DCOPF-Based LMP Simulation: Algorithm, Comparison With ACOPF, and Sensitivity

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***Abstract*—The locational marginal pricing (LMP) methodology has become the dominant approach in power markets. Moreover, the dc optimal power flow (DCOPF) model has been applied in the power industry to calculate locational marginal prices (LMPs), especially in market simulation and planning owing to its robustness and speed. In this paper, first, an iterative DCOPF-based algorithm is presented with the fictitious nodal demand (FND) model to calculate LMP. The algorithm has three features: the iterative approach is employed to address the nonlinear marginal loss; FND is proposed to eliminate the large mismatch at the reference bus if FND is not applied; and an offset of system loss in the energy balance equation is proved to be necessary because the net injection multiplied by marginal delivery factors creates doubled system loss. Second, the algorithm is compared with ACOPF algorithm for accuracy of LMP results at various load levels using the PJM 5-bus system. It is clearly shown that the FND algorithm is a good estimate of the LMP calculated from the ACOPF algorithm and outperforms the lossless DCOPF algorithm. Third, the DCOPF-based algorithm is employed to analyze the sensitivity of LMP with respect to the system load. The infinite sensitivity or step change in LMP is also discussed.**

***Index Terms*—DCOPF, energy markets, fictitious nodal demand (FND), locational marginal pricing (LMP), marginal loss pricing, optimal power flow (OPF), power markets, power system planning, sensitivity analysis.**

I. Introduction

THE locational marginal pricing (LMP) methodology has been the dominant approach in power markets to calculate electricity prices and to manage transmission congestion. LMP has been implemented or is under consideration at a number of ISOs such as PJM, New York ISO, ISO-New England, California ISO, and Midwest ISO [1]–[3].

Locational marginal prices (LMPs) may be decomposed into three components: marginal energy price, marginal congestion price, and marginal loss price [5], [6]. Several earlier works [7]–[11] have reported the modeling of LMPs, especially in marginal loss model and related issues. Reference [7] points out the significance of marginal loss price, which may differ up to 20% among different zones in New York Control Area based on actual data. Reference [8] presents a slack-bus-independent approach to calculate LMPs and congestion components. Reference [9] presents a real-time solution without repeating a traditional power flow analysis to calculate loss sensitivity for any market-based slack bus from traditional Energy Management System (EMS) products based on multiple generator slack buses. Reference [10] demonstrates the usefulness of dc power flow in calculating loss penalty factors, which has a significant impact on generation scheduling. The authors of [10] also point out that it is not advisable to apply predetermined loss penalty factors from a typical scenario to all cases. Reference [11] presents LMP simulation algorithms to address marginal loss pricing based on the dc model.

From the viewpoint of generation and transmission planning, it is always crucial to simulate or forecast LMPs, which may be obtained using the traditional production (generation) cost optimization model, given the data on generation, transmission, and load [4], [5]. Typically, dc optimal power flow (DCOPF) is utilized for LMP simulation or forecasting based on production cost model via linear programming (LP) owing to LP's robustness and speed. The popularity of DCOPF lies in its natural fit into the LP model. Moreover, various third-party LP solvers are readily available to plug into DCOPF model to reduce the development effort for the vendors of LMP simulators. In industrial practice, DCOPF has been employed by several software tools for chronological LMP simulation and forecasting, such as ABB's GriveView™, Siemens' Promod®, GE's MAPS™, and PowerWorld [12], [13]. In addition, other literature shows the acceptability of dc model in power flow studies if the line flow is not very high, the voltage profile is sufficiently flat, and the R/X ratio is less than 0.25 [14].

It should be noted that in this paper the production cost is assumed to have a linear model. A quadratic cost curve can be represented with piecewise-linear curves to allow the application of LP.

In this paper, first, an iterative DCOPF-based algorithm is presented with the fictitious nodal demand (FND) model to calculate LMPs. The algorithm has three features: the iterative approach is employed to address the nonlinear marginal loss; FND is proposed to eliminate the large mismatch at the reference bus if FND is not applied; and an offset of system loss in the energy balance equation is proved to be necessary because the net injection multiplied by marginal delivery factors creates doubled system loss. Section II reviews the LMP calculation with delivery factors. Section III discusses the observation of a large nodal mismatch at the reference bus. Section IV presents a new algorithm for LMP simulation based on FND model to eliminate the nodal mismatch.

Second, the proposed DCOPF-FND-based algorithm is compared with ACOPF algorithm for accuracy of LMP results at various load levels using the PJM 5-bus system. It is shown that

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the FND algorithm is a good estimate of LMPs calculated from the ACOPF algorithm and outperforms the lossless DCOPF algorithm. This is discussed in Section V.

Third, the DCOPF-FND-based algorithm is employed to analyze the sensitivity of LMP with respect to the system load. A simple, explicit equation of LMP sensitivity is presented and validated. Also, a special case of infinite sensitivity under the step change of LMP is discussed. If the operating point is close to the critical load level of LMP step change, the sensitivity is less reliable and may not be applied to a large variation of load. This is discussed in Section VI.

Finally, concluding remarks are presented in Section VII.

II. DCOPF Model Considering Losses

Earlier studies of LMP calculation with DCOPF ignore the line losses. Thus, the energy price and the congestion price follow a perfect linear model with a zero loss price. However, challenges arise if nonlinear losses need to be considered in LMP calculations.

This section first reviews the lossless DCOPF model. Then, the marginal loss factor and marginal delivery factor based on generation shift factor (GSF) are discussed. Lastly, an iterative DCOPF model considering losses is presented and discussed.

A. DCOPF Without Losses

The generic DCOPF model [4] without the consideration of losses can be modeled as the minimization of the total production cost subject to energy balance and transmission constraints. The voltage magnitudes are assumed to be unity and reactive power is ignored. Also, it is assumed that there is no demand elasticity. This model may be written as LP

$$\text{Min} \quad \sum_{i=1}^{N} c_i \times G_i \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} G_i = \sum_{i=1}^{N} D_i \tag{2}$$

$$\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i) \le Limit_k, \quad \text{for } k = 1, 2, \ldots, M \tag{3}$$

$$G_i^{\min} \le G_i \le G_i^{\max}, \text{ for } i = 1, 2, \ldots, N \tag{4}$$

where

N	number of buses;
M	number of lines;
c_i	generation cost at Bus $i(\$/MWh)$;
G_i	generation dispatch at Bus $i(MWh)$;
$G_i^{\max}, G_i^{\min}$	max. and min. generation output at Bus i;
D_i	demand at Bus $i(MWh)$;
GSF_{k-i}	generation shift factor to line k from Bus i;
$Limit_k$	transmission limit of Line k.

It should be noted that the mathematical formulation in this paper assumes that each bus has one generator and one load for simplicity of discussion. Actual implementation can be more complicated considering multiple generators and loads may be connected to a bus. It should also be noted that the actual GSF values depend on the choice of slack bus, although the line flow in (3) based on GSF is the same with different slack buses.

B. Loss Factor and Delivery Factor

The key to consider marginal loss price is the marginal loss factor, or just loss factor (LF) for simplicity, and the marginal delivery factor, or just delivery factor (DF). Mathematically, they can be written as

$$DF_i = 1 - LF_i = 1 - \frac{\partial P_{Loss}}{\partial P_i} \tag{5}$$

where

DF_i	marginal delivery factor at Bus i;
LF_i	marginal loss factor at Bus i;
P_{Loss}	total loss of the system;
$P_i = G_i - D_i$	net injection at Bus i.

The loss factor and delivery factor can be calculated as follows. Based on the definition of loss factor, we have

$$P_{Loss} = \sum_{k=1}^{M} F_k^2 \times R_k \tag{6}$$

$$\frac{\partial P_{Loss}}{\partial P_i} = \frac{\partial}{\partial P_i}\left(\sum_{k=1}^{M} F_k^2 \times R_k\right) \tag{7}$$

where

F_k	line flow at line k;
R_k	resistance at line k.

In the linear dc network, a line flow can be viewed as the aggregation of the contribution from all power sources (generation as positive source and load as negative source) based on superposition theorem. This can be written as

$$F_k = \sum_{j=1}^{N} GSF_{k-j} \times (G_j - D_j) = \sum_{j=1}^{N} GSF_{k-j} \times P_j. \tag{8}$$

Equation (8) can be utilized to further expand LF as

$$\begin{aligned}\frac{\partial P_{Loss}}{\partial P_i} &= \sum_{k=1}^{M} \frac{\partial}{\partial P_i}\left(F_k^2 \times R_k\right) \\ &= \sum_{k=1}^{M} R_k \times 2F_k \times \frac{\partial F_k}{\partial P_i} \\ &= \sum_{k=1}^{M} 2 \times R_k \times GSF_{k-i} \times \left(\sum_{j=1}^{N} GSF_{k-j} \times P_j\right).\end{aligned} \tag{9}$$

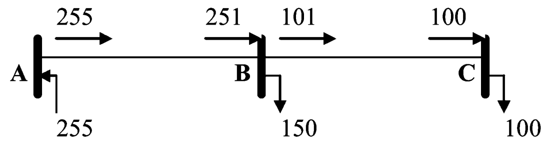

Fig. 1. Three-bus system with Bus B as the reference bus.

Interestingly, the loss factor at a bus may be positive or negative. When it is positive, it implies that an increase of injection at the bus may increase the total system loss. If it is negative, it implies that an increase of injection at the bus may reduce the total loss. For example, Fig. 1 shows a simple three-bus system with Bus B as the reference bus. If there is a hypothetical injection increase at Bus A, and the increased injection is absorbed by the reference bus (or the two load buses proportionally), the line flows as well as the losses will increase. Hence, the loss factor at Bus A is positive. If there is a hypothetical injection increase at Bus C and it is absorbed by the reference bus (or the two load buses proportionally), this will reduce the Line BC flow and then reduce the system loss. Thus, the loss factor at Bus C is negative.

Consequently, if loss factor is positive, the corresponding delivery factor is less than 1. If marginal loss factor is negative, marginal delivery factor is greater than 1.

C. DCOPF Algorithm Considering Marginal Loss

As shown in (9), loss factor depends on the net injection P_j, which is the actual dispatch minus the load at Bus j. On the other hand, generation dispatch may be affected by loss factors since different generators may be penalized differently based on their loss factors.

Since P_j is unknown before performing any dispatch, one way to address this is to have an estimation of dispatch to obtain an estimated LF at each bus. Then, the estimated loss factors will be used to obtain new dispatch results. This logic reasoning leads to the proposed iterative DCOPF approach. In other words, in the $(l+1)$th iteration, the dispatch results from the lth iteration is used to update the estimated DF_i and P_{loss}. Here, in each iteration, an LP-based DCOPF is solved. The iterative process is repeated until the convergence stop criteria are reached. After convergence, the LMPs can be obtained easily from the final iteration. In addition, the estimated DF_i and P_{loss} from the next-to-last iteration will be the same as the final values. It should be noted that the very first iteration is a lossless DCOPF in which the estimated loss is zero. The algorithm can be formulated as follows:

$$\text{Min} \quad \sum_{i=1}^{N} c_i \times G_i \tag{10}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} DF_i^{est} \times G_i - \sum_{i=1}^{N} DF_i^{est} \times D_i + P_{loss}^{est} = 0 \tag{11}$$

$$\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i) \le Limit_k, \quad \text{for } k \in \text{all lines} \tag{12}$$

$$G_i^{\min} \le G_i \le G_i^{\max}, \text{ for } i \in \text{all generators} \tag{13}$$

where

DF_i^{est} delivery factor at Bus i from the previous iteration;

P_{Loss}^{est} $= P_{loss}$ from the previous iteration.

It is not surprising that this iterative algorithm gives more accurate results with longer running time than the lossless DCOPF. However, the number of iterations is acceptable. The tests in later Sections show that the iterative DCOPF (with the proposed FND model in Section IV) needs four iterations to converge for the PJM 5-bus system, even if a very low tolerance of 0.001 MW is applied for high accuracy. When compared with ACOPF, the iterative DCOPF model is still much faster than ACOPF, which may be up to 60 times slower than DCOPF [15]. In addition, ACOPF requires care in preparing accurate input data to make it converge. Therefore, the iterative DCOPF model is advantageous compared to ACOPF, especially for simulation and planning purpose.

It should be noted that in real-time operation, delivery factors can be quickly obtained from real-time SCADA/EMS data. Unfortunately, this is not a viable option for simulation or a planning study. Therefore, it is necessary to identify a feasible approach such as iteration to obtain more accurate delivery factors for simulation and planning purpose. This is consistent with previous work [10], which shows that it is not advisable to apply penalty factors from a typical scenario with dc model to all other cases.

After obtaining the optimal solution of generation dispatch, the LMP at any Bus B can be calculated with the Lagrangian function. This function and LMP can be written as

$$\begin{aligned}\psi = &\left(\sum_{i=1}^{N} c_i \cdot G_i\right) \\ &- \lambda\left(\sum_{i=1}^{N} DF_i \cdot G_i - \sum_{i=1}^{N} DF_i \cdot D_i + P_{loss}\right) \\ &- \sum_{k=1}^{M} \mu_k \left(\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i) - Limit_k\right)\end{aligned} \tag{14}$$

$$\begin{aligned}LMP_B = \frac{\partial \psi}{\partial D_B} &= \lambda \cdot DF_B + \left(\sum_{k=1}^{M} \mu_k \times GSF_{k-B}\right) \\ &= \lambda + \left(\sum_{k=1}^{M} \mu_k \times GSF_{k-B}\right) + \lambda\,(DF_B - 1)\end{aligned} \tag{15}$$

where

LMP_B $= LMP$ at Bus B;

λ = Lagrangian multiplier of (11) = energy price of the system = price at the reference bus;

μ_k = Lagrangian multiplier of (12) = sensitivity of the kth transmission constraint.

From (15), LMP can be easily decomposed into three components: marginal energy price, marginal congestion price and marginal loss price. The LMP formulation can be written as (16)–(19), which are consistent with [1], [2]

$$LMP_B = LMP^{energy} + LMP_B^{cong} + LMP_B^{loss} \tag{16}$$

$$LMP^{energy} = \lambda \tag{17}$$

$$LMP_B^{cong} = \sum_{k=1}^{M} GSF_{k-B} \times \mu_k \tag{18}$$

$$LMP_B^{loss} = \lambda \times (DF_B - 1). \tag{19}$$

III. On the Equality Constraints of Energy Balance

It should be noted that P_{loss}^{est} in (11) is used to offset the doubled system loss caused by the (marginal) loss factor, LF, and the (marginal) delivery factor, DF. The inclusion of P_{loss}^{est} eliminates the overestimated loss issue reported in the previous work [11]. This is consistent with the fact that the marginal loss (injection multiplied by marginal loss factor) is twice the actual loss (also referred to as the average loss) in the dc model, since the line loss is linearly related to the square of bus injection. A rigorous proof of the validity of (11) is given as follows:

$$\begin{aligned}
&\sum_{i=1}^{N} DF_i \times G_i - \sum_{i=1}^{N} DF_i \times D_i \\
&\quad = \sum_{i=1}^{N} DF_i \times (G_i - D_i) = \sum_{i=1}^{N} DF_i \times P_i \\
&\quad = \sum_{i=1}^{N} (1 - LF_i) \times P_i \\
&\quad = \sum_{i=1}^{N} \left(1 - \frac{\partial P_{loss}}{\partial P_i}\right) \times P_i \\
&\quad = \sum_{i=1}^{N} P_i - \sum_{i=1}^{N} \left(\frac{\partial P_{loss}}{\partial P_i} \times P_i\right) \\
&\quad = \sum_{i=1}^{N} P_i - \sum_{i=1}^{N} \left(\left(\sum_{k=1}^{M} 2R_k \times F_k \times GSF_{k-i}\right) \times P_i\right) \\
&\quad = \sum_{i=1}^{N} P_i - \sum_{k=1}^{M} \left(2R_k \times F_k \times \sum_{i=1}^{N} (GSF_{k-i} \times P_i)\right) \\
&\quad = \sum_{i=1}^{N} P_i - \sum_{k=1}^{M} (2R_k \times F_k \times F_k) \\
&\quad = \sum_{i=1}^{N} P_i - 2 \cdot \sum_{k=1}^{M} \left(R_k \times F_k^2\right) \\
&\quad = P_{loss}^{schd} - 2P_{loss}^{act} = -P_{loss}
\end{aligned} \tag{20}$$

where

$P_{loss}^{schd} = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} (G_i - D_i)$ scheduled loss;

$P_{loss}^{act} = \sum_{k=1}^{M} (R_k \times F_k^2)$ actual loss.

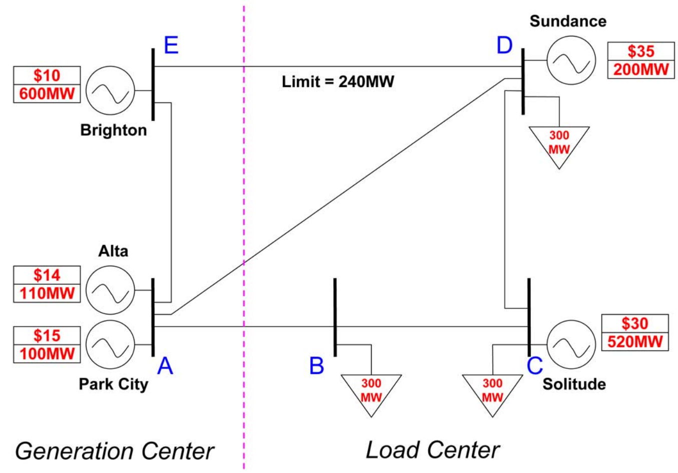

Fig. 2. Base case of the PJM 5-Bus example.

TABLE I
LINE IMPEDANCE AND FLOW LIMITS

	A-B	A-D	A-E	B-C	C-D	D-E
R (%)	0.281	0.304	0.064	0.108	0.297	0.297
X (%)	2.81	3.04	0.64	1.08	2.97	2.97
Limit (MW)	999	999	999	999	999	240

In the aforementioned derivation, P_{loss}^{schd} represents the system net injection at all buses. Therefore, it is called the scheduled loss of the entire system. Meanwhile, the actual loss is represented by P_{loss}^{act}, which is the sum of the actual losses at all lines. After the iterative approach converges, the scheduled loss should be equal to the actual loss, or $P_{loss}^{schd} = P_{loss}^{act} = P_{loss}$.

From the aforementioned derivation, it is apparent that the net injection multiplied by loss factor, that is, $\sum_{i=1}^{N}(\partial P_{loss}/\partial P_i \times P_i)$, doubles the system loss. This suggests that (11) must include an extra deduction of system loss (P_{loss}^{est}) when marginal loss factors are applied. Computationally, the actual loss value from the previous iteration is used for the current iteration to keep the linearity of the optimization formulation. The convergence criteria, i.e., the dispatch of each generator, will ensure the convergence of P_{loss}, i.e., $P_{loss}^{schd} = P_{loss}^{act}$. Equation (11) may be verified with the sample system shown in Fig. 2 for illustration. The system is slightly modified from the PJM 5-bus system [1] and will be used for the rest of this paper. The generation cost at Sundance is modified from the original \$30/MWh to \$35/MWh to differentiate its cost from the Solitude unit for better illustration.

The system can be roughly divided into two areas, a generation center consisting of Buses A and E with three low-cost generation units and a load center consisting of Buses B, C, and D with 900 MWh load and two high-cost generation units. The transmission line impedances are given in Table I, where the reactance is obtained from [1] and the resistance is assumed to be 10% of the reactance. Here only the thermal flow limit of Line ED is considered for illustrative purpose.

TABLE II
VERIFICATION OF (11) TO AVOID DOUBLED LOSSES CAUSED BY MARGINAL DELIVERY FACTORS

	With P^{est}_{loss} in Eq. (11)	Without P^{est}_{loss} in Eq. (11)
Load	900	900
Scheduled gen.	908.81	917.61
Scheduled losses	8.81	17.61
Actual line losses	8.81	8.81
Error	**0%**	**99.9%**

TABLE III
DISPATCH RESULTS FROM THE ITERATIVE DCOPF

	G (MW)	L (MW)	Line inj. (MW)	Mismatch (MW)
Bus A	210	0	-210	0
Bus B	0	300	300	0
Bus C	0	300	300	0
Bus D	124.88	300	183.92	8.80
Bus E	573.92	0	-573.92	0

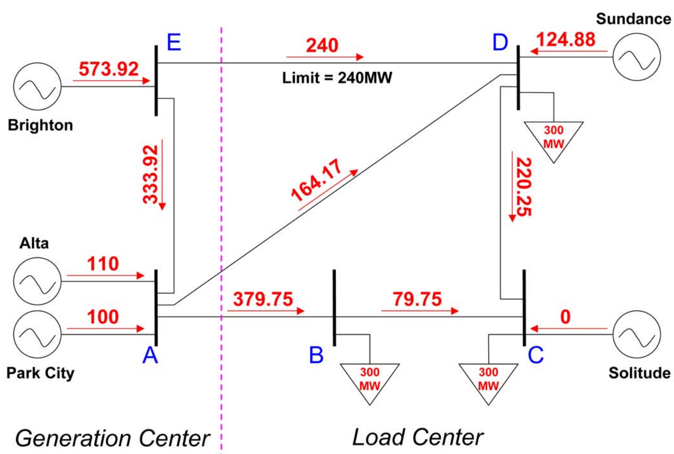

Fig. 3. Dispatch results for the base case.

Table II clearly shows that the dispatch will give doubled losses if P_{loss}^{est} is excluded from (11). The result is more reasonable if P_{loss}^{est} is included.

IV. ITERATIVE DCOPF ALGORITHM WITH FND FOR LOSSES

A. Mismatch at the Reference Bus in the Previous Model

The aforementioned model addresses the marginal loss price through the delivery factors. However, the line flow constraints in (12) still assume a lossless network. Meanwhile, the system energy balance constraint in (11) enforces that the total generation should be greater than the total demand by the average system loss. This leads to a mismatch at the reference bus because the amount of the mismatch has to be absorbed by the system reference bus. If the amount of demand is a large amount like a few GW, the system loss may be in the scale of tens to hundreds of MW. It is not accurate to have all the loss absorbed by the reference.

Taking the PJM 5-bus sample system, the dispatch result is shown in Table III and Fig. 3. The result shows the nodal mismatch, defined as *Nodal Generation − Nodal Demand + Injections from all connected lines*. Although all buses except the reference Bus D have zero mismatches, the mismatch at Bus D is relatively large because it absorbs the total system loss of 8.80 MW. This is a centralized loss model, which means that all losses are centrally absorbed by the reference bus.

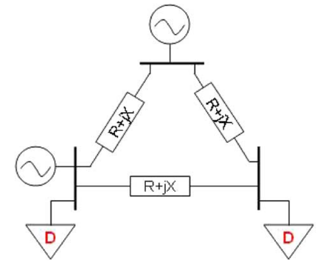

Fig. 4. System with line resistance.

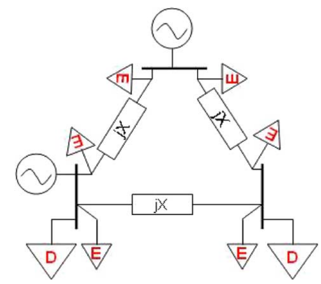

Fig. 5. System with FND to represent line losses.

B. FND-Based Iterative DCOPF Algorithm

To address the mismatch issue at the reference bus, it is desirable that the line losses are represented in the transmission lines. Since line flow is represented with GSF in LP-based DCOPF, it is challenging to include line losses without losing the linearity of the model.

This paper employs the concept of fictitious nodal demand (FND) to represent the losses of the lines connected to a bus. The FND is similar yet different from the fictitious load and fictitious midpoint bus model in [16]. [16] uses the fictitious load and midpoint bus to partition an inter-area tie line and eventually models multi-area OPF. This research work does not need the fictitious midpoint bus and uses a different representation of the fictitious loss model, as shown in (21). More important, FND is applied here to distribute system losses among each individual line to eliminate a significant mismatch at the reference bus. The FND model is illustrated in Figs. 4 and 5. With this approach, the loss in each transmission line is divided into two equal halves, attached to both buses of that line. Each half is represented as if it is an extra nodal demand. For each bus, the total of all equivalent line losses is the proposed FND.

Here the FND at Bus i is written as E_i, defined as follows:

$$E_i = \sum_{k=1}^{M_i} \frac{1}{2} \times F_k^2 \times R_k \tag{21}$$

where M_i is the number of lines connected to Bus i.

The line flow F_k can be obtained from the FND calculation in the previous iteration. Here, F_k is calculated as

$$F_k = \sum_{j=1}^{N} GSF_{k-j} \times (G_j - D_j - E_j^{est}). \quad (22)$$

The loss factor calculation may use the same (9); however, the value of F_k will be different under the new approach of FND.

Therefore, the new iterative DCOPF formulation, which replaces (10)-(13), can be formulated as

$$\text{Min} \quad \sum_{i=1}^{N} c_i \times G_i \quad (23)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} DF_i^{est} \times G_i - \sum_{i=1}^{N} DF_i^{est} \times D_i + P_{loss}^{est} = 0 \quad (24)$$

$$\sum_{i=1}^{N} GSF_{k-i} \times \left(G_i - D_i - E_i^{est}\right) \le Limit_k, \quad \text{for } k \in \text{all lines} \quad (25)$$

$$G_i^{\min} \le G_i \le G_i^{\max}, \text{ for } i \in \text{all generators}. \quad (26)$$

When the above formulation converges using the generation dispatch of each unit (G_i) as the convergence criterion, other parameters such as the line flows (F_k), the delivery factors (DF_i), and the system loss (P_{loss}) will converge as well. The Appendix shows the proof of the convergence feature of this new algorithm.

The detailed procedure of this FND-based iterative DCOPF algorithm is given as follows:

1) Set $\mathrm{LF_i^{est}} = 0$, $\mathrm{DF_i^{est}} = 1$, $\mathrm{E_i^{est}} = 0$ (for $\mathrm{i} = 1, 2, \ldots, \mathrm{N}$) and $\mathrm{P_{loss}^{est}} = 0$.

2) Perform generation dispatch using (23)–(26).

3) Update $\mathrm{LF_i^{est}}$, $\mathrm{DF_i^{est}}$, $\mathrm{E_i^{est}}$, and $\mathrm{P_{loss}^{est}}$ using (5), (6), (9), and (21).

4) Perform another dispatch using (23)–(26).

5) Check the results of the dispatch of each generator with the previous dispatch. If the difference at one or more buses is greater than the predefined tolerance, go to Step 3. Otherwise, go to Step 6.

6) Calculate the three LMP components using (16)–(19).

The result of the proposed new iterative DCOPF model is shown in Fig. 6. The total loss is distributed to each *individual* line. At each bus, the nodal generation, plus incoming flows from connected lines, and then minus nodal demand is equal to the FND, which represents half of the losses in all connected lines. Therefore, the system loss is distributed in each line and numerically represented by the FND at each bus. The mismatch at the reference bus, like at any other bus, is just 50% of the losses of all connected lines, not the total system loss.

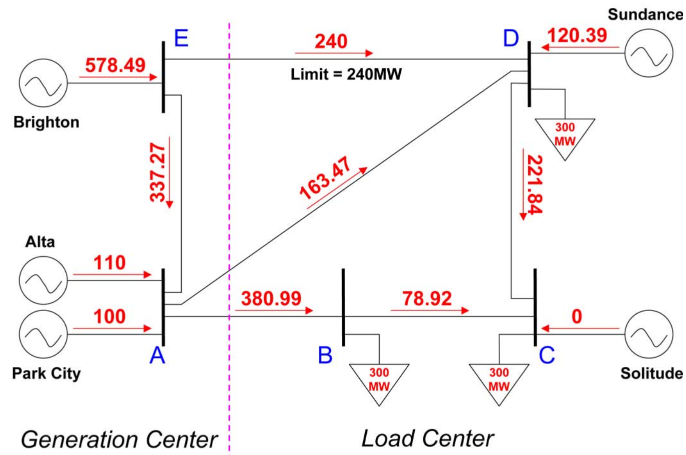

Fig. 6. Dispatch results with the FND approach.

V. Benchmarking the FND and Lossless DCOPF Algorithms With ACOPF-Based Algorithm

In this section, the ACOPF algorithm is briefly discussed. Then, the FND and lossless DCOPF algorithms are benchmarked with the ACOPF algorithm using the PJM 5-bus system. Finally, results are analyzed and explained.

A. ACOPF-Based LMP Algorithm

As a comparison, a model based on ACOPF [4] is presented. Although this is not typical for market price simulation because of its relatively slow speed and convergence problem in a fairly large system, it is presented here for the purposes of comparison and illustration.

Generally, the ACOPF model can be presented as minimizing the total generation cost, subject to nodal real power balances, nodal reactive power balances, transmission limits, generation limits, bus voltage limits, and so on. Details may be found in [4]. The LMP at each bus from ACOPF formulation is the Lagrange multiplier of the equality constraint of the nodal real power balance [15], [17].

B. Test Results From PJM 5-Bus System

This section gives the test results with the slightly modified PJM 5-bus system as shown in Section III and Fig. 2. In the ACOPF run, all loads are assumed to have 0.95 lagging power factors. The generators are assumed to have a reactive power range between 150 MVar capacitive to 150 MVar inductive such that reactive power will not be a limiting issue. ACOPF is implemented with MATPOWER package [18].

LMP calculations are performed using the lossless DCOPF algorithm, the FND-based iterative DCOPF algorithm, and the ACOPF algorithm in the previous subsection. The LMP results from the two DCOPF algorithms are benchmarked with the ACOPF under various load levels from 1.0 to 1.3 per unit of the base-case load (900 MWh). Tests are performed with step size of 0.0025 p.u. load increase. All bus loads are varied proportionally, and the same power factor is kept at each bus for the ACOPF case. Test results show that the FND algorithm quickly converges in 4–5 iterations for the PJM 5-bus case even if a low tolerance of 0.001 MW is applied for high accuracy.

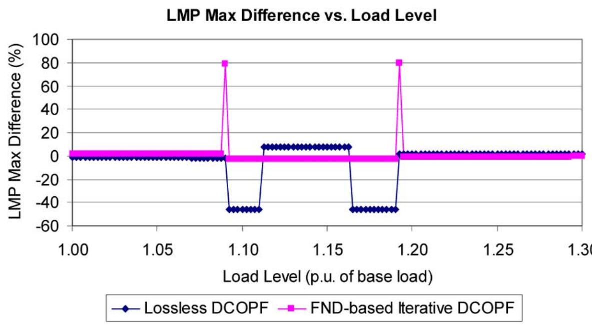

Fig. 7. Maximum difference of LMP in percentage between each DCOPF algorithm and the ACOPF for the PJM 5-bus system.

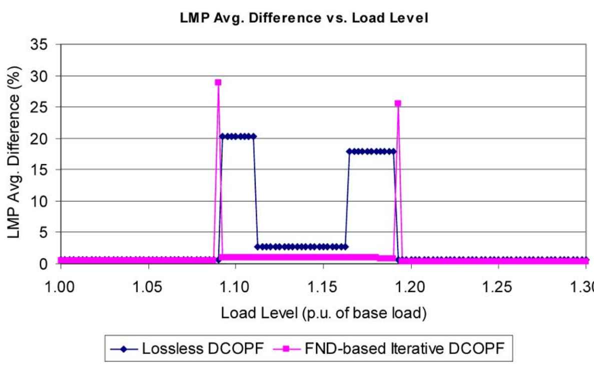

Fig. 8. Average difference of LMP in percentage between each DCOPF algorithm and ACOPF for the PJM 5-bus system.

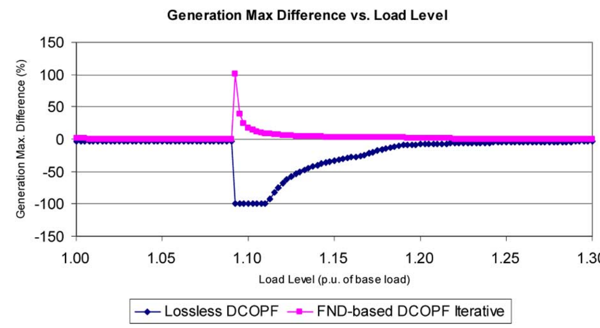

Fig. 9. Maximum difference of generation dispatch between each DCOPF algorithm and the ACOPF for the PJM 5-bus system.

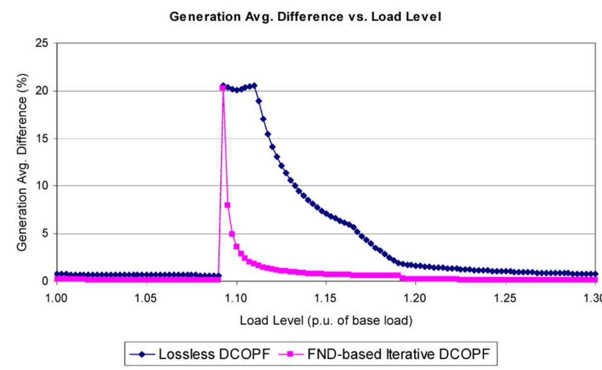

Fig. 10. Average difference of generation dispatch between each DCOPF algorithm and ACOPF for the PJM 5-bus system.

Figs. 7 and 8 plot the maximum difference (MD) and the average difference (AD) of nodal LMPs between two models. The MD and AD of LMP at a given load level are given as

$$MD_{LMP}(\%) = \pm \max\left\{\left|\frac{LMP_i^{(1)} - LMP_i^{(2)}}{LMP_i^{(2)}}\right| \times 100\right\}_{i\in\{1,2,\ldots,N\}}, \quad (27)$$

$$AD_{LMP}(\%) = \frac{\sum_{i=1}^{N}\left|\frac{LMP_i^{(1)} - LMP_i^{(2)}}{LMP_i^{(2)}}\right| \times 100}{N} \quad (28)$$

where $LMP_i^{(1)} = LMP$ from the lossless DCOPF algorithm or the FND algorithm, and $LMP_i^{(2)} = LMP$ from the ACOPF algorithm.

Sign of MD is determined by the sign of $\left(LMP_i^{(1)} - LMP_i^{(2)}\right)$.

The MD and AD of the generation dispatch, similar to those for LMP in (27)–(28), are also presented in Figs. 9 and 10.

As Figs. 7–10 show, LMP from the lossless DCOPF algorithm matches ACOPF results for 82% of all load levels tested. This is consistent with the results reported in the earlier work [15]. However, the lossless DCOPF causes some significant errors at 18% load levels.

The FND algorithm outperforms the lossless DCOPF algorithm when using ACOPF as a benchmark for LMP as well as generation dispatch. For example, the LMP results from the FND algorithm are very close to the ACOPF LMP results with exceptions at only two particular load levels: 1.0900 and 1.1925 per unit of the base load. As a comparison, the LMP from the lossless DCOPF produces significant errors in two bands of load levels: $[1.0900, 1.1125]$ and $[1.1625, 1.1925]$. Similar observations can be made for generation dispatch. Since the lossless DCOPF ignores the line loss, it is not surprising that it performs much more poorly than the FND-based iterative DCOPF algorithm.

Further tests in IEEE 30-bus System are also performed. The results are very similar to the results from the PJM 5-bus system. For instance, the FND DCOPF algorithm gives a much closer approximation than the lossless DCOPF algorithm in all four measures, MD of LMP, AD of LMP, MD of generation dispatch, and AD of generation dispatch.

C. Discussion of the LMP and Dispatch Difference

Here, the causes and implications of the LMP differences between the FND-based iterative DCOPF algorithm and the ACOPF algorithm are discussed.

The significant LMP differences between dc model and ac model are due to the approximation in dc model, which causes the different sets of marginal units and eventually the different prices. For example, when the load level is 1.09 per unit of the base load, the dispatch results are shown in Table IV. With the ACOPF model, the marginal units are Sundance and Brighton with the latter being dispatched at very close to its maximum capacity. However, in the FND-based iterative DCOPF, the Brighton unit is now dispatched at its maximum capacity, and the Solitude unit is dispatched for a very small amount. Thus, the marginal unit for the FND algorithm is Sundance

TABLE IV
GENERATION DISPATCH RESULTS FROM DCOPF AND ACOPF

	Max. Cap (MW)	Cost ($/MWh)	FND-based DCOPF	ACOPF
Alta	110	14	110.00	110.00
Park City	100	15	100.00	100.00
Solitude	520	30	**0.49**	0.00
Sundance	300	35	**180.39**	**179.94**
Brighton	600	10	600.00	**599.79**
Total			990.88	989.72

and Solitude. Therefore, the different marginal units lead to the LMP difference because they determine the overall LMP. It should be noted that once the ACOPF and the FND algorithms identify the same marginal units at 1.1925 p.u. load level, the prices will be very close.

It should be noted that a large difference at a particular load level is almost unavoidable for any approximate approach to calculate LMP owing to the step-change nature of marginal units. However, as shown in Figs. 7 and 8, a better approximation should be capable of giving a good LMP prediction for a broader range of load levels.

This observation has a practical implication for real systems. The test system can be roughly divided into two areas, a generation center consisting of Buses A and E and a load center consisting of Buses B, C, and D. In the generation center, there are abundant low-cost generation resources, while in the load center there are many loads with expensive generators. The change of marginal units may cause considerable LMP differences when the generation center approaches its maximum export. Since the test system with a generation center and a load center is a typical case in many systems, it is reasonable to conclude that when the units in the low-cost, net-exporting area are approaching its maximum capacity, it is very likely that the difference in DCOPF and ACOPF may lead to significant price difference because two approaches may give different marginal units. Special care such as verification with ac model may be necessary for system planners if dc model is the primary approach.

As for the generation dispatch results shown in Figs. 9 and 10, the FND algorithm is very close to ACOPF algorithm for most cases except the load levels between 1.09 and 1.10. The difference is not as big as it looks because Figs. 9 and 10 show relative differences. When a unit is dispatched as a small value, for example 0.5 MWh in ACOPF, the difference percentage is as large as 100% when the FND algorithm gives 1.0 MWh. In this case, the large relative difference is not very surprising.

In addition, large dispatch difference does not necessarily correspond to large LMP difference. As long as the dispatch difference occurs at the same marginal unit(s), the LMP difference between ACOPF and an approximation algorithm is not big enough to be noteworthy.

VI. SENSITIVITY ANALYSIS OF LMP WITH RESPECT TO LOAD

The previous section shows that the FND-based iterative DCOPF algorithm is a trustable approximation, especially when compared with lossless DCOPF, of the ACOPF-based LMP. This section will examine the sensitivity of the LMP w.r.t. load changes based on the FND-based iterative DCOPF.

TABLE V
μ, DF AND LMP WITH RESPECT TO DIFFERENT LOAD LEVELS AT BUS B

Load @ B	μ of Line ED	DF @ B	LMP @ B	DF @ C	LMP @ C
300	50.98634	1.011301	24.30337	1.013040	27.32212
303	50.98628	1.011411	24.30721	1.013120	27.32494
306	50.98622	1.011520	24.31105	1.013200	27.32776
309	50.98617	1.011630	24.31490	1.013280	27.33058
312	50.98611	1.011739	24.31874	1.013361	27.33340
315	50.98605	1.011848	24.32258	1.013441	27.33621
318	50.98599	1.011958	24.32643	1.013521	27.33903
321	50.98593	1.012067	24.33027	1.013601	27.34185
324	50.98587	1.012177	24.33411	1.013682	27.34467
327	50.98581	1.012286	24.33796	1.013762	27.34749
330	50.98575	1.012396	24.34180	1.013842	27.35031

A. Sensitivity When There is No Change of Marginal Unit(s)

Based on DCOPF formulation, the sensitivity is due to the loss model. In other words, if there is no loss considered, the LMP should remain unchanged if there is a very small change of demand (as long as there is no new marginal unit). Hence, the sensitivity of LMP is zero in this case.

The possible nonzero sensitivity of LMP in DCOPF must be attributed to the loss model. When load grows, loss grows quadratically. The change of load will lead to a change of not only *DF* but λ and μ also. This is because the change of delivery factor in the DCOPF model shall lead to a new λ and μ, when load is varied. In summary, the sensitivity can be written as

$$\frac{\Delta LMP_i}{\Delta D_j} = \frac{\Delta\left(DF_i \cdot \lambda + \sum_{k=1}^{M} \mu_k \cdot GSF_{k-i}\right)}{\Delta D_j}. \tag{29}$$

Hence, we have

$$\frac{\Delta LMP_i}{\Delta D_j} = \lambda \cdot \frac{\Delta DF_i}{\Delta D_j} + DF_i \cdot \frac{\Delta \lambda}{\Delta D_j} + \sum_{k=1}^{M}\left(GSF_{k-i} \cdot \frac{\Delta \mu_k}{\Delta D_j}\right). \tag{30}$$

Generally, $(\Delta\lambda/\Delta D_j) \neq 0$ and $(\Delta\mu_k/\Delta D_i) \neq 0$. This makes the case with loss different from lossless case.

Table V shows the μ of Line ED, the DF at Bus B, the LMP at Bus B, the DF at Bus C, and the LMP at Bus C w.r.t. Bus B Load from 300 to 330 MW. It can be easily verified that each variable is linearly related to Bus B Load in Table V. In addition to Table V, the energy component of LMP, or λ, is $35/MWh, constantly. However, this is a special case because the marginal unit is located at the reference bus, thus λ is constant and $(\Delta\lambda/\Delta D_j) = 0$ here. As stated earlier, λ is usually not a constant in DCOPF model with loss, that is, $\Delta\lambda/\Delta D_j \neq 0$. Also, the GSFs of line DE to Bus B and Bus C are -0.2176 and -0.1595, respectively. With all the above data, (30) can be verified. Taking Bus B as an example (i.e., $i = j = \text{Bus B}$), we have the following calculation based on the 2nd and 3rd columns in Table V

$$\begin{aligned}\lambda \cdot \frac{\Delta DF_i}{\Delta D_j} &= 35 \times \frac{(1.012396 - 1.011301)}{30}\\ &= 1.2775 \times 10^{-3}\\ \frac{\Delta\mu_{\mathrm{DE}}}{\Delta D_j} \cdot GSF_{DE-i} &= \frac{(50.98575 - 50.98634)}{30} \times (-0.2176)\\ &= 0.0043 \times 10^{-3}.\end{aligned}$$

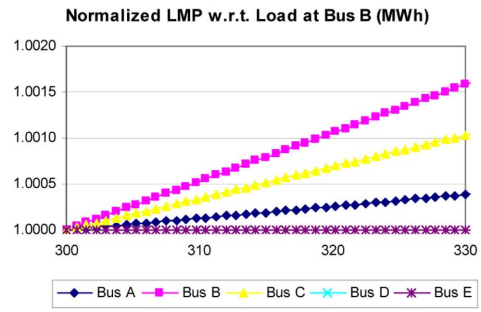

Fig. 11. LMP normalized to the base case at each bus with respect to load at Bus B. The LMPs of base case for the five buses are 15.86, 24.30, 27.32, 35.0, and 10.0 \$/MWh, respectively.

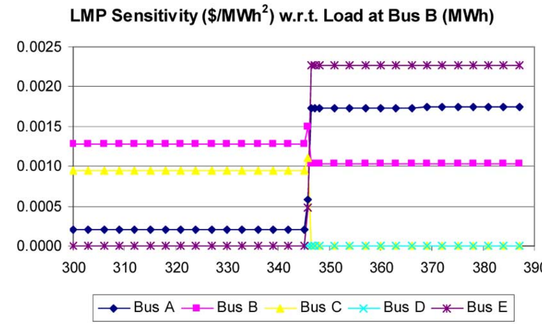

Fig. 12. LMP sensitivity w.r.t. load at Bus B ranging from 300 to 390 MWh.

Therefore, we have

$$\lambda \cdot \frac{\Delta DF_i}{\Delta D_j} + DF_i \cdot \frac{\Delta \lambda}{\Delta D_j} + \sum_{k=1}^{M}\left(GSF_{k-i} \cdot \frac{\Delta \mu_k}{\Delta D_j}\right)$$
$$= (1.2775 + 0 + 0.0043) \times 10^{-3}$$
$$= 1.2818 \times 10^{-3}\left(\frac{\$}{\mathrm{MWh}^2}\right).$$

On the other hand, from the 4th column in Table V, we have

$$\frac{\Delta LMP_i}{\Delta D_j} = \frac{(24.34180 - 24.30337)}{30}$$
$$= 1.2810 \times 10^{-3}\left(\frac{\$}{\mathrm{MWh}^2}\right).$$

Hence, the LMP sensitivity $(\Delta LMP_i/\Delta D_j)$ is essentially the same as the value computed with the right-hand side in (30). This verifies (30). Similar verification can be easily obtained for Bus C using the 2nd, 5th, and 6th columns. Also, Fig. 11 shows the LMPs at all buses w.r.t. Bus B load between 300 and 330 MW. The LMPs at each bus are normalized to the base case when Bus B load is 300 MW.

As shown in Fig. 11, the LMP at marginal unit buses such as Bus E is constant, which equals to the cost of the local unit Brighton, because this unit is always a marginal one when Bus B Load is between 300 to 330 MW. Thus, any load change at Bus E will be solely provided by Brighton, and the sensitivity of LMP at Bus E is zero. This is also the case for Bus D because the local unit Sundance is also a marginal one.

For nonmarginal-unit buses (A, B and C in this study), the LMPs increase linearly as the load increases. Since the loss is a quadratic function of the load, the generation is a quadratic function of the load as well. If there is no change of marginal units (i.e., due to very small change of load), the dispatch cost is quadratically related to the load. The LMP, defined as incremental cost over incremental load, should be a linear function of load, as shown in Fig. 11. Therefore, the LMP sensitivity at a bus without any marginal unit should be a non-zero constant, if there is no change of marginal unit.

B. Sensitivity When There is a Change of Marginal Unit(s)

Fig. 12 shows the normalized LMP sensitivity when load at Bus B is varied in a wider range from 300 to 390 MW. Again, other loads remain unchanged for simplicity. Fig. 12 shows a stiff change of LMP sensitivity, when Bus B Load increases from 346.50 to 347.25 MWh (the simulation step size is 0.75 MWh). The stiff change is due to the change of marginal units from Brighton and Sundance to Solitude and Sundance, which changes the nodal LMP prices significantly.

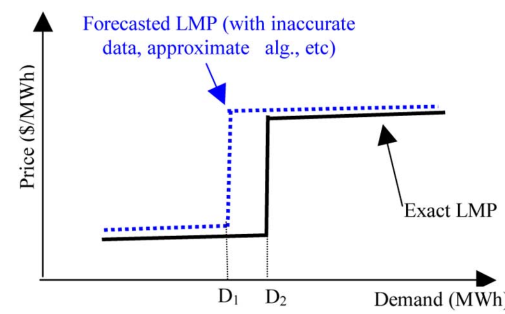

Fig. 13. Forecasted LMP and exact LMP.

The step change pattern has implications for the applicability of LMP sensitivities. If some of the present marginal units are near their generation limits or some transmission lines are nearly congested, the LMP sensitivities calculated in the present operating point are less reliable for calculating future LMP at a different load level because a step change of LMP may occur even with a small load growth.

It should be noted that this step change or infinite sensitivity is not caused by the dc model. Even with the ac model, the step change still occurs due to the dependence of LMPs upon the source of marginal generation. If the load increases to a level that causes congestion to occur at a new location, then there will be a new marginal unit that leads to a step change in price. This has important implications for the present LMP methodology: there is a significant uncertainty or risk in LMP forecasting due to inaccurate data or approximate LMP algorithms. This is illustrated in Fig. 13 in which the LMP error is relatively significant when the load is between D_1 and D_2. Hence, all approximate LMP algorithms cannot completely eliminate the relatively considerable error in a range of load levels in which a step change in LMPs occurs. However, a better approximation algorithm shall be able to narrow the range of LMP errors, as demonstrated by the proposed FND-based DCOPF as opposed to the lossless DCOPF.

C. Discussion of LMP Sensitivity

Reference [17] presents a generalized, ACOPF-based model for LMP sensitivity w.r.t. load and other variables. A matrix formulation needs to be solved to calculate LMP sensitivity eventually; therefore, there is no direct, explicit expression available from [17] for LMP sensitivity to load. This paper does not intend to supplant the work in [17]; instead, this research work does present an explicit formulation, (30), of LMP sensitivity to load, based on DCOPF with Delivery Factor, which is neither applicable nor necessary to the ACOPF model. Hence, with the concept of Delivery Factor, the LMP sensitivity to load in dc model is straightforward and simple by easily providing an overview of LMP sensitivity. This is reasonable considering the simplifications of the dc model. The observed results match the analytical (30) and clearly show that the LMP sensitivity is related to the loss component, is linear in the sensitivity of delivery factors, and is a constant numerically. In addition, an important observation is that the LMP sensitivity is numerically small if there is no new marginal unit, while it is infinite if there is a change of marginal unit(s).

VII. Concluding Discussions

The proposed FND algorithm may be further simplified by executing only the first two iterations. Basically, the first iteration is essentially a lossless DCOPF run to give an estimation of delivery factors, FND, and system loss. Then, another DCOPF is performed based on the estimation. The reason for this simplification is that this research found that an initial estimation of delivery factors and losses have a bigger impact of LMP and dispatch than later iterations to refine delivery factors, FND, and system loss. This can reduce the computational effort since it does not require the algorithm to run till convergence. Therefore, it fits a simulation or planning purpose well if the accuracy is reasonably acceptable. The tests on the PJM 5-bus system and IEEE 30-bus system show that the two-iteration simplification of the FND algorithm produces results very close (less than 4% error in the maximum difference of nodal LMPs) to the fully converged FND algorithm. Nevertheless, this is a heuristic observation and needs further research to be credibly applied to much larger, real systems.

The proposed FND-based iterative DCOPF should be applicable to security (contingency) constrained optimal power flow, that is, SCOPF or CCOPF, because there is no mathematical difference between SCOPF and OPF, despite more computational complexity. In general, additional arrays of GSFs under contingency scenarios may be added to model contingency constraints. The security limit can be modeled similar to the line limits presented in (25).

In summary, this paper first presents loss factors and delivery factors based on GSF. The offset of system loss in the energy balance equality constraint is rigorously proved. Then, the challenge of a considerable nodal mismatch at the reference bus is presented. The mismatch issue is tackled with the proposed FND model, in which the total loss is distributed to each *individual* line and there is no nodal mismatch.

This paper also presents a comparison of LMP results from the lossless DCOPF, the FND-based DCOPF, and the ACOPF algorithms. The results indicate that FND-based iterative DCOPF gives better results than lossless DCOPF and represents a better approximation of ACOPF LMPs.

In addition, this paper presents a simple and explicit formulation of LMP sensitivity with respect to load based on the FND algorithm. Without the loss component, the LMP sensitivity is zero if load is varied in a small range. The LMP sensitivity may be infinite (i.e., a step change in LMP) when the load grows to a critical level to lead to a new marginal unit. This step-change nature presents uncertainty and risk in LMP forecast, especially considering the possible data inaccuracy or algorithm approximation. Therefore, future research could explore approaches for smoothing out these step changes using penalty or rebate functions on constraints, and evaluate whether such approaches would ease forecasting of prices while preserving correct economic signals.

Appendix

Proof: If G_i converges after the $(l+1)$th iteration, F_k, E_i, DF_i, *and* P_{loss} all converge for the FND-based DCOPF algorithm.

Solution: G_i converges $\Rightarrow \left|G_i^{l+1} - G_i^l\right| < \varepsilon$ for all G_i.

1) Since line loss in an individual line is a small portion of the line flow, we have

$$F_k = \sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i - E_i)$$
$$= (1 + a_k)\left[\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i)\right]$$

where $a_k =$ the ratio of line loss to line flow of the kth line. Typically, a_k is a small positive number less than 10%

$$\because D_i^{l+1} = D_i^l = D_i$$
$$\therefore \left|F_k^{l+1} - F_k^l\right| = \left|(1 + a_k) \times \sum_{i=1}^{N}\left(GSF_{k-i} \times \left(G_i^{l+1} - G_i^l\right)\right)\right|$$
$$\therefore \left|F_k^{l+1} - F_k^l\right| < (1 + a_k) \times N \times \max\{|GSF_{k-i}|\} \times \varepsilon = b_k \times \varepsilon$$

where $b_k = (1 + a_k) \times N \times \max\{|GSF_{k-i}|\}$.

2) E_i is the FND, thus we have

$$G_i + F_{i\Sigma} - D_i - E_i = 0 \text{ or } E_i = G_i + F_{i\Sigma} - D_i$$

where $F_{i\Sigma} =$ all line flows injecting into Bus i.

Since $\left|G_i^{l+1} - G_i^l\right| < \varepsilon$, $\left|F_k^{l+1} - F_k^l\right| < b_k \times \varepsilon$, and $D_i^{l+1} = D_i^l = D_i$, we have

$$\left|E_i^{l+1} - E_i^l\right| < \varepsilon + M_i \times \max\{b_k\} \times \varepsilon$$
$$= (1 + M_i \times b) \times \varepsilon$$

where $M_i =$ number of lines connected to Bus i; $b =$ the maximum b_k of all lines connected to Bus i.

3)
$$\because \left|DF_i^{l+1} - DF_i^l\right| = \left|\sum_{k=1}^{M}\left(2 \times R_k \times GSF_{k-i} \times (F_k^{l+1} - F_k^l)\right)\right|$$
$$\therefore \left|DF_i^{l+1} - DF_i^l\right| < 2M \times \max\{|R_k \times GSF_{k-i} \times b_k|\} \times \varepsilon = c_i \times \varepsilon$$

where $c_i = 2M \times \max\{|R_k \times GSF_{k-i} \times b_k|\}$.

4)
$$\because P_{loss} = \sum_{i=1}^{N}(G_i - D_i)$$
$$\therefore \left|P_{loss}^{l+1} - P_{loss}^l\right| = \left|\sum_{i=1}^{N}\left(G_i^{l+1} - G_i^l\right)\right| < N \times \varepsilon.$$

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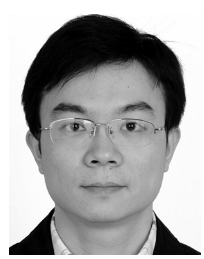

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