An Aumann-Shapley Approach to Allocate Transmission Service Cost Among Network Users in Electricity Markets

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***Abstract*—This work presents a new methodology for the allocation of transmission service cost among network users in energy markets. The proposed method is based on an optimization/game-theoretic framework (Aumann-Shapley) that retains the desirable properties of other existing methodologies such as the Average Participations Factors (APF) and Long Run Marginal Costs (LRMC). The approach is shown to be computationally feasible and presents desirable characteristics in terms of economic coherence and isonomy. Computational results are presented for the Brazilian power system and compared with those obtained by three other methodologies: LRMC, APF, and the current method adopted in Brazil.**

***Index Terms*—Aumann-Shapley pricing, cooperative game theory, transmission pricing, transmission service cost allocation.**

I. Introduction

A UNIVERSAL characteristic of the restructured power industry environment is competition in generation expansion, that is, investors are free to decide when and where to build new capacity. Investors typically decide on the type and size of a new plant through the comparison of its investment and operation costs with the projected revenues from energy sales (spot market plus supply contracts). The plant *siting* decision, however, depends on an additional parameter, which is the cost of transporting the energy to load centers. For obvious reasons, it is neither feasible nor economical to build independent transmission systems for each generation—load pair. Therefore, the transmission grid becomes an open-access *service*, used by generators and consumers.

As with any other service, the transmission investment and operation costs should be recovered though some kind of "service charge." Short-term congestion revenues resulting from locational marginal prices (LMP) would ideally recover the transmission costs, and in addition provide efficient siting signals for new generation investments. However, for several reasons (discrete nature of transmission investments, economies of scale, security constraints, etc.), congestion revenues in practice only recover about 20% of the total fixed transmission service cost (see, e.g., [1]). This leads to the well-known revenue reconciliation problem [1], which is usually solved by the creation of additional *fixed charges* at each node. These charges are known as *open access transmission tariffs* (OATT).

The focus of this work is the calculation of OATTs which not only are "fair" in the sense that the charge in each generation or load bus is proportional to the degree of use of the shared transmission resources, but which also provide efficient *siting* signals, i.e., induce investors to build generation facilities at sites that lead to the best overall use of the generation-transmission system.

These siting signals are particularly important in countries where there is a high load growth and transmission costs represent a significant part of the overall investments. This is usually the case of emerging countries with significant hydroelectric resources, such as Brazil, Colombia, and Peru. In those countries, new hydro plants—which are usually far from load centers and thus require heavy transmission investments—compete against new gas-fired thermal plants, which are closer to load centers, with correspondingly smaller transmission costs.

The case of Brazil, where 85% of the 100 GW installed generation capacity comes from hydro plants, illustrates this investment challenge. The country's surface area is roughly the same as the US, and is interconnected by 85 000 km of high-voltage (230–765 kV) transmission facilities. With yearly electricity load growth rates of 5%, US$ 6 billion in generation investments, mostly for new hydro plants, are required each year. The related transmission investments are substantial: 14 000 km of new HV transmission lines have been built in the past five years, and 40 thousand km more are planned for the next decade. The key economic issue for new generation investment is competition between hydro plants, which have lower operating costs but may be as far as 2500 km from load centers, and gas-fired plants, which have lower investment costs, higher operation costs, but are closer to the load. Because the OATT component for distant hydro plants may reach 10 US$/MWh, about 20% of the total energy cost, it is easy to see that the transmission charges may "tip the scale" in favor of one technology over another. OATTs are also critical for the location of new industrial loads. For example, aluminum producers must decide whether they should be sited closer to the alumina mines or to the energy production source.

In the next sections, an overview of the OATT methodologies and of the proposed approach of this work is presented.

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II. Overview of OATT Methodologies

As discussed in [2], [3], and [9], many different methods to allocate transmission costs among network users have been applied worldwide. Several methodologies for the pricing of transmission services have been proposed, mostly based on marginal pricing [4], [8] or on grid-usage [5]–[7], [14]. More recently [10] proposed an alternative method based on nodal prices control in order to recover the network costs. However, as discussed in [2], [3], [9] there is still no consensus on the most adequate scheme.

The allocation of transmission costs is an instance of the general problem of allocating costs (or benefits) among a *coalition* of agents that cooperate on the construction of a shared resource, such as a road. The most classical way to solve this type of problem is through coalitional, or cooperative, game theory [11], [12]. This is discussed next.

A. Cooperative Game Theory

Most allocation methods in cooperative game theory try to calculate the *incremental impact* of each agent (or subcoalitions of agents) on the total cost of the shared resource. For example, let T^* be the cost of the optimal transmission network for all generation and load agents. Suppose that agent i leaves the coalition, and that $T_{-\mathrm{i}}$ is the cost of the redesigned transmission network. The difference $\delta T_{\mathrm{i}} = T^* - T_{-\mathrm{i}}$ is a measure of the incremental impact of agent i on the overall transmission cost. An allocation scheme based on one-at-a-time incremental impacts would calculate $\{\phi_{\mathrm{i}} = [(\delta T_{\mathrm{i}})/(\Sigma_{\mathrm{all}j}\delta T_{\mathrm{j}})]\}$, where ϕ_{i} is the fraction of the total cost T^* that is allocated to agent i (by construction, $\Sigma_{\mathrm{all}i}\phi_{\mathrm{i}} = 1$).

Other game-theoretic schemes take into account the impact of *combinations* of agents. For example, the set of linear constraints known as the core of a cooperative game requires the calculation of the impacts of all agents one by one, by twos, by threes, etc. In turn, the Aumann-Shapley (AS) allocation requires the calculation of all possible orders of entrance of agents into the coalition (agent i first, then second, etc., then last; agent j first while agent i is fixed in second, and so on). As mentioned, the AS allocation is the basis of our proposed OATT scheme.

B. Simplified Transmission Cost Calculation

Because it is not computationally feasible to re-design the optimal transmission network several times, in order to simulate the impact δT_i of a given agent (or combinations of agents), most OATT methodologies *approximate* these cost variations as follows. Initially, one defines a *dispatch scenario* $\{g, d\}$ where g is a vector of generations (MW) in each bus, and d is a vector of bus loads. Given the dispatch scenario and the network parameters (grid topology and susceptance of each line), a linearized power flow model is solved to produce the flows f_{k} (MW) in each circuit k.

The total network cost (\$) is estimated as $T^* = \Sigma_{\mathrm{all}k} c_{\mathrm{k}} \times f_{\mathrm{k}}$, where c_{k} is the "unit cost" (\$/MW) of circuit k.[1] In this case, the incremental impact δT_{i} is estimated as $\Sigma_{\mathrm{all}k} c_{\mathrm{k}} \times \delta f_{\mathrm{ki}}$, where δf_{ki} is the change in the flow of circuit k resulting from a change in the injection of agent i (bus generation or load).

[1]As it will be discussed in Section IV, T^* recovers a transmission service cost proportionally to the grid usage f_{k}.

C. Existing OATT Schemes

As it is well known, total generation and load should be balanced in the linear power flow model. Therefore, when simulating the impact of a generation increment on circuit flows, in order to apply an OATT methodology, one has to specify the load(s) that compensate this increase.

One way to characterize OATT methodologies is on how they specify these balanced {generation; load} sets. For example, the MW-mile method [13] is based on the specification of a balanced generation-load pair, usually associated to a physical supply contract. A linear power flow is then run for the entire network, but with only the specified generation-load pair as injections; the "wheeling charge" is then calculated for the resulting circuit flows, $\Sigma_{\mathrm{all}k} c_{\mathrm{k}} \times \delta f_{\mathrm{ki}}$, as seen previously. The well-known "superposition property" of linear networks ensures that the addition of the circuit flows resulting from the different power flow runs for all {generation; load} pairs restores the flow pattern of the original run. As a consequence, the sum of wheeling charges recovers the total network cost T^*.

The MW-mile method cannot be applied in systems where supply contracts are not based on physical deliveries, but are financial instruments (otherwise, transmission tariffs would be affected by changes in contracts). The Long Run Marginal Cost (LRMC) methodology [8] was one of the first to be applied in such financial-based systems. The LRMC scheme is also based on the circuit flows resulting from a given generation-load pair, and on the network superposition property. However, only the generation is explicitly specified; the balancing load is represented implicitly as the system "economic slack bus" [15]. As a consequence, the flow variations can be written directly as linear combinations of the injections. It is also possible to calculate all tariffs in a single run. The ease of implementation and interpretation has motivated the application of the LRMC methodology in several countries, such as Colombia, UK and Brazil. Section V discusses the LRMC method in detail.

The use of a fixed "slack bus" is adequate in countries where most of the load is concentrated in a single center, such as the cities Buenos Aires (Argentina) and Santiago (Chile). However, if the country has different regions with significant local load and generation, the injection in a given region may cause increments in circuit flows all around the country, as they follow the electric paths to the (electrically) distant "slack bus." As a consequence, the resulting tariffs are sometimes nonintuitive, with generators that are close to load centers in a given region receiving high tariffs even if the region exports a small fraction of its power.

The limitations of the LRMC have been recognized for some time, and a few alternatives have been proposed to alleviate them. In the Average Participation Factor (APF) method [16]–[18], for example, a different set of loads is used to balance the generation injection in each different bus. These loads are determined by a heuristic scheme that tries to minimize their electrical distance to the injection bus. The APF scheme matches the "engineering intuition" that the circuit flows go to the loads "closest" to the injection, and produces tariffs which are more consistent and stable than the LRMC scheme in multi-region systems. The APF method is presented Section VI.

One limitation of the APF scheme is that, while it does follow the superposition property, the flows used to calculate the tariff associated to a given {generation; load} set do not correspond to those of a power flow run.

III. Proposed Methodology: Overview

The methodology proposed in this paper to allocate the transmission fixed charges is based on an optimization/game-theoretic framework that retains the desirable properties of both APF and LRMC methods. It is described in Section VII. The optimization scheme provides an economic justification for the "engineering intuition" of the APF method, while retaining the desirable network superposition properties of the LRMC scheme. In turn, the game-theoretic framework provides a theoretical justification for the allocation rules.

The basic idea is to define an *entrance order* for the generation injections. Each generator that enters selects a set of matching loads by solving an optimization problem that minimizes its transmission tariffs. The selected loads are then "frozen" (i.e., cannot be used by the next generators in the entrance order) and the process is repeated until all generation injections (and, by definition all loads) have been covered. The optimization scheme is similar in purpose to the APF criterion, that is, to identify loads that are electrically close to the injection. However, the network characteristics (2nd Kirchhoff law) are fully represented.

The key question in the method is how to determine the entrance order. It is easy to see that the first generation to enter has the widest set of loads to choose from, whereas the last to enter is "stuck" with the "leftover" loads. The entrance order problem has been solved in game theory by Shapley [12], [19]: determine the tariffs for all possible entrance orders, that is, all generators have a "shot" at being first, second etc., and then calculate the average tariff over all combinations.

At first sight, the Shapley scheme is computationally prohibitive for realistic systems. However, an extension of the method called Aumann-Shapley [20] scheme reduces the computation effort to the solution of a set of linear optimization programs, which is quite feasible to implement, also assuring a set of desirable economic properties [20]–[24].

The remainder of this paper is organized as to define and detail mathematically the aforementioned OATT schemes. Section IV defines the transmission service cost. Sections V and VI review three methods for its allocation, including a description of the Brazilian approach. Section VII presents the proposed methodology, focusing on AS pricing. Section VIII illustrates the methodologies through a simple example and Section IX applies the methods to the Brazilian system. Finally, Section X concludes and suggests future work.

IV. Transmission Service Cost

This work considers a competitive electricity market environment where investors freely decide to construct generating units and compete for energy sales contracts with customers. It is considered that market agents have open-access to the transmission grid with no point-to-point power transactions and, therefore, no "wheeling" rates. The transmission service charges depend on the amount of power being injected or removed in each node, and on the location of that node.

As discussed in Section I, the problem to be analyzed in this work is related to the allocation of transmission service costs among network users. In other words, given the (fixed) transmission service cost, the question is how to allocate it among grid users (in the form of fixed OATT) in an efficient and fair way.

The first step in this direction is to define the total transmission service cost (TSC). This cost is calculated as the sum of the annual revenue for each transmission facility (circuits and transformers), also known as grid element, permitted by the regulator.

However, the total amount T^* that *can be* recovered from the network usage (elements loading) is defined as the weighted average of element flows, where the weight is the element's unitary cost:[2]

$$T^* = \sum_{k=1}^{K} c_k \times |f_k| \qquad (\$) \tag{1}$$

where k is the element index, K is the number of elements; c_k is the element k unit cost (\$/MW) and f_k is the power flow in element k (MW). This is the cost to be allocated among network users.

All methodologies begin with a unique balanced power flow scenario (reference generations $\{g_i\}$ and demands $\{d_i\}$), respecting transmission constraints). Adopting a lossless dc linearized power flow representation, the flow in each element is a linear function on bus loads and generations:

$$f_k = \sum_{i=1}^{N} \beta_{ki}(d_i - g_i), \quad k = 1, \ldots, K \tag{2}$$

where β_{ki} is the sensitivity of flow in element k with respect to injection in bus i; g_i is the bus i generation (MW), d_i is the bus i load (MW) and N is the number of buses.

Substituting (2) in (1) allows expressing the transmission service cost as a linear function of bus loads and generations

$$T^* = \sum_{k=1}^{K} c_k \times \left| \sum_{i=1}^{N} \beta_{ki}(d_i - g_i) \right| \tag{3}$$

where g and d are N-dimensional vectors whose components are, respectively, actual bus generation and load.

A. Recovery of Transmission Costs

An important issue is the full recovery of all fixed transmission costs through allocation methods based on the grid use. All proposed methodologies recover T^*, which is the service cost based on the network usage, instead of TSC.

As discussed in [24], the construction of an electric grid with optimal dimensioning is impossible in practice. Because of the discrete nature of elements and reliability constraints (which creates redundancy and reduces element loadings—the

[2]The unit cost is calculated by the permitted annual revenue of each element divided by its capacity.

grid looks "under-used"), power flows are usually smaller than the corresponding element limits. Therefore, if grid use is not 100% of the element's capacity, T^* will be smaller than TSC and an additional complementary charge will be needed to recover the difference. Several complementary charges have been formulated and applied in the literature [4], [16], [22]. In this work a postage stamp scheme is proposed as a complementary charge in order to guarantee the total costs recovery. This method equally shares the fixed cost payments among grid users. Although it presents many disadvantages, most of them related to *fairness* aspects, it will be used here in order to allow a comparison with the Brazilian OATT, which is calculated using a postage stamp scheme as the complementary charge.

The postage stamp scheme is implemented by applying a unique additive correction tariff (π_c) to all agents, calculated by equally sharing the difference between recovered costs from this method and total costs among all users

$$\pi^c = \frac{TSC - \sum_{i=1}^{N} \pi_i^g \times g_i - \sum_{i=1}^{N} \pi_i^d \times d_i}{\sum_{i=1}^{N} (g_i + d_i)} \quad (\$/\mathrm{MW}) \qquad (4)$$

where TSC is the annual total transmission service cost (\$), N is number of buses, π_i^g and π_i^d are, respectively, the generation and demand tariff in bus i calculated using the different methodologies (\$/MW), g_i is the generation in bus i (MW) and d_i is the load in bus i (MW).

Observe that since methodologies referred to in this work need the postage stamp method as a final complementary charge, the solutions of the referred cost-allocation methodologies do not fully solve the revenue reconciliation problem described earlier.

Finally, it is also assumed in this work that total cost is allocated between generation and demand, on a 50%-50% basis.

B. Transmission Cost Allocation Criteria

Once the transmission service cost to be recovered from network use is defined, different methods can be applied for its allocation among the users, i.e., to determine π^g and π^d.

This allocation has to meet two basic properties: i) *fairness*—in the sense that the charges in each generation or load bus should be proportional to the degree of use of the shared transmission resources and ii) *economic efficiency*—the charges should provide efficient siting signals which induce investors to build generation facilities at sites that lead to the best overall use of the generation-transmission system.

V. LRMC Methodology

The Long Run Marginal Cost (LRMC) scheme is based on the circuit flows resulting from a given generation-load pair, and on the network superposition property. The method aims at allocating the TSC proportionally to the marginal contribution of each user to the cost of an ideal transmission network constructed to match supply and demand, i.e., by reflecting the variation in transmission investment cost I resulting from an injection variation in each node i[3] [8]

$$\pi_i = \frac{\partial I}{\partial d_i} \qquad (\$/\mathrm{MW}). \qquad (5)$$

This derivative can be calculated through sensitivity factors of the line flows as a function of incremental power injected in each bus. These factors can be obtained with the linear power-flow model, and they constitute the well-known "β Sensitivity Matrix." Transmission charges then result proportional to incremental flows produced by agents in each grid element and its respective unitary cost, expressed in \$/MW. The LRMC approach is applied in countries such as Brazil, Colombia, and the U.K. In the case of Brazil, there are some particularities that will be discussed in Section V-A.

The tariffs π for generators and demands are calculated as

$$\pi_i^g = -\pi_i^d = \sum_{k=1}^{K} \beta_{ki} \times c_k \quad (\$/\mathrm{MW}) \qquad (6)$$

where β_{ki} is the sensitivity of flow in the element k as function of a marginal injection at bus i and c_k is the unitary cost of element k (\$/MW).

The LRMC method calculates a tariff closer to the "ideal", because it allocates transmission costs to those agents that "stress" the network and, as a consequence, will cause reinforcement. Because the method is based on economic principles, it is easy to be understood by regulators and technical personnel.

An important aspect of the LRMC is that the utilization of the β_{ki} factors implicitly assumes that all incremental injections supply 1 MW of demand at an "economic" slack bus. The difficulty is that, in practice, the "economic" slack bus is not explicitly chosen and its location can jeopardize some agents far from it. As shown in [15], a change of the slack bus is equivalent to a change in the cost allocation between generation and demand. Since this proportion is already defined (50%-50% in this paper), the "economic" slack bus is also implicitly defined.

As discussed in Section II-C, the use of a fixed "slack bus" is adequate in countries where most of the load is concentrated in a single center. However, if the country has different regions with significant local load and generation, the injection in a given region may cause increments in circuit flows all around the country, as they follow the electric paths to the (electrically) distant "slack bus." As a consequence, the resulting tariffs are sometimes nonintuitive, with generators close to local load centers in a given region receiving high tariffs even if the region exports a small fraction of its power. In addition, "negative" tariffs are possible with the LRMC for agents whose injections cause counter-flows. Negative tariffs usually cause opposition from system agents, who interpret it as a cross-subsidy.

A. Brazilian Method (a Variant of LRMC)

The current methodology adopted in Brazil (BRA) is based on LRMC with some particularities [25]. Most of them are re-

[3]The methodology applies for both generation and load, which correspond, respectively, to positive and negative injections.

lated to an attenuation of the locational signals of the tariffs. This is done by means of a multiplicative factor $(FPond)$, which is a factor between 0 and 1 (corresponding to the element loading in p.u.), that is applied to reduce the loading of some elements.

In this sense, the tariffs are calculated as

$$\pi_i^g = -\pi_i^d = \sum_{k=1}^{K} \beta_{ki} \times c_k \times FPond_k \quad (\$/\text{MW}). \tag{7}$$

One of the implications of $FPond_k$ factor is a differentiated treatment for elements with low loading (typically elements for energetic optimization, such as long tie-lines between hydro basins or regions), where the flow direction may vary significantly depending on the hydrology. In other words, an element with a low loading has a low $FPond_k$ and thus the amount collected through the allocation method is consequently *smaller*. Thus, the locational signals of tariffs are attenuated in the BRA and a great portion of the investments of these elements is equally shared among all agents through the postage stamp scheme.

In summary, besides the need of an "economic" slack bus, the BRA method has a particularity that reduces the locational signals provided by the LRMC method. These issues have motivated the Brazilian regulator to search for more attractive methodologies and the correspondent research has resulted in this work.

VI. Average Participation Factors Method

The limitations of the LRMC have been recognized for some time, and some alternatives have been proposed to alleviate them. The Average Participation Factor (APF) method [16]–[18] is the most attractive of them.

The methodology based on APF deals with the general transportation problem of how the flows are distributed in a meshed network. Practically, the only requirement for the input data is that Kirchhoff's First Law must be satisfied.

Once the power flow scenario is determined, the main idea of this methodology is to determine the participation share of the agents (generators and loads) in the flow of all elements. In this sense, it will be possible to trace the flow of electricity from a generator to its consumption buses.

The principle adopted to trace these flows is the proportional sharing principle. The assumption made in this principle is that the network bus is a perfect mixer of all incomings flows so that it is impossible to determine which incoming electron goes into which outgoing element. As it is common sense that electricity is indistinguishable, it may be assumed that each element flow f_k leaving a bus i can be decomposed in M shares, where M is the number of incoming flows (generators and elements injecting power in bus i). The sizes of these shares have the same proportion of the incoming flows have in the total power P_i injected in bus i. This principle is illustrated in the example of Fig. 1.

In this example, there are four circuits connected to bus i (two injecting and two withdrawing power). For each injection, it is possible to calculate its percentage participation in $\mathrm{P_i}$. For instance, circuit j-i has 40% of responsibility in $\mathrm{P_i}$ and circuit k-i, 60%. In this way, the 30 MW leaving bus i in circuit i-l can be divided into a flow of $30 \times 40\% = 12$ MW attributed to the injection j-i and $30 \times 60\% = 18$ MW attributed to the injection k-i. The same can be done for all outgoing flows.

This approach intuitively induces a solution algorithm that runs through all the paths from generators in each bus i to all consumption buses j calculating the participation share $\phi(j, i)$ and consequently all elements. In this way, the tariff becomes

$$\pi_i^g = \frac{1}{g_i} \sum_{k=1}^{K} c_k \times f_k \times \phi(\delta_k, i) \quad (\$/\text{MW}) \tag{8}$$

where $\phi(\delta_k, i)$ is the participation factor of a generator located in bus i in bus δ_k. Also, δ_k is the bus from which element k withdraws power.

In practice, according to the power system being considered, the algorithm to find all paths from each generator to all its consumption buses can be computationally infeasible and therefore it is not used. Bialek in [17] formulated a matricial algorithm that is computationally efficient and robust to calculate the participations factors $\phi(j, i)$.

The APF method has attractive properties: it is very simple, straightforward to apply, matches the "engineering intuition" that the circuit flows go to the loads "closest" to the injection, and produces tariffs which are more consistent and stable than the LRMC scheme in multi-region systems. The method has been recently considered for implementation to allocate inter TSO costs in the European Union [26].

As mentioned in Section II-C, one limitation of the APF scheme is that, while it does follow the superposition property, the flows used to calculate the tariff associated to a given {generation; load} set do not correspond to those of a power flow run.

VII. Proposed Methodology: The Aumann-Shapley Scheme

A. Motivation: Incremental Cost Allocation

An intuitive allocation approach is to allow each agent (generation injection or load) to choose its "optimal" network use (that minimizes its transmission tariffs) and allocate for each agent the incremental difference between the transmission costs when agents are successively added to the system: the difference between the total costs *with* and *without* the agent is directly allocated to it.

For example, in the case of generation,[4] if a generator in bus 1 is the first to use the network, it is free to choose which load will be supplied. Since the generator will pay for the incremental transmission service cost, it can choose to supply the load that will result in lowest grid use (and thus lowest transmission cost). This can be obtained through the solution of the following linear programming problem (LPP):

$$T_1 = Min \sum_{k=1}^{K} c_k \times \left| \left(\sum_{i=1}^{N} \beta_{ki} \delta_i \right) - \beta_{k1} g_1 \right| \tag{9}$$

[4]The procedure for consumers is similar, although, in this case, the decision variables are generations.

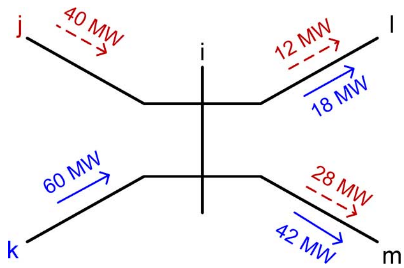

Fig. 1. Proportional sharing principle.

Subject to

$$\sum_{i=1}^{N} \delta_i = g_i \tag{10}$$

$$-\overline{f}_k \le \left(\sum_{i=1}^{N} \beta_{ki}\delta_i\right) - \beta_{\mathrm{k}1} g_1 \le \overline{f}_k,\ \forall k \tag{11}$$

$$\delta_i \le d_i, \quad i = 1, \ldots, N. \tag{12}$$

The decision variables of the LPP are $\{\delta_i, i = 1, \ldots, N\}$, each one representing the load in bus i supplied by the generator in bus 1. The objective function of (9) minimizes the actual cost of the network usage for generator 1 given that it is free to choose its optimal use of the network.[5] Constraint (10) says that the sum of supplied demands must be equal to generator's dispatch g_1, constraint (11) represents the flow limits in each circuit and constraint (12) for each bus says that the generator may supply at most the corresponding bus load. The objective function value T_1 in the optimal solution gives the cost allocated to the first generator 1.

Now suppose that the second generator uses the network. The same linear optimization problem is solved, now considering that the first one is already using the network

$$T_2 = Min \sum_{k=1}^{K} c_k \times \left| \left(\sum_{i=1}^{N} \beta_{ki}\delta_i\right) - \beta_{\mathrm{k}1} g_1 - \beta_{\mathrm{k}2} g_2 \right| \tag{13}$$

Subject to

$$\sum_{i=1}^{N} \delta_i = g_1 + g_2 \tag{14}$$

$$-\overline{f}_k \le \left(\sum_{i=1}^{N} \beta_{ki}\delta_i\right) - \beta_{\mathrm{k}1} g_1 - \beta_{\mathrm{k}2} g_2 \le \overline{f}_k,\ \ \forall k \tag{15}$$

$$\delta_i \le d_i, \quad i = 1, \ldots, N. \tag{16}$$

The service cost allocated to the second generator will be the incremental cost difference

$$\Delta T_2 = T_2 - T_1. \tag{17}$$

These incremental allocation steps can be repeated until the last generator is allocated. It is easy to see that in the end (last problem) the total transmission service cost (limited by the elements loading) is allocated, since once the optimization problem is solved for all generators the objective function value is the total service cost.

The OATT paid by the generator located in bus i is given by

$$\pi_i^g = \frac{\Delta T_i}{\overline{g}_i} \quad (\$/\mathrm{MW}). \tag{18}$$

B. Shapley Procedure

The previous procedure is intuitive and easy to implement. However, an important aspect is missing: the order of generators' entrance. If the order is changed, the cost allocation will be different. Obviously, all generators would prefer to be the first to enter—when there are more "degrees of freedom" to choose which load to supply to minimize transmission costs (and receive a lower OATT)—rather than the last ones.

Shapley [19] eliminated the limitations of the incremental cost approach by repeating the procedure for all possible combinations of entrance orders and calculating the average service costs allocated to each generator. In other words, the scheme is intuitively "fair": all the generators would have the same opportunity to be the first—and also the last ones—thus avoiding the drawback due to the arbitrary entrance order of generators. Although the Shapley allocation has many attractive characteristics, it presents two important limitations.

i) *Lack of isonomy* (also known as the comparability requirement): The method depends on the size of generators, and so the allocation process is affected by agents aggregation and is dependent on their relative size. For example, the sum of the allocated costs of two generators located at the same bus that produce 20 MW each one, can differ from the total that could be allocated to only one generator with 40 MW.

ii) *Computational feasibility*: Due to its combinatorial nature, the problem size increases fast with the number of agents (the total number of permutations for a case with N agents is equal to N!).

The Shapley approach was applied in the context of transmission cost allocation in [7] but its application is limited to small systems. In order to overcome the aforementioned difficulties, the next section introduces a subsequent development based on the Aumann-Shapley pricing, thus avoiding both the lack of isonomy and the computational burdens of the Shapley approach.

C. Aumann-Shapley Procedure

In order to correct the limitation i) of the Shapley allocation scheme, a suggested approach is to allow smaller agents to enter after just a *fraction* of the larger agent has been served. In other words, the generator j is "split" into two subagents, j_a and j_b and then the Shapley procedure is repeated as if each subagent were an individual agent. This is the basis of the Aumann-Shapley (AS) allocation method.

The AS [20]–[23] allocation is a generalization of the Shapley principle and can be seen as a limiting process of agent "splitting" and permutations of entrance orders as the size of each subagent goes to zero. The idea of the AS allocation procedure is to divide all the generations g_j in infinitesimal segments,

[5] This cost is given by the product between the circuit costs and their respective power flows, as shown in (1). The power flows are calculated according to (2) considering that demands are decision variables and that in this problem only generator 1 is participating ($g_i = 0, i \neq 1$).

and then apply Shapley's approach as if each segment were an individual agent (calculate the difference in the transmission service costs as agents are successively added to the system), calculating the average of the incremental service costs of the resources when they grow uniformly from zero up to their current values. By using marginal cost information, the allocation provides fair, economically efficient results (see [21]–[23]).

In this sense, note that problem (13)–(16) represents the variation of the transmission service cost caused by the entrance of a second generator. In this problem, both generators (1 and 2) are explicitly represented, as are their transactions to any bus i. In the AS allocation procedure all generators are equally split and there is no need to explicitly represent a specific transaction. Therefore, the AS allocation procedure becomes the following parametric linear optimization problem parameterized by the "size" of the generators

$$T(\lambda) = Min \sum_{k=1}^{K} c_k \times \left| \sum_{i=1}^{N} \beta_{ki}(\delta_i - \lambda g_i) \right| \tag{19}$$

Subject to

$$\sum_{i=1}^{N} \delta_i = \sum_{i=1}^{N} \lambda g_i \tag{20}$$

$$-\overline{f}_k \leq \sum_{i=1}^{N} \beta_{ki}(\delta_i - \lambda g_i) \leq \overline{f}_k \tag{21}$$

$$\delta_i \leq d_i, \quad i = 1, \ldots, N. \tag{22}$$

The parameter λ varies continuously within the interval [0,1], thus applying in the same proportion the splitting of all generators. Therefore, the drawback i) for the Shapley allocation disappears. In addition, note that, as opposed to LRMC and BRA methods, in the AS scheme all generators are equally free to choose which demand to supply instead of supplying a fixed demand located in the "economic" slack bus. As mentioned earlier, the objective function (19) reflects the network flows caused by the transaction between λg and all demands d and does not change as a function of the slack. Observe that the optimization scheme is similar in purpose to the APF criterion, that is, to identify loads that are electrically close to the injection. However, the network characteristics (2nd Kirchhoff law) are fully represented.

As detailed in the Appendix, in order to solve (19)–(22), a numerical integration can be applied, where parameter λ is discretized. Parametric linear optimization problems are usually solved by the same linear programming algorithm used to solve a simple linear optimization problem, being available in commercial simplex packages. Basically, initially one solves problem (19)–(22) using the simplex method with $\lambda = 1$, saving the basis. From thereon, when decreasing λ, once one reaches a value such that the optimal basis changes, a few simplex iterations are required to find a new basis; this process goes on until $\lambda = 0$. The computational implementation of the AS method is described in greater details in Annex A.

Finally, observe from the solution procedure above that although the objective function of the AS is composed by the sum of usage cost in each circuit, tariffs are calculated based on the dual variables of (19)–(22) and, therefore, also reflect the expansion marginal costs as the LRMC (good locational signals but without defining an economic slack bus).

D. Aumann-Shapley: Computational Feasibility

An interesting question is that the number of agents and the number of permutations considerably increases in the AS approach. Thus, its computational difficulty seems to be even greater. However, it is shown in [20] that the AS method allows the problem to have an *analytical solution* if each agent is divided into infinitesimal parts and therefore it does not depend on the entrance order of the agents. The total transmission service cost allocated among network users through the AS procedure results from the solution of the following path-integral for each agent

$$\Psi_j = b_j \int_0^1 \frac{\partial f(\lambda b)}{\partial b_j} \tag{23}$$

where λ is the integration parameter; b_j is the vector of resources of agent j (generation or load) and $f(\cdot)$ is a function that represents the marginal cost of transmission service for a given λ. It can be noted that the AS allocations correspond to the average of the marginal costs of the resources when they grow uniformly from zero to their current values. As mentioned, in this work this integral is numerically calculated by discretizing the variable λ in the interval [0,1] through the optimization problem (19)–(22). By using parametric linear programming, the computational time is reduced.

In this sense, drawback ii) of the Shapley method disappears and the AS becomes computationally feasible.

E. Aumann-Shapley: Summary and Other Applications

The AS is a special case of a "path-based" cost allocation method, as discussed by Friedman and Moulin [23]. The AS cost allocation was originally developed in the context of nonatomic games [20] and was proved in [21] and [22] to be an additive, weakly aggregation invariant and monotonic allocation method. The AS provides fair, economically efficient allocation since it uses marginal cost information. Thus, it can provide an attractive alternative to transmission pricing. Methodologies under the same AS principles have also been applied to the allocation of costs for transmission losses [27], reactive support [28], transmission congestion cost [29] and the allocation of firm energy rights among hydro plants [30].

VIII. EXAMPLE

The calculation of OATT for generators and loads will be illustrated in an example with a small system for the LRMC, APF and AS methodologies. The system has five buses, two generators and three loads. Fig. 2 shows the unitary costs of the circuits, the topology of the grid and the power flow. For didactic purposes, the tariffs will also be calculated by the Shapley method. The Brazilian method will not be represented because it is a simple variant of the LRMC method.

1) LRMC Procedure: As already discussed, the LRMC methodology is based on sensitivity factors, which depend on a slack bus that is implicitly chosen through the sharing

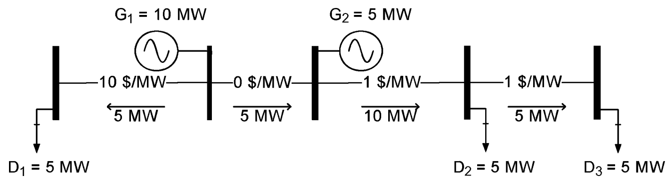

Fig. 2. Sample system.

TABLE I
INITIAL TARIFFS FOR THE LRMC METHOD

	MW	π ($/MW)	P ($)		MW	π ($/MW)	P ($)
G_1	10	10	100	D_1	5	0	0
G_2	5	10	50	D_2	5	-9	-45
				D_3	5	-8	-40
Total	15		150	Total	15		-85

TABLE II
FINAL TARIFFS FOR THE LRMC METHOD

	MW	π ($/MW)	P ($)		MW	π ($/MW)	P ($)
G_1	10	2.2	21.7	D_1	5	7.8	39.2
G_2	5	2.2	10.8	D_2	5	-1.2	-5.8
				D_3	5	-0.2	-0.8
Total	15		32.5	Total	15		32.5

of payments criteria among generators and demands [15]. As initial step, D_1 is chosen as electrical slack bus. In this sense, initial transmission tariffs (π) and the transmission service cost allocated to the agents (P) are presented in Table I.

From the initial tariffs, it can be noted that generators and loads are not sharing the total payments (P) following a 50%/50% allocation criteria. Therefore, as discussed in [15], a α value equal to -7.833 needs to be added to the tariffs in order to allow a 50%/50% allocation cost criteria. This value can be interpreted as a "displacement" of the slack bus from D_1 to the "economic center of gravity" of the system, i.e., where the total payments of generators is equal to total payments of demands. The final tariffs are presented in Table II. It can be observed that generators and loads now equally share total transmission costs.

2) Average Participation Factors Procedure: As already mentioned, tariffs in APF method are calculated by tracing power flows according to the proportional sharing principle. Observe that all circuit costs are divided by two because of the 50%/50% allocation criteria adopted in this work.

In this sense, generators' tariffs can be calculated as

$$\begin{aligned} T_{G1} &= 5\ \text{MW} \times \frac{10}{2}\$/\text{MW} + 50\% \times (10\ \text{MW} + 5\ \text{MW}) \\ &\quad \times \frac{1}{2}\ \$/\text{MW} \\ T_{G1} &= \mathbf{28.75}\$ \\ T_{G2} &= 50\% \times (10\ \text{MW} + 5\ \text{MW}) \times \frac{1}{2}\$/\text{MW} = \mathbf{3.75}\ \$ \\ \pi_{\text{G1}} &= \frac{28.75\ \$}{10\ \text{MW}} = \mathbf{2.88}\ \$/\text{MW} \\ \pi_{\text{G2}} &= \frac{3.75\ \$}{5\ \text{MW}} = \mathbf{0.75}\ \$/\text{MW}. \end{aligned}$$

TABLE III
FINAL TARIFFS FOR THE APF METHOD

	MW	π ($/MW)	P ($)		MW	π ($/MW)	P ($)
G_1	10	2.88	28.8	D_1	5	5.0	25.0
G_2	5	0.75	3.8	D_2	5	0.5	2.5
				D_3	5	1.0	5.0
Total	15		32.5	Total	15		32.5

TABLE IV
FINAL TARIFFS FOR THE SHAPLEY METHOD

	MW	π ($/MW)	P ($)		MW	π ($/MW)	P ($)
G_1	10	1.88	18.8	D_1	5	5.0	25.0
G_2	5	2.75	13.8	D_2	5	0.5	2.5
				D_3	5	1.0	5.0
Total	15		32.5	Total	15		32.5

Using a similar procedure for consumers, the final tariffs for loads are: $\pi_{\text{D1}} = \mathbf{5.00}\ \$/\text{MW}$, $\pi_{\text{D2}} = \mathbf{0.50}\ \$/\text{MW}$ and $\pi_{\text{D3}} = \mathbf{1.00}\ \$/\text{MW}$.

The final tariffs are presented in Table III.

3) Shapley Procedure: The Shapley procedure is based on the combination of all entrance orders of generators G_1 and G_2 to supply loads D_1, D_2 and D_3. Thus, for generation tariff, the two possible entrance alternatives will be examined. Observe that, again, all circuit costs should be divided by 2 because of the 50%/50% allocation criteria adopted.

a) G_1 chooses first its demand and then G_2 decides: G_1 chooses to supply D_2 and D_3 and its allocated cost is calculated as

$$T_{G1}^{a} = 10\ \text{MW} \times \frac{1}{2}\$/\text{MW} + 5\ \text{MW} \times \frac{1}{2}\$/\text{MW} = \mathbf{7.5}\ \$$$

Then, G_2 supplies D_1. Its allocated cost is calculated as:

b) G_2 chooses first its demand and then G_1 decides: G_2 chooses to supply D_2:

$$T_{G2}^{b} = 5\ \text{MW} \times \frac{1}{2}\$/\text{MW} = \mathbf{2.5}\ \$$$

G_1 supplies D_1 and D_3:

$$T_{G1}^{b} = 5\ \text{MW} \times \frac{10}{2}\$/\text{MW} + 5\ \text{MW} \times \frac{2}{2}\$/\text{MW} = \mathbf{30}\ \$$$

Final tariffs are calculated as the average of previous values

$$\begin{aligned} T_{G1} &= (7.5 + 30)/2 = 18.75\ \$ \rightarrow \\ \pi_{G1} &= 18.75/10 = \mathbf{1.88}\ \$/\text{MW} \end{aligned}$$

$$\begin{aligned} T_{G2} &= (25 + 2.5)/2 = 13.75\ \$ \rightarrow \\ \pi_{G2} &= 13.75/5 = \mathbf{2.75}\ \$/\text{MW}. \end{aligned}$$

Using a similar procedure for consumers, the final tariffs for loads are: $\pi_{\text{D1}} = \mathbf{5.00}\ \$/\text{MW}$, $\pi_{\text{D2}} = \mathbf{0.50}\ \$/\text{MW}$ and $\pi_{\text{D3}} = \mathbf{1.00}\ \$/\text{MW}$.

The final tariffs are presented in Table IV.

TABLE V
AUMANN-SHAPLEY RESULTS

Step	G_1 [MW]	G_2 [MW]	P_1 [\$]	P_2 [\$]	Demand Supplied [MW]
1	0.33	0.17	0.17	0.08	0.5
2	0.33	0.17	0.17	0.08	1.0
3	0.33	0.17	0.17	0.08	1.5
4	0.33	0.17	0.17	0.08	2.0
5	0.33	0.17	0.17	0.08	2.5
10	0.33	0.17	0.17	0.08	5.0
15	0.33	0.17	0.33	0.17	7.5
20	0.33	0.17	0.33	0.17	10.0
30	0.33	0.17	1.67	0.83	15.0
Total	10.00	5.00	21.67	10.83	

Supplying:
D_1
D_2
D_3

TABLE VI
FINAL TARIFFS FOR THE AS METHOD

	MW	π (\$/MW)	P (\$)		MW	π (\$/MW)	P (\$)
G_1	10	2.17	21.7	D_1	5	5.0	25.0
G_2	5	2.17	10.8	D_2	5	0.5	2.5
				D_3	5	1.0	5.0
Total	15		32.5	Total	15		32.5

4) Aumann-Shapley Procedure: Table V presents the results for Aumann-Shapley procedure in some steps of the discrete integration. The variable λ was discretized in 30 equal steps $(\Delta\lambda = 0.0333)$, which means that the generators were split in 30 small ones. As already mentioned, for very small step sizes, the entrance order of generators is not important and it can be neglected: G_1 is always the first to choose the demands to supply.

In the first splitting (steps 1 to 10), G_1 and G_2 are free to choose the demands to supply. For obvious reasons they choose the closest (and thus cheapest) demand (D_2). In this sense, P_1 and P_2 represent the incremental transmission costs for the generators. From this point onwards, i.e., along the next discretizations, G_1 and G_2 decide to supply the closest demands that are still available; however they have less degrees of freedom, since some of the demands have already been supplied in the first discretizations. In this sense, D_3 becomes the cheapest available demand and is chosen in steps 11 to 20. Finally, D_1 is supplied in steps 21 to 30. The final tariffs are calculated as

$$\pi_{G1} = \frac{21.67\ \$}{10\ \text{MW}} = \mathbf{2.17}\ \$/\text{MW}$$
$$\pi_{G2} = \frac{10.83\ \$}{5\ \text{MW}} = \mathbf{2.17}\ \$/\text{MW}.$$

Using a similar procedure for consumers, i.e., demands are free to choose which generator will supply their demand, the final tariffs are: $\pi_{D1} = \mathbf{5.00}\ \$/\text{MW}$, $\pi_{D2} = \mathbf{0.50}\ \$/\text{MW}$ and $\pi_{D3} = \mathbf{1.00}\ \$/\text{MW}$.

The final tariffs are presented in Table VI.

5) Analysis of the Results: By analyzing the generator's tariffs (π_G) in the example, some interesting aspects of the discussed methodologies can be identified, as it can be observed in Table VII. For instance, given that the two generators are connected in very close buses, intuitively, it can be expected that their tariffs present similar behavior, which is actually verified in the AS method. In this sense, both APF and Shapley methods present limitations related to fairness aspects, since G_1 and G_2 have very different tariffs. As already discussed in Section VII, the Shapley procedure is dependent on agents size and this limitation is evident in the previous example. In the case of APF methodology, the method allocation is affected by the counterflow between G_1 and G_2 buses. The only method to present negative tariffs in the example was the LRMC. The AS method proposes a new allocation criteria that overcomes some limitations of the other discussed methods, which were pointed out in the previous example.[6]

TABLE VII
COMPARISON RESULTS FOR GENERATORS

		LRMC		APF		Shapley		AS	
	MW	π (\$/MW)	P (\$)	π (\$/MW)	P (\$)	π (\$/MW)	P (\$)	π (\$/MW)	P (\$)
G_1	10	2.20	21.7	2.88	28.8	1.88	18.8	2.17	21.7
G_2	5	2.20	10.8	0.75	3.8	2.75	13.8	2.17	10.8
Total	15		32.5		32.5		32.5		32.5

Finally, another aspect to be pointed out is that the four discussed methods allow the total recovery of transmission costs. It can be noted that a total amount of \$65 is recovered by the tariffs applied for generators and consumers, and this amount represents the sum of all circuits loadings weighted by their unitary cost (\$/MW). In this example, the circuits were considered as 100% loaded and there is no need of additional complementary charge.

IX. CASE STUDY: BRAZILIAN NETWORK

In this section, the Aumann-Shapley methodology for transmission cost allocation will be applied in the Brazilian power system, with data for the year 2006.

A. Brazil: Power System Overview

Brazil started its power sector reform in 1996. As in many countries worldwide, the new rules were designed to encourage competition in generation and retailing. Conversely, transmission and distribution remained regulated activities, with provisions for open access. Other reform ingredients included the creation of an Independent System Operator (ONS), a short-term electricity market (CCEE), a regulatory agency (ANEEL) and a planning research institute (EPE), as well as the privatization of distribution and some generation companies. The transmission expansion is based on a centralized environment and new (candidate) reinforcements are proposed by the government (Mines and Energy Ministry) and approved by the regulator. ANEEL is responsible for carrying out public auctions for their concession, where investors bid for annual revenues to build and operate the circuits [31]. Transmission costs are then allocated among network users using the BRA method described previously.

The country has an installed capacity of 100 GW (2006), where hydro generation accounts for 85%, for a peak demand near 65 GW. The hydro system is composed of several large reservoirs, capable of multi-year regulation (up to five years), organized in a complex topology over several basins. Thermal generation includes nuclear, natural gas, coal and diesel plants.

[6]In the case of AS method, negative tariffs for some agents can occur. However, in practice, huge cross payments, as those that LRMC provides in some cases, do not occur, as it was observed in the Brazilian case study.

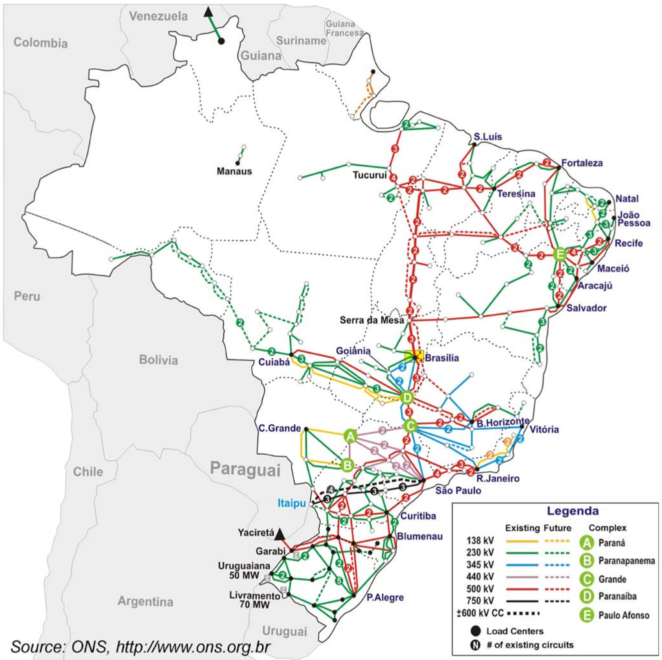

Fig. 3. Brazilian (main) interconnected system.

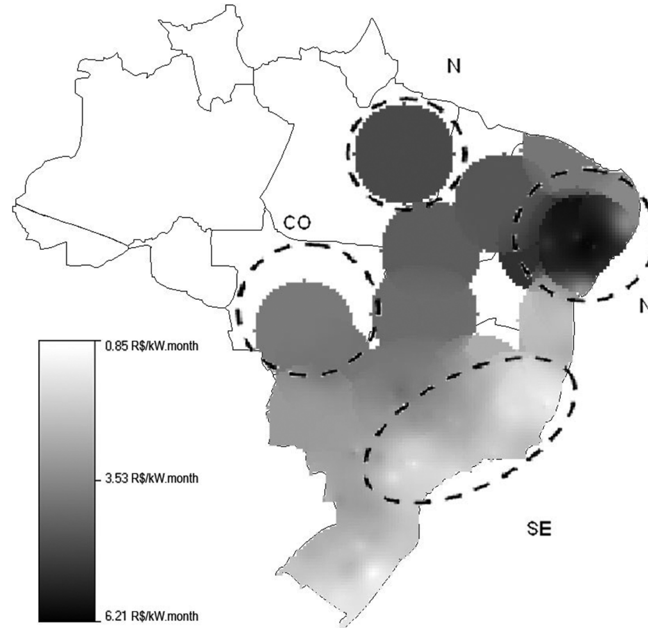

Fig. 4. OATT: Brazilian methodology.

The country is fully interconnected at the bulk power level by a 85 000 km meshed high-voltage transmission network (Fig. 3), with voltages ranging from 230 kV to 765 kV A.C., plus two 600 kV dc links connecting the Itaipú power plant (14 000 MW) to the main grid. The main direct international interconnections are the back-to-back links with Argentina (2200 MW), plus some smaller interconnections with Uruguay (70 MW) and Venezuela (200 MW).

B. Main Results

All methodologies previously discussed were implemented and applied to allocate transmission service cost for the Brazilian power system. The simulations were carried out considering data for year 2006, i.e., grid topology and unitary costs (allowed revenues) of elements. The generation dispatch and load scenario was also obtained from the official data. Generators and loads share the payment on a 50%-50% basis in all methods. For the sake of simplicity, only generation payments will be shown.

For the case of the AS method, generators were split into 10 000 subagents, i.e., a $\Delta\lambda$ equal to 0.0001 was adopted. This value represents a good balance between the robustness of results and computational efficiency, and it was decided based on previous experience of the authors. With this discretization value, the AS approach for the Brazilian system, with approximately 4000 buses and over 5000 circuits, can be solved in about 40 min in a PC Pentium IV 2.0 GHz with 512 Mb of RAM memory. Transmission capacity constraints were fully considered in the intermediate steps of the AS procedure.

As mentioned before, a postage stamp scheme is applied to all methods in order to allow a total recovery of transmission cost. In the case of Brazil, the locational signal recovers only 35% of the required revenue; the "postage stamp" complement covers the remaining 65%. As discussed, one reason for this under-recovery is that the nodal charge scheme tends to recover an amount related to the average loading of the transmission circuits. Because networks usually have redundancy in order to allow for circuit outages ("N $-$ 1" criterion), the grid looks "under-used" when power flows are considered for "base case" conditions alone (all circuits available). This is especially relevant in Brazil, because of hydro predominance and the need to allow the transfer of huge power blocks in different directions, depending on hydrological conditions, resulting in low average loading of long tie lines. As a consequence, the locational signals in the final transmission tariff are significantly weakened.

Given that the circuits' loadings are the same in all cases, the postage-stamp scheme has the same impact in the tariffs for all methodologies, except for the BRA method,[7] and thus does not distort the comparative analysis. Regardless, the results' breakdown (locational and postage stamp) will be presented.

Figs. 4 and 5 present contouring plots of generation transmission tariffs in Brazil for the BRA and AS methods. Dark areas represent those with higher tariffs and lighter areas represent those with smaller tariffs. Areas with the same behavior and tariffs with similar values can be noted. Energy export areas can also be identified by means of higher tariffs. Observe that Figs. 3 and 4 have different tariff scales, but this is irrelevant for the purposes of this first analysis: the objective is to highlight that both methods are able to identify higher and lower areas (even with different scales).

Four main tariff areas can be identified.

i) Northeast (NE) region: represents the São Francisco Basin's area, where the most important hydro power plants of the Northeast region are located. Therefore, this area has the highest tariffs of the Northeast region,

[7]Particularly, in the case of BRA, the postage-stamp scheme has a more important impact on the tariffs due to the Fpond Factor mentioned in Section III-A that reduces the locational signal when compared to the other methods.

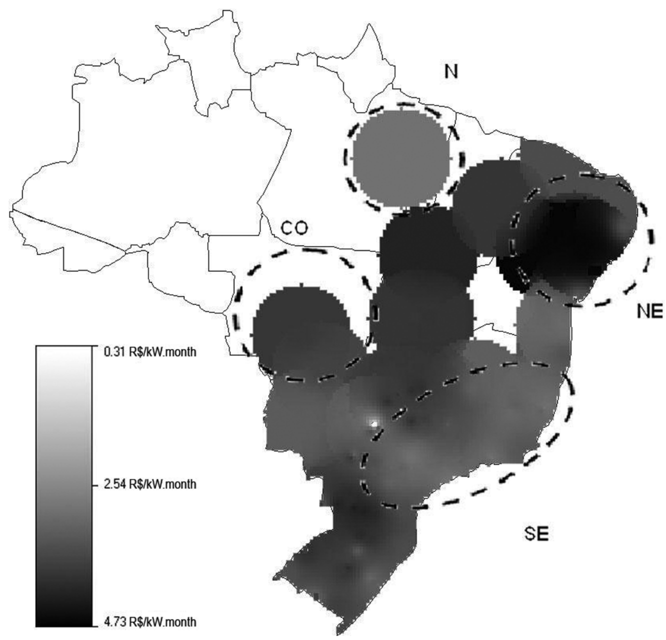

Fig. 5. OATT: Aumann-Shapley method.

which reflects the effective use of the electric grid by these power plants.

ii) Southeast (SE) region: represents the most important load center in the Southeastern region, close to the cities of Rio de Janeiro and São Paulo. Consequently, the network usage is smaller and this region presents the cheapest tariffs for generators as well as adequate signals to new generation connections.

iii) Center-West (CO) region: represents Mato Grosso with tariffs higher than SE. Besides the energy exporter behavior of this area, these tariffs can also be explained as a result of a weak electric grid in this region, where elements have high loading indices.

iv) North (N) region: represents the North region, which is characterized by important hydro power plants, low demand and a net energy exporting behavior. This results in a high network usage and thus high tariffs.

Table VIII presents the resulting total tariffs (locational + postage stamp components) for some important hydro (denoted by UHE) and thermal (denoted by UTE) generators in Brazil.

Some of the selected generators are: (a) Angra 2 (1309 MW) and Termorio (1036 MW), which are, respectively, a nuclear and a natural gas power plants located close to an important load center (Rio de Janeiro city) in Area 2; (b) Tucuruí (7983 MW), which is an important hydro power plant in the North region; (c) Sobradinho (1050 MW), which is one of the most important hydro power plants of the Northeast region; (d) Itá (1450 MW), which is a hydro generator located in the south of country; and (d) Furnas (1210 MW) and A. Vermelha (1388 MW), which are hydro plants located in SE region.

It can be noted that tariffs for generators located in North and Northeast regions, such as Tucuruí and Sobradinho, are smaller in the AS method than the BRA and LRMC methods. This happens because these agents are located far from the "economic"

TABLE VIII
GENERATORS' TARIFFS (R$/kW.Month)

Bus	Name	Area	AS	BRA	LRMC	APF
5827	UTE TermoBahia	NE	3.08	3.15	5.66	2.55
5151	UTE TermoPE	NE	3.07	3.43	6.59	2.77
5654	UTE Fortaleza	NE	3.12	3.81	4.98	2.97
5061	UHE Xingo	NE	4.61	5.53	9.32	4.10
6294	**UHE Sobradinho**	**NE**	**4.67**	**5.28**	**8.74**	**5.04**
4203	UTE TermoRio	SE	2.45	1.88	-0.16	2.38
11	**UTE Angra II**	**SE**	**2.75**	2.10	0.51	**2.67**
3962	UTE NorteFlu	SE	2.99	2.32	0.73	3.20
16	UHE Furnas	SE	3.09	2.63	1.66	3.44
500	UHE Agua Vermelha	SE	3.23	3.01	2.02	3.28
904	UHE Ita	SE	3.82	2.60	2.70	3.96
501	UHE Ilha Solteira	SE	0.31	3.02	0.16	0.35
1107	UHE Itaipu	SE	3.55	2.94	2.20	3.56
4596	UTE Cuiaba	CO	2.82	3.75	4.95	2.28
21	UHE Manso	CO	3.56	3.73	6.43	3.99
4804	UHE Guapore	CO	3.80	4.12	6.86	3.24
4809	UHE Jauru	CO	3.80	4.12	6.85	3.24
7110	UHE Peixe Angical	N	3.90	4.21	5.18	4.17
7206	UHE Lajeado	N	4.01	4.36	5.80	4.29
6419	**UHE Tucurui**	**N**	**2.45**	**4.76**	**5.11**	**2.41**

TABLE IX
TARIFFS BY REGION AND GENERATOR TYPE (R$/kW.Month)

Area	Type	AS	BRA	LRMC	APF
NE	Hydro	4.54	5.42	9.00	4.48
	Thermal	3.12	3.47	5.78	2.78
	Total	4.33	5.13	8.53	4.23
SE	Hydro	3.03	2.91	***2.06***	3.08
	Thermal	2.69	***2.11***	***0.50***	2.65
	Total	2.98	2.80	***1.84***	3.02
S	Hydro	3.81	2.75	3.21	3.81
	Thermal	3.14	***1.77***	***1.15***	3.07
	Total	3.70	2.59	2.88	3.69
N	Hydro	2.45	4.75	5.10	2.38
	Thermal	-	-	-	-
	Total	2.45	4.75	5.10	2.38

slack bus (located in the SE region) but very close to local demands. Therefore, the AS method captures that the electrons of these generators can meet a close demand, while in the BRA and LRMC any injection in these buses are compensated in this "economic" slack bus. In addition, for the BRA and LRMC "negative" tariffs are possible for the agents whose injections cause counter-flows. This is observed for the case of Termorio.

The APF method provides results with a similar behavior of those obtained with the AS. It can be shown, however, that there are cases where generators located close to one another are charged very different tariffs in the APF. This happens for the same reasons discussed in the example of Section VIII.

Table IX shows the average total tariff, separated by region and generator type. In general, it can be noted that tariffs in regions N and NE are higher (these are exporting regions) and the hydro generators have higher tariff than the thermal ones. This fact occurs because the hydro generators are usually located in distant areas from the load centers, and all methodologies were able to identify this characteristic.

In this case study, the postage stamp complement is equal to 2.29 R$/kW.month and all bold values in Table IX indicate that a negative locational tariff would be obtained without the complement. It can be seen that the locational signal for the LRMC methodology indicate negative tariffs for hydro

TABLE X
TARIFFS BY REGION AND GENERATOR TYPE [P.U. OF AVERAGE TARIFF (TOTAL) IN EACH REGION]

Area	Type	AS	BRA	LRMC	APF
NE	Hydro	0.89	1.06	1.76	0.87
	Thermal	0.61	0.68	1.13	0.54
	Total	0.84	1.00	1.66	0.82
SE	Hydro	1.08	1.04	0.74	1.10
	Thermal	0.96	0.75	0.18	0.95
	Total	1.06	1.00	0.66	1.08
S	Hydro	1.47	1.06	1.24	1.47
	Thermal	1.21	0.68	0.44	1.18
	Total	1.43	1.00	1.11	1.42
N	Hydro	0.52	1.00	1.07	0.50
	Thermal	-	-	-	-
	Total	0.52	1.00	1.07	0.50

and thermal generators in SE and for thermals in the South (S) region. In the case of the BRA, due to the FPond factor, only thermal generators from SE and S regions have negative locational tariffs in the average tariff.

Table X shows all values of Table IX divided by the average total tariff of the corresponding region, in order to compare the behavior of all methodologies in each region. Results higher than this corresponding reference were highlighted in gray.

Particularly for LRMC and BRA these higher tariffs (Table IX) can also be explained because of the allocation methodology, which is based on power injections in each bus and the compensations of theses injections at an "economic" slack bus. As mentioned, this "economic" slack bus is implicitly chosen in the economic gravity center of the system, i.e., where the total generation payments are equal to total load payments. In Brazil, this gravity center is located close to the Southeast region and this may explain the higher tariffs in the North and Northeast regions, since they are more distant from it. The necessity of this "economic" slack bus is pointed out as one of the most important deficiencies of these methods, since this does not represent a real system's behavior.

Both AS and APF methods are not based on power injections in each bus and do not need an "economic" slack bus to compensate for these injections. Hence, they give lower tariffs for the North and Northeast regions than the other methods. This characteristic reflects a more isonomic treatment of the tariffs of these methods, thus capturing the real behavior of electric systems: it is known that an increase of power at a determined generator tends to supply the closest loads (electric physical laws). This behavior is explicitly considered in the AS method and it can be pointed to as one of the most interesting aspects of this methodology (as well as the APF). It can also be interpreted as providing better incentives towards economic efficiency (since tariffs will be closer to the true marginal costs).

From the analysis of the results, it can be noted that the only method to present negative tariffs is the LRMC. Negative tariffs are usually not well seen by the market agents since they represent a cross payment, and they can be pointed as one important deficiency of this method. In the case of the Brazilian method, the "forced" attenuation of locational signals of the tariffs results in a decrease of the total amount recovered by the LRMC method, a consequent increase of the postage stamp component, and the absence of final negative tariffs.

X. CONCLUSIONS

The allocation of the fixed transmission service costs among network users has recently become more important. As discussed in this work, there is not a unique way to carry out this allocation: each method has its advantages and disadvantages. This work contributes to the field of transmission pricing by presenting an alternative and new methodology for energy transmission costs allocation. The methodology proposed in this paper is based on an optimization/game-theoretic framework (Aumann-Shapley) that retains the desirable properties of the well known APF and LRMC methods. The optimization scheme provides an economic justification for the "engineering intuition" of the APF method, while retaining the desirable network superposition properties of the LRMC scheme. In turn, the game-theoretic framework provides a theoretical justification for the allocation rules. Computational results were presented for the Brazilian power system and compared with those obtained by three other methodologies: LRMC, BRA and APF. The AS scheme has been studied and is being considered for implementation by the Brazilian Electricity Regulatory Board (ANEEL) as the transmission pricing method for the country.

Finally, one important issue is the variance of the OATT with respect to the generation dispatch scenario, mainly in hydro based countries such as Brazil, where dispatch conditions varies significantly with hydrology. Different dispatch conditions result in different flows and OATT. This issue was not investigated in this work and is currently being analyzed by the authors for a future work. Alternatives include treating the payments of generators and loads as average payments for several dispatches or to calculate yearly deviations (positive or negative) between the actual transmission tariffs and the estimated tariffs and allocating them to consumers. The same challenge applies for changes in the demand conditions.

APPENDIX
AUMANN-SHAPLEY IMPLEMENTATION

This appendix presents a formulation for implementing the AS allocation scheme described in Section VII. The allocation implementation is presented for generators; a similar procedure can be carried out for loads.

For ease of presentation, the AS allocation formulation $T(\lambda)$ is represented next:

$$T(\lambda) = Min \sum_{k=1}^{K} c_k \times \left| \sum_{i=1}^{N} \beta_{ki}(\delta_i - \lambda g_i) \right| \quad \text{(A.1)}$$

Subject to

$$\sum_{i=1}^{N} \delta_i = \sum_{i=1}^{N} \lambda g_i \quad \text{(A.2)}$$

$$-\overline{f}_k \leq \sum_{i=1}^{N} \beta_{ki}(\delta_i - \lambda g_i) \leq \overline{f}_k \quad \text{(A.3)}$$

$$\delta_i \leq d_i, \quad i = 1, \ldots, N. \quad \text{(A.4)}$$

The first step for the implementation is to replace the network representation by an alternative but equivalent one. In model (A.1)–(A.4), the network is implicitly represented by means of the sensitivity matrix β (compact network model). In the alternative model, the network variables, circuit flow (f_k) and bus voltage angle θ_i, will be explicitly represented.

Moreover, in order to replace the absolute value in the objective function, the flow variable—that can assume both positive and negative values—is split and represented as the difference between two nonnegative variables, f_k^+ and f_k^-.

Applying all changes, the model with full network representation becomes

$$T(\lambda) = Min \sum_{k=1}^{K} c_k \times \left(f_k^+ + f_k^-\right) \tag{A.5}$$

Subject to

$$\sum_{k\ in\ \Omega_i} \left(f_k^+ - f_k^-\right) + \delta_i = \lambda g_i, i = 1,\ldots,N \tag{A.6}$$

$$f_k^+ - f_k^- - \gamma_k(\theta_{sk} - \theta_{ek}) = 0, k = 1,\ldots,K \tag{A.7}$$

$$f_k^+ - f_k^- \leq \overline{f}_k, k = 1,\ldots,K \tag{A.8}$$

$$f_k^+ - f_k^- \geq -\overline{f}_k, k = 1,\ldots,K \tag{A.9}$$

$$\delta_i \leq d_i, i = 1,\ldots,N \tag{A.10}$$

where Ω_i is the set of circuits connected to bus i, γ_k is the susceptance of circuit k, s_k and e_k the starting and ending nodes of circuit k. Constraints (A.6) and (A.7) represents the first and second Kirchhoff laws, (A.8) and (A.9) represent the circuit flow upper and lower limits and (A.10) represents that the demand supplied in each bus must be lower than the bus load.

By applying the primal-dual equality between (A.5) and its dual, the objective function in the optimal solution can be written as

$$T(\lambda) = \sum_{i=1}^{N} \lambda g_i \times \pi\lambda_i + \sum_{k=1}^{K} \overline{f}_k \times \left(\pi f_k^+ + \pi f_k^-\right) + \sum_{i=1}^{N} \pi d_i \times d_i \tag{A.11}$$

where $\pi\lambda_i$, $\pi\gamma_k$, πf_k^+, πf_k^- and πd_i are the dual variables associated to constraints (A.6), (A.7), (A.8), (A.9) and (A.10), respectively.

If an incremental increase $\Delta\lambda_m$ (small enough not to change the multipliers) is applied, we have

$$\begin{aligned} \Delta T_m &= T(\lambda + \Delta\lambda_m) - T(\lambda) \\ \Delta T_m &= \sum_{i=1}^{N} \Delta\lambda_m g_i \times (\pi\lambda_i)_m \end{aligned} \tag{A.12}$$

where $(\pi\lambda_i)_m$ denotes the multiplier obtained in step m.

Thus, the parcel of increase in T that can be attributed to a generator i is given by the corresponding term in the summation of (A.12), which leads to the allocation of ΔT_m for each agent i.

The process can be repeated for each $\Delta\lambda_m$, where for each new segment the basis of the previous optimal solution is recovered and the optimization problem is reoptimized with the dual-simplex method. This way, the OATT for each generator T_i can be obtained through the integral of the incremental costs ΔT_m for each segment $\Delta\lambda_m$.

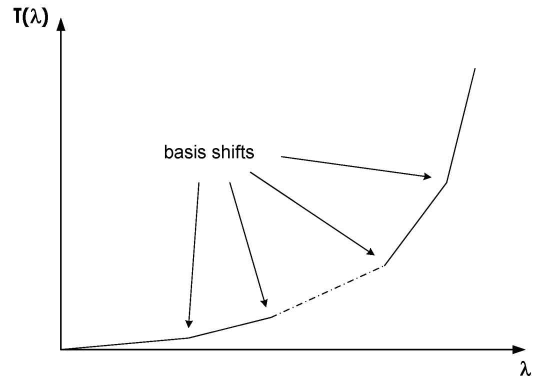

Fig. 6. Parameterized transmission cost.

Assuming a discretization of λ in M segments, this integral is given by

$$T_i = \sum_{m=1}^{M} \Delta\lambda_m \times (\pi\lambda_i)_m. \tag{A.13}$$

Below we discuss two possible discretization approaches regarding the size of the incremental increase $\Delta\lambda_m$ that can be used in the AS procedure.

A. Constant Discretization Step

The straightforward approach to implement the Aumann-Shapley procedure is to discretize parameter λ in equal step sizes $\Delta\lambda$. In this approach, the accuracy of the discretization is directly related to the step size chosen (good accuracy requires very small discretization of λ).

B. Variable Discretization Step

This approach avoids the discretization of the parameter λ in equals step sizes $\Delta\lambda$ chosen arbitrarily, independently of the precision of the discretization.

Initially, observe that function $T(\lambda)$ is piecewise-linear with respect to λ, as shown in Fig. 6.

It can be seen that $T(\lambda)$ is an increasing function of λ: as λ increases, the least-cost supply options is reduced and the total cost increases. When a load in a closer bus is fully supplied or the flow in an element reaches its limit (a new active constraint), the model must find another solution. The kinks shown in Fig. 6 represent these transitions.

This suggests that the discretization of λ can be linked to the location of the basis shifts, and the segments do not need to be of equal length. From LP sensitivity analysis we know, for each constraint, how much the right-hand side coefficients (RHS) can be changed without causing a change in the optimal basis.

This scheme provides a better discretization for λ and, in theory, it can speed up the convergence process: it is not necessary to determine the step size of λ in advance, and its infinitesimal discretization ("brute force") is avoided.

The procedure is implemented as follows: after the first LP solution is obtained, a new value for λ is calculated for each of the power balance constraint of the LP (A.6)

$$\lambda = max\left(\frac{R(PB_i)}{g_i}\right) \tag{A.14}$$

where $R(PB_i)$ is the minimal value of the RHS of the power balance constraint i, PB_i, for which the present solution basis remains optimal, and g_i is the RHS of PB_i.

In this sense, one can ensure that each new value for λ will lead to the closest kink as illustrated in Fig. 6.

C. Computational Comparison Between Constant and Variable Discretization Steps

In this work, both schemes were implemented for the Brazilian case study and had similar results. However, a CPU time of 15 hours (using the same computer applied in the case study of Section IX) was observed for the variable step approach, as opposed to a CPU time of 40 min for the constant discretization step.

The reason for this difference has to do with the size of the case study: the Brazilian system has about 3500 buses and 5000 circuits resulting in a LP problem of about 30 000 constraints with a complex feasible region. Thus, the sensitivity analysis gains are less significant and still demands a computational effort that cannot be neglected.

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