bus system. Section III presents the basic idea of CLMP. Section IV first describes a brute-force approach to calculate CLMP by testing LMP at many different load levels; then presents an efficient algorithm to obtain the next or previous critical load level where a step change of LMP occurs; and finally proposes the Future Limit Risk (FLR) price under the new CLMP approach. Section V further illustrates the CLMP with two case studies using the PJM five-bus system. Section VI extends the discussions and concludes the paper.

II. Step Changes of LMP When Load Changes

The present LMP methodology may lead to step changes when load grows. This can be verified with the test results from the PJM five-bus sample system [1]. The system is shown in Fig. 1. The system may be roughly divided into two areas, a generation center consisting of Buses A and E with three low-cost generation units and a load center consisting of Buses B, C, and D with 300 MW load at each bus and two high-cost generation units. The transmission line parameters are given in Table I. Several slight modifications to the original PJM five-bus system are made for better illustration: the output limit of the Alta unit is reduced from 110 MW to 40 MW, while the output limit of the Park City unit is increased from 100 MW to 170 MW; the cost of Sundance unit at Bus D is changed from \$30/MWh to \$35/MWh to differentiate its cost from the Solitude unit; and Line AB is assumed to have 400 MW limit. These changes are made such that there will be reasonably more binding limits within the investigated range of load levels. Also, there will not be two binding limits that occur at very close load levels. Hence, better illustration will be achieved when the price curves versus load levels are drawn.

TABLE I
Line Impedance and Flow Limits

Line	AB	AD	AE	BC	CD	DE
X (%)	2.81	3.04	0.64	1.08	2.97	2.97
Limit (MW)	400	--	--	--	--	240

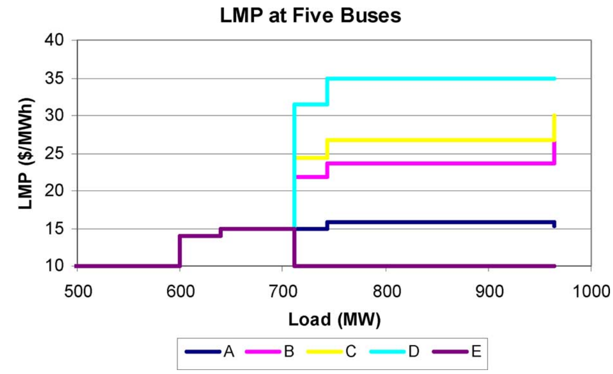

Fig. 2. LMP at all buses with respect to different system loads.

In the discussion below, the system load change is assumed to be distributed to all bus loads in proportion to their base case load. Also, as commonly assumed, the load remains the same within 1 hour, so energy (in MWh) will be numerically the same as power (in MW). In addition, lossless DCOPF is employed as stated previously, so LMP here consists of energy price and congestion price.

Fig. 2 shows the nodal LMP with respect to the system load, uniformly distributed at all three load buses. The figure clearly shows that there are several step changes of LMP when the system load grows from 500 MW to 1000 MW. For example, if the load is less than 600 MW, all buses have a LMP of \$10/MWh. When the system load is even slightly over 600 MW, the price will have a stiff increase because the cheapest Brighton unit reaches its limit at 600 MW and the new marginal unit is the \$14/MWh Alta unit. Similarly, when the system load reaches 711.81 MW, there will be another LMP step change due to a new binding transmission limit of Line DE. Since this is a binding transmission limit, so the step change of

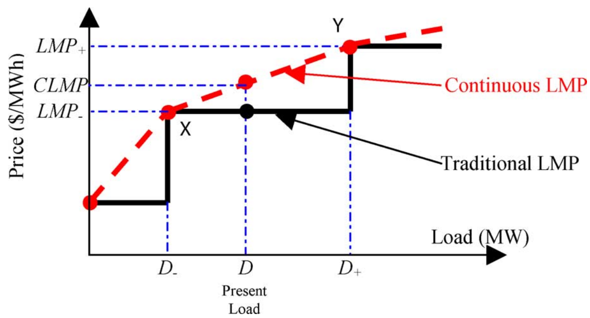

Fig. 3. LMP and CLMP curves.

LMP is different at each bus. It should be noted that in the real application, this sudden, step change of price can make market participants financially unprepared and give them misleading price signals. In fact, LMP remains constant between two load levels corresponding to two adjacent binding limits (i.e., between the previous step change and the next step change). In other words, LMP remains unchanged with respect to load growth, as long as the next binding limit has not been reached. Therefore, LMP neither provides readily available information about how close the market is to the next critical load level at which the LMP will have a step change, nor utilizes this information to set nodal price. Hence, the step change, implying an infinite sensitivity at some critical load levels, presents risk and uncertainty for market participants and operators. For example, assume the actual price in real-time operation may indicate a step change at 100 MW, while a market participant forecasts the price to have a step change at 95 MW due to a slight inaccuracy of input data or an approximate LMP algorithm. Then, this will potentially lead to a significant error from the participants viewpoint when the load is between 95 MW and 100 MW.

III. Basics of the Proposed CLMP Methodology

As stated previously, the cause of this step change is due to a new binding constraint, either a transmission limit or a generation limit, when load grows to a critical level. In addition, there is no information about how far the current load level is away from the next (or the previous) critical load level, which may lead to a new binding constraint and hence a new price with a step change.

The paper attempts to solve the step change issue in the current LMP methodology by a modified pricing methodology, Continuous LMP or CLMP, to remove possible sudden changes when the load increases to a critical level leading to a new binding constraint or decreases to a critical level leading to the disappearance of an existing binding constraint. For clarification purpose, the present LMP methodology is also referred to as the *Traditional LMP* from here on forward.

Fig. 3 shows the diagram of Traditional LMP with respect to the different load levels. It shows that the current load is D. The Traditional LMP is LMP_- for loads in $[D_-, D]$. The LMP remains LMP- for loads in $[D, D_+)$. If the load reaches D_+, the Traditional LMP jumps to LMP_+ due to a new binding limit. It should be noted that the price remains LMP_- for load in $[D_-, D_+)$ because all binding constraints remain unchanged and the lossless DCOPF is a linear model.

The amount of load change may be distributed to all load buses based on a given pattern. In the discussion below, it is assumed that the system load change is distributed to all load buses in proportion to their base case load, though in practice any pattern of load variation may be applied.

To avoid the step change in the LMP curve, this paper proposes to use the LMP_- from the Traditional LMP as the CLMP price at the exact load of D_- and LMP_+ as the CLMP price at the exact load of D_+. Assume these two points are X (D_-, LMP_-) and Y (D_+, LMP_+). Then, we may draw a straight line between X and Y. Line XY will be the new Continuous LMP curve as shown with the dash line in Fig. 3. Then, we can easily obtain the new price, *CLMP*, at load D using linear interpolation. If we have the same procedure for all other different load levels, we will eventually obtain a piece-wise-linear, continuous price curve without any step changes.

The proposed Continuous LMP methodology with an efficient algorithm has the features summarized below:

- The CLMP curve will not have step changes when load grows. The load level where step change occurs in Traditional LMP methodology will be a continuous point in Continuous LMP with respect to load growth.
- The CLMP methodology will introduce a new pricing component, Future Limit Risk Price or $\mathrm{CLMP}^{\mathrm{FLR}}$, to create a continuous "penalty" due to the next future binding limit.
- An efficient algorithm will be presented to quickly calculate CLMP at any given load.

IV. Implementation of CLMP

The above is the conceptual description of the Continuous LMP methodology. Although the concept is straightforward, it is challenging to obtain the continuous, straight line without intense computational effort. This is discussed in detail in the following subsections.

A. Brute-Force Approach

To obtain the Continuous LMP curve, we may have a straightforward, brute-force approach described as follows.

1) Run multiple Traditional LMP calculation at various load levels, increasing or decreasing relative to the current load level of interest, to obtain the Traditional LMP curve with respect to load.
2) Find two connecting points (like X and Y in Fig. 3) to create a Continuous LMP curve with respect to different load levels. Here, the two connecting points correspond to the adjacent LMP step changes when the load varies.
3) Find the new CLMP corresponding to the actual load level of interest using linear interpolation.

This brute-force approach may take a lot of effort to calculate due to the large amount of OPF runs at many different load levels. If the step change occurs at a load level much greater than the current load level, many trial runs may be needed. It cannot efficiently predict the next (or previous) critical load level.

B. Basic Idea of the Proposed CLMP Algorithm

As we can observe from Fig. 3 and the brute-force approach, the key of the Continuous LMP is to find the *previous* and the *next* critical load levels, D_+ and D_-, where there is a sudden change based on the Traditional LMP due to load variation. The discussion below will take load increase as an example to find D_+ for illustration. Similar approach can be applied to find D_- as load decreases.

Since the next binding constraint occurs at D_+, there is no new binding constraint between D and D_+. Hence, there is no new marginal unit.

Therefore, if there is a load growth, ΔD (i.e., D_+ minus D), the present marginal units shall provide all matching amount of load growth. The output of marginal units shall grow proportionally when load increases from D to D_+. This can be written as

$$\frac{\Delta MG_i}{\Delta D} = \alpha_i \tag{9}$$

where $MG_i =$ marginal unit.

This sensitivity of marginal unit output with respect to load variation should be a constant value because the DCOPF model is based on Linear Programming model. Hence, the change of each marginal generation with respect to a load change should also be linear. As a matter of fact, α_i is zero for all non-marginal units. Only marginal units have nonzero values.

A special case of α_i is a no-congestion case. That is, if there is no congestion in the system, a small change of load will be absorbed by the only marginal generator. Therefore, α_i is 1 for the only marginal unit and 0 for all other units.

Based on the linearity of DCOPF-based Linear Programming, the sensitivity shall remain constant when load grows until a new constraint reaches its limit. This implies that a new marginal unit is needed. Hence, from the present load level, all marginal units should grow at their specific sensitivity. This feature can be used to find the amount of load change that causes the next new constraint, either transmission or generation, that will be violated. The total load when this new binding limit occurs will be the new critical load, D_+.

Next, the detailed analytical algorithm will be discussed and described how to identify D_+ or D_- efficiently and therefore to obtain Continuous LMP easily.

C. Details of the Proposed Algorithm of Continuous LMP

1) To Obtain the Sensitivity of Marginal Generation Output Versus Load: When OPF is solved at the present load level D (or D_Σ from here and forward to distinguish it from bus load D_i as shown in (10)), we can obtain the marginal unit set, $\{\mathrm{MG}_i | i = 1, 2, \ldots, N_{MG}\}$, and the non-marginal unit set $\{NG_i | i = 1, 2, \ldots, N_{NG}\}$. Thus, based on lossless DCOPF, i.e., (1)–(4) with $DF_i = 1$ at all buses and $P_{loss} = 0$, we may rewrite all binding limits as follows:

$$\sum_{i=1}^{N_{MG}} MG_i + \sum_{i=1}^{N_{NG}} NG_i = D_\Sigma \tag{10}$$

$$\sum_{i=1}^{N_{MG}} GSF_{k-i} \times MG_i + \sum_{i=1}^{N_{NG}} GSF_{k-i} \times NG_i - \sum_{i=1}^{N} GSF_{k-i} \times D_i = Limit_k \tag{11}$$

where $D_\Sigma = \sum_{i=1}^{N} D_i =$ the system total load.

If there is a very small change of load (without change of marginal units and binding constraints), the present binding constraints can be written in (12)–(13)

$$\sum_{i=1}^{N_{MG}} (MG_i + \partial MG_i) + \sum_{i=1}^{N_{NG}} NG_i = (D_\Sigma + \partial D_\Sigma) \tag{12}$$

$$\sum_{i=1}^{N_{MG}} GSF_{k-i} \times (MG_i + \partial MG_i) + \sum_{i=1}^{N_{NG}} GSF_{k-i} \times NG_i - \sum_{i=1}^{N} GSF_{k-i} \times (D_i + f_i \cdot \partial D_\Sigma) = Limit \tag{13}$$

where $f_i = \partial D_i / \partial D_\Sigma =$ participating factor of load D_i.

In fact, any load variation pattern may be employed for the participating factor f_i, which represents the contribution ratio from load at Bus i to the total load variation. In the discussion below, f_i is assumed to be proportional to the base case load for easy illustration. Hence, we have $f_i = \partial D_i / \partial D_\Sigma = D_i / D_\Sigma$ here.

With mathematical operations between (12) and (10) and between (13) and (11), we have

$$\sum_{i=1}^{N_{MG}} \left(\frac{\partial MG_i}{\partial D_\Sigma} \right) = 1.0 \tag{14}$$

$$\sum_{i=1}^{N_{MG}} GSF_{k-i} \times \partial MG_i = \left(\sum_{i=1}^{N} GSF_{k-i} \times f_i \right) \times \partial D_\Sigma. \tag{15}$$

Therefore, from the above equations, we can easily obtain $\partial MG_i / \partial D_\Sigma$ by solving a standard linear matrix equation. Apparently, the value of $\partial MG_i / \partial D_\Sigma$ should be a constant as shown in (9), i.e., $\partial MG_i / \partial D_\Sigma = \alpha_i$.

It should be noted that the number of marginal units should equal to the number of binding transmission constraints plus 1 [5]. The "1" here accounts for the equality constraint for energy balance. For example, if there is no binding transmission constraint, there should be only one marginal unit. If there is one binding transmission limit, there should be two marginal units. Therefore, (15) corresponds to $N_{MG}-1$ equations, and (14)–(15) have a unique solution in general.

2) To Obtain the Next and Previous Critical Load Levels Corresponding to LMP Step Changes: With $\partial MG_i / \partial D_\Sigma$ obtained, we can then solve the *next* (or *previous*) critical load level where a new binding limit occurs (or an existing binding limit becomes unbinding). The following discussion is based on the *next* binding limit for illustrative purposes. Similar approach can be employed for the *previous* critical load level.

To do so, we need to examine *all unbinding* constraints in which the marginal unit(s) is involved. This can be written as

$$\sum_{i=1}^{N_{MG}} GSF_{k-i} \times MG_i + \sum_{i=1}^{N_{NG}} GSF_{k-i} \times NG_i - \sum_{i=1}^{N} GSF_{k-i} \times D_i \le Limit_k \tag{16}$$

$$MG_i^{\min} \le MG_i \le MG_i^{\max}. \tag{17}$$

First, the unbinding transmission constraint in (16) is considered. Assuming the next new binding limit occurs at $D_\Sigma+\Delta D_\Sigma$ that corresponds to $MG_i+\Delta MG_i$, we have

$$\sum_{i=1}^{N_{MG}} GSF_{k-i}\times(MG_i+\Delta MG_i)+\sum_{i=1}^{N_{NG}} GSF_{k-i}\times(NG_i) \\ -\sum_{i=1}^{N} GSF_{k-i}\times(D_i+f_i\cdot\Delta D_\Sigma)=Limit_k \quad (18)$$

where $\Delta D_\Sigma=$ the maximum allowed load growth before another limit is reached.

The above equation can be simplified to

$$\sum_{i=1}^{N_{MG}} GSF_{k-i}\times(\Delta MG_i)-\sum_{i=1}^{N} GSF_{k-i}\times f_i\times\Delta D_\Sigma \\ =Limit_k-LF_{k0} \quad (19)$$

where

$$LF_{k0}=\sum_{i=1}^{N_{MG}} GSF_{k-i}\times MG_i+\sum_{i=1}^{N_{NG}} GSF_{k-i}\times NG_i \\ -\sum_{i=1}^{N} GSF_{k-i}\times D_i \\ =\sum_{i=1}^{N} GSF_{k-i}\times(G_i-D_i).$$

In the above equation, LF_{k0} is the kth line flow when the system load is D_Σ and $Limit_k-LF_{k0}$ is the remaining capacity of the kth line when the system load is D_Σ.

Since $\Delta MG_i/\Delta D_\Sigma=\partial MG_i/\partial D_\Sigma=\alpha_i$ for all load levels in $[D_\Sigma, D_{\Sigma+})$, we can obtain ΔD_Σ as

$$\Delta D_\Sigma=\frac{Limit_k-LF_{k0}}{\left[\sum_{i=1}^{N_{MG}}(GSF_{k-i}\times\alpha_i)-\sum_{i=1}^{N}(GSF_{k-i}\times f_i)\right]}. \quad (20)$$

This can be repeated for each unbinding transmission constraint to obtain the corresponding ΔD_Σ that causes the transmission constraint binding. Hence, we will have a set of ΔD_Σ, $\{\Delta D^T_{\Sigma,1},\Delta D^T_{\Sigma,2},\ldots,\Delta D^T_{\Sigma,U}\}$, each representing the allowed load change to cause the unbinding transmission constraint to be binding.

The same process can be repeated for the generation limit of each marginal unit as shown in (17). This can be handled in a similar way to the transmission limit. It should be noted that α_i may be negative in the case that the output of a low-cost generator has to be reduced when load grows because of transmission congestion. Considering

$$MG_i+\Delta MG_i=MG_i^{\max} \text{ or } MG_i+\Delta MG_i=MG_i^{\min}.$$

Then, we have

$$\Delta D_\Sigma=\frac{\Delta MG_i}{\alpha_i}=\begin{cases}\frac{MG_i^{\max}-MG_i}{\alpha_i}, & \text{if } \alpha_i>0\\ \frac{MG_i^{\min}-MG_i}{\alpha_i}, & \text{if } \alpha_i<0\end{cases}. \quad (21)$$

Hence, we have another set of ΔD_Σ, $\{\Delta D^G_{\Sigma,1},\Delta D^G_{\Sigma,2},\ldots,\Delta D^G_{\Sigma,N_{MG}}\}$, calculated from all unbinding, marginal generators' limits. Each ΔD_Σ represents the allowed load growth before reaching a new generation limit.

The two sets of ΔD_Σ, $\{\Delta D^T_{\Sigma,1},\Delta D^T_{\Sigma,2},\ldots,\Delta D^T_{\Sigma,U}\}$ and $\{\Delta D^G_{\Sigma,1},\Delta D^G_{\Sigma,2},\ldots,\Delta D^G_{\Sigma,N_{MG}}\}$, should be compared and the lowest value should be chosen as the actual allowed load growth, ΔD_Σ, before reaching the next binding limit. With ΔD_Σ obtained, we can easily obtain the next critical load level $D_{\Sigma+}$, which is equal to $D_\Sigma+\Delta D_\Sigma$. Also, we can obtain ΔMG_i, which is equal to $\alpha_i\times\Delta D_\Sigma$.

Similarly, we can obtain the *previous* critical load level $D_{\Sigma-}$, at which an existing binding limit, either generation or transmission, becomes unbinding. If this limit is a transmission limit, it becomes unbinding simultaneously as the disappearance of a marginal unit. Hence, we only need to calculate the reduced load, ΔD_Σ, to identify which marginal (unbinding) unit will become non-marginal (binding). The new output of the unit should be either at its lower bound if α_i is greater than 0 or at the upper bound if α_i is less than 0. Hence, we have

$$\Delta D_\Sigma=\frac{\Delta MG_i}{\alpha_i}=\begin{cases}\frac{MG_i-MG_i^{\min}}{\alpha_i}, & \text{if } \alpha_i>0\\ \frac{MG_i-MG_i^{\max}}{\alpha_i}, & \text{if } \alpha_i<0\end{cases}. \quad (22)$$

The final allowed ΔD_Σ will be the minimum of all ΔD_Σ's obtained for each marginal unit. And, the previous critical load level, $D_{\Sigma-}$, is equal to $D_\Sigma-\Delta D_\Sigma$. Again, there is no need to check the transmission constraints, since the disappearance of a binding transmission limit is always simultaneously accompanied by the disappearance of a marginal generation unit.

3) To Obtain CLMP and Future Limit Risk (FLR) Price: To obtain the new CLMP at this load level, we need to run the DCOPF and LMP calculation at the new critical load level $D_{\Sigma+}$. Practically, to avoid numeric problem, we may consider running the DCOPF and LMP calculation at a slightly higher load level such as $D_\Sigma+\Delta D_\Sigma+\varepsilon$, where ε is a very small number. On the other hand, the LMP at $D_{\Sigma-}$ is the same as the LMP at the current load level D_Σ as discussed before.

Therefore, the new price at the load level D_Σ can be easily obtained using linear interpolation as follows:

$$CLMP=LMP_-+\frac{D_\Sigma-D_{\Sigma-}}{D_{\Sigma+}-D_{\Sigma-}}\times(LMP_+-LMP_-) \quad (23)$$

where

LMP_- Traditional LMP at $D_{\Sigma-}$;

LMP_+ Traditional LMP at $D_{\Sigma+}$.

In fact, for any load level D_x between $D_{\Sigma-}$ and $D_{\Sigma+}$, the new price with the CLMP methodology can be calculated from the piece-wise linear curve. It can be written as

$$CLMP_x=LMP_-+\frac{D_x-D_{\Sigma-}}{D_{\Sigma+}-D_{\Sigma-}}\times(LMP_+-LMP_-) \\ \text{for any } D_x\in[D_{\Sigma-},D_{\Sigma+}]. \quad (24)$$

As shown in the above approach as well as in Fig. 3, there is a price uplift at D_Σ in CLMP, compared with the Traditional LMP. The increased part can be viewed as the "penalty" cost,

TABLE II
GSF OF LINE AB AND ED

	A	B	C	D	E
Line AB	0.1939	-0.4759	-0.349	0	0.1595
Line DE	0.3685	0.2176	0.1595	0	0.4805

which increases as the load moves closer to the next binding limit. Hence, this price component is named Future Limit Risk price, or $CLMP^{\rm FLR}$, which can obtained from (23)

$$CLMP^{\rm FLR} = \frac{D_\Sigma - D_{\Sigma-}}{D_{\Sigma+} - D_{\Sigma-}} \times (LMP_+ - LMP_-). \quad (25)$$

This is the proposed fourth component of CLMP. The CLMP price shall include this component, in addition to the three components in the Traditional LMP. Thus, the Continuous LMP may be decomposed into four components, energy price, congestion price, loss price, and FLR price. The first three components are the same as in the Traditional LMP methodology, while the fourth component is to smooth the price curve with respect to load variation.

Since this new LMP component is based on the LMP at $D_{\Sigma-}$ and $D_{\Sigma+}$, the previous and the next binding limit, respectively, it is also a marginal price. Also, $(D_\Sigma - D_{\Sigma-})/(D_{\Sigma+} - D_{\Sigma-})$ may be viewed as the weight of the relative distance between the current load level D_Σ and the next (or previous) critical load level $D_{\Sigma+}$ (or $D_{\Sigma-}$). Hence, FLR may serve as an indication of how close the present system state is to the next constraint.

Although the CLMP and the proposed FLR price ($\rm CLMP^{FLR}$) may require the buyers to pay slightly more, this will be justified by a less risky and more competitive market due to the continuous price curve as load grows. Hence, this will reduce the volatility in market price and benefit all buyers and other participants in the long run. In general, the step change issue in the Traditional LMP can make the participants, especially short-term traders and providers, to guess and even gamble whether the congestion will occur or not in the next trading period, such as in the next 5 min. With the proposed CLMP, this risk of sudden, stiff price change will be much reduced. Since smaller providers are more vulnerable to risk, a less risky market will make more small-to-medium sized providers willing to participate the market. Therefore, CLMP will reduce the chance that the market may go to undesired oligopoly or monopoly. Hence, reduced risk in price will stimulate more small-to-medium sized participants, which are desired by an ideal competitive market model.

D. Summary of the Proposed CLMP Algorithm

The whole CLMP algorithm can be summarized as follows.

1) Solve the Traditional LMP.

2) Apply (14) to the energy balance constraint and (15) to all binding transmission limits to obtain $\partial MG_i/\partial D_\Sigma = \alpha_i$.

3) Apply (20) to each unbinding transmission limit to calculate the possible load change ΔD_Σ before reaching the limit of each transmission line.

4) Apply (21) to each marginal unit to calculate the possible load change ΔD_Σ before reaching the limit of each marginal generator.

5) Find the minimum of all ΔD_Σ values calculated in Steps 3 and 4. It is the actual allowed load growth before reaching another binding limit.

6) Find the allowed load decrease, at which an existing binding limit becomes unbinding, using (22).

7) Find *CLMP* using (23) and $CLMP^{\rm FLR}$ using (25).

The proposed CLMP algorithm is performed by solving a few linear matrix equations such as (14)–(15) and (20)–(21). And, there is no need to repeat many OPF runs as in the brute-force approach. Therefore, compared with the OPF run in the Traditional LMP, the additional computational effort for the proposed CLMP is dimensionally less than the OPF and should not be an issue for implementation.

It should be noted that with CLMP methodology, the price curve will have a different growth rate in the left or right of a critical load level, while the Traditional LMP price curve has a sudden change, as shown in Fig. 3. The critical load level is still useful in CLMP, as it indicates the different price growth rate, i.e., the slope of the different pieces of the piece-wise-linear CLMP curves. As a by-product, the results of the proposed CLMP algorithm also indicate the previous and next critical load levels.

It should be also noted that the same technique of the proposed CLMP algorithm can be used to efficiently predict the next or previous critical load level in the Traditional LMP. Hence, it will be very easy to identify how far the current load is away from the load level at which the next binding limit occurs or an existing binding limit becomes unbinding. This should be useful information for congestion management under the Traditional LMP.

V. CASE STUDIES

In this section, the PJM five-bus system will be employed to illustrate how to calculate CLMP and FLR price. Two different load levels, 630 MW and 900 MW, will be studied. We assume that the system load change is distributed to each nodal load proportional to its base case load. Therefore, the load change is equally distributed at Buses B, C, and D since each has 300 MW load in the base case. Again, the same slight modifications are made here as in Section II for better illustration purposes.

The Generation Shift Factors of Line AB and ED with respect to all buses are shown in Table II.

A. Load at 630 MW

When the system load is 630 MW (or 70% of the base case load), the results from DCOPF are listed as follows.

Unit Dispatch (MW): 30, 0, 0, 0, 600;
LMP ($/MWh): 14, 14, 14, 14, 14;
$\text{Line AB Flow (MW)} = 274.77;$
$\text{Line ED Flow (MW)} = 220.14.$
Note 1: The unit dispatches are shown in the order of Alta, Park City, Solitude, Sundance, and Brighton.

TABLE III
LMPs AT $D_{\Sigma-}$ AND $D_{\Sigma+}$ AND $\mathrm{CLMP^{FLC}}$ AND CLMP AT $\mathrm{load} = 630$ MW

Bus	A	B	C	D	E
LMP @ 600	14.0000	14.0000	14.0000	14.0000	14.0000
LMP @ 640	15.0000	15.0000	15.0000	15.0000	15.0000
CLMP @ 630	14.7500	14.7500	14.7500	14.7500	14.7500
CLMP^{FRC} @ 630	0.7500	0.7500	0.7500	0.7500	0.7500

Note 2: The first two generators are at Bus A, while there is no generator at Bus B. So, the five generators do not correspond to the five buses on a one-to-one match of generators and buses.

As the results show, there is no transmission congestion. The only marginal unit is Alta. Hence, $\alpha_1 = \partial MG_i/\partial D_\Sigma = 1$ for Alta and 0 for all other units. Therefore, applying this sensitivity to the unbinding constraints, i.e., (16)–(17), we have MG_1 (Alta) reaches its upper limit before the other two unbinding transmission constraints reach their limits. Therefore, we have $MG_1^{\max} - MG_1$ as the maximum load growth before reaching a new binding limit. This can be written as $\Delta D_{\Sigma+} = (MG_1^{\max} - MG_1)/\alpha_1 = (40-30)/1 = 10$ MW.

Therefore, $D_\Sigma+$ is 640 MW ($= 630 + 10$). Since this is a simple case, it can be easily verified that no transmission line will be congested at $D_{\Sigma+}$.

Similarly, considering the unit's minimum limit, we have

$$\Delta D_{\Sigma-} = \frac{MG_1 - MG_1^{\min}}{\alpha_1} = \frac{30-0}{1} = 30 \text{ MW}.$$

Hence, $D_{\Sigma-}$ is 600 MW ($= 630 - 30$). This is the final value, and there is no change of the binding transmission constraint.

LMP at $D_{\Sigma-}$ is the same as D_Σ, and LMP at D_{Σ_+} can be easily obtained by running a new DCOPF. Then, CLMP can be obtained using linear interpolation in (23).

The LMP at $D_{\Sigma-}$ and $D_{\Sigma+}$ are shown in the second and third rows in Table III. The new CLMP and $\mathrm{CLMP^{FLR}}$ are shown in the last two rows in Table III. Please note that the Traditional LMP in [600, 640) MW is invariable to load. Hence, the LMP at 630 MW is the same as LMP at 600 MW.

B. Load at 900 MW

When the system load is 900 MW (or 1.0 per unit of the base case load), the results from DCOPF give the following information.

Unit Dispatch (MW): 40, 170, 0, 116.08, 573.92;
LMP ($/MWh): 15.8256, 23.6798, 26.6985, 35.0000, 10.0000;
Line AB Flow (MW) $= 379.75$;
Line ED Flow (MW) $= 240$.

The DCOPF and LMP results show that there is one congested transmission line, Line ED, and hence there are two marginal units, Sundance and Brighton.

Hence, from binding constraints related to marginal units, i.e., (14)–(15), we have

$$\partial MG_4 + \partial MG_5 = \partial D_\Sigma$$
$$0 \times \partial MG_4 + 0.4805 \times \partial MG_5 = 0.1257 \times \partial D_\Sigma$$

Therefore, we have

$$\frac{\partial MG_4}{\partial D_\Sigma} = 0.7384 \quad \text{and} \quad \frac{\partial MG_5}{\partial D_\Sigma} = 0.2616.$$

TABLE IV
LMPs AT $D_{\Sigma-}$ AND $D_{\Sigma+}$ AND $\mathrm{CLMP^{FLC}}$ AND CLMP AT $\mathrm{load} = 900$ MW

Bus	A	B	C	D	E
LMP @ 742.80	15.8256	23.6798	26.6985	35.0000	10.0000
LMP @ 963.94	15.2379	28.1815	29.9998	35.0000	10.0000
CLMP @ 900	15.4078	26.8799	29.0453	35.0000	10.0000
CLMP^{FRC} @ 900	-0.4178	3.2001	2.3468	0.0000	0.0000

Putting the above sensitivity into the unbinding transmission constraint (Line AB) as shown in (20), we have

$$\sum_{i=1}^{N_{MG}} (GSF_{k-i} \times \alpha_i) = 0 \times 0.7384 + 0.1595 \times 0.2616 = 0.0417$$
$$\sum_{i=1}^{N} (GSF_{k-i} \times f_i) = (-0.4759) \times \frac{1}{3} + (-0.3490) \times \frac{1}{3} + 0 \times \frac{1}{3} = -0.2750.$$

The present Line AB flow is 379.75 MW. Thus, we have

$$\Delta D_\Sigma = \frac{400 - 379.75}{(0.0417 - (-0.2750))} = 63.94 \text{ MW}.$$

This is the maximum load growth before Line AB reaches its 400 MW limit.

As for the generation limits, the possible load growth before a marginal unit reaches its limits can be found with (21).

For Generator 4 (Sundance):

$$\Delta D_\Sigma = \frac{MG_4^{\max} - MG_4}{\alpha_4} = \frac{200 - 116.08}{0.7384} = 113.65.$$

For Generator 5 (Brighton):

$$\Delta D_\Sigma = \frac{MG_5^{\max} - MG_5}{\alpha_5} = \frac{600 - 573.92}{0.2616} = 99.69.$$

The above results show that Line AB should reach the limit before the two marginal generators reach their own generation limit, because $63.94 < 99.69 < 113.65$. Hence, the next critical load level, $D_{\Sigma+}$ is 963.94 MW ($= 900 + 63.94$), corresponding to a new binding limit at Line AB.

Similarly, the previous critical load level, $D_{\Sigma-}$, is 742.80 MW. At which the output of Generator 4 (Sundance) will reduce to 0 (binding at lower limit), while the output of Generator 5 (Brighton) will reduce to 532.80 MW, and Line ED still remains congested.

The LMP at $D_{\Sigma-}$ and $D_{\Sigma+}$ for this case and the new CLMP and $\mathrm{CLMP^{FLR}}$ are shown in Table IV. Please note that the traditional LMP in [742.80, 963.94) MW is invariable to load.

C. Complete CLMP Curve

The complete curve of the proposed CLMP with respect to load from 500 MW to 1000 MW is shown in Fig. 4. If compared with Fig. 2 of the Traditional LMP, the difference between CLMP and LMP is the fourth component in CLMP, $\mathrm{CLMP^{FLR}}$.

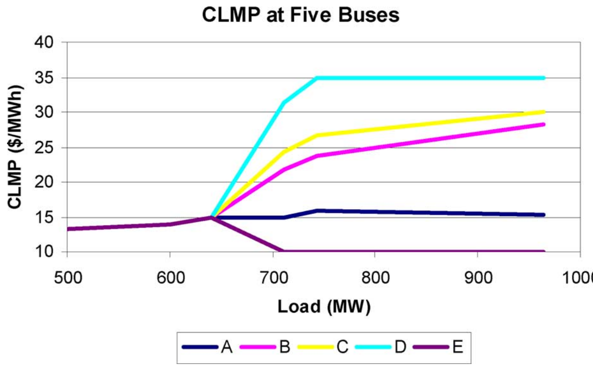

Fig. 4. CLMP at all buses with respect to different system loads.

VI. Discussions

A. Pattern of Load Variation

The real-world load variation pattern, especially in short-term such as in every 5 min, may be roughly represented with a linear equation like $D_i = a_i \times t + b_i$, where a_i is the rate of load variation at Bus i, t is the time interval of energy market, and b_i is the fixed amount of the load at Bus i. Hence, the load variation pattern affecting the CLMP is the parameter a_i, rather than D_i in the proposed algorithm in Section IV. That is, f_i is equal to a_i/a_Σ, rather than D_i/D_Σ. Nevertheless, this should not affect the main mathematical kernel of the proposed algorithm. Certainly, the discussion of using other patterns, possibly with more dynamic and accurate models, will make itself an interesting extension of the proposed idea and is beyond the main scope of this paper.

B. Redistribution of FLR

The total FLR collected from all loads can be redistributed to loads based on their participating factor, i.e., f_i. Hence, if a load is relatively constant and makes little contribution to the load increase, it may pay very little FLR. A load with a higher growth rate will pay more for FLR since it makes more contribution to the future binding constraint. If combined with the load variation model in Part A in this section, this redistribution of FLR may stimulate each load to maintain a relatively stable load curve.

C. Variant Model of CLMP and FLR

If a market designer does not want to have increased total consumer payment due to FLR, this can be addressed with a slight variant of the proposed version of CLMP and FLR. The variant is to use a set of different interpolation points. As shown in Fig. 5, a piece-wise-linear curve may be created such that X is the average of the Traditional LMPs at the left and right of D_-, Y is the average of the Traditional LMPs at the left and right of D_+, and Z is the point where the two triangles have equal areas. Certainly, FLR will be a credit instead of a payment if the load is between the load levels corresponding to X and Z, respectively. With this variant version of CLMP and FLR, the net FLR will be very close to zero over a long duration (such as a month or a year), with a reasonable assumption that the load levels over a long duration will follow a roughly even distribution between

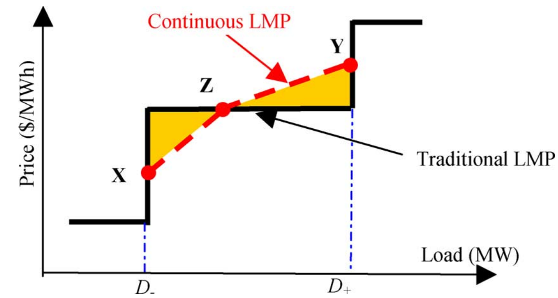

Fig. 5. Variant to the proposed CLMP curve.

D_- and D_+. Hence, a continuous price curve can be essentially achieved without extra payment from the consumers, compared with the Traditional LMP. This approach may be combined with the redistribution of FLR proposed in the Part B in this section.

D. Step Change in LMP and ACOPF

The step change feature in LMP is not caused by the Linear Programming-based DCOPF. In fact, it is caused by the marginal feature in LMP definition. Since LMP is the incremental cost to serve the next incremental load, there will be always a step change in price when a limit becomes binding if the system load grows across a critical level. The reason is that a high-cost unit will be dispatched as a new marginal unit, and the new price will be determined (at least at some locations) by the cost of the new unit. Hence, the price change (at least at some locations) across the critical load level should be determined by the marginal cost difference between the new marginal unit and the old marginal unit, which is always a step change in theory. Hence, regardless of the LMP calculation model (ACOPF, DCOPF or any approximation algorithms), the step change feature should always exist.

E. Security (Contingency) Constraints

Although this paper does not explicitly address security (contingency) constraints, they can be easily included by adding more arrays of Generation Shift Factors under contingency scenarios. Therefore, the security limits can be modeled similar to line limits modeled in (3), as shown in many SCOPF models.

VII. Conclusions

The proposed CLMP methodology and the efficient algorithm based on DCOPF is an enhanced pricing methodology to smooth the step changes under the Traditional LMP methodology. A new price component, Future Limit Risk (FLR) price, is introduced to eliminate the step change and set a continuous function to charge market participants for their contribution to the next future binding limit. Therefore, it reduces the risk of a sudden, step change in electricity price, mitigates price risk with an additional means, and may be of interest to future works in market design and research. In addition, the key of the proposed CLMP algorithm, i.e., the calculation of the next and previous critical load levels, can be used for the Traditional LMP methodology to efficiently find how far the system load is away from the next (or previous) binding limits as the load increases (or decreases).

As previous mentioned, the current work ignores the loss component in the LMP methodology. This may be addressed in the future research.

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