

Decentralized Congestion Management for Multilateral Transactions Based on Optimal Resource Allocation

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Abstract—This paper proposes a decentralized model for dc load flow based congestion management for the forward markets via Optimal Resource Allocation (ORA). The available thermal capacities of possible congested transmission lines are considered as commonly shared resources for all bilateral and multilateral transactions in the market. In our model, each transaction maximizes its profit under the limits of transmission line capacities allocated by the ISO. The ISO searches the optimal allocation of line capacities to each transaction. Finally, the same market efficiency (social welfare) as centralized optimization can be reached. The ORA-based decentralized approach is more suitable for power market environment, since it always keeps the intermediate solution feasible during the iterations and does not require each transaction to submit their private and sensitive information. The mathematical model, computation procedure and relevant proof are presented. The computer test results from the IEEE 30 bus system illustrate the effectiveness of proposed approach.

Index Terms—Congestion management, decentralized optimization, optimal resource allocation, power market.

I. INTRODUCTION

ELECTRICITY markets have been developing rapidly in many parts of the world. However, the design of perfect power markets is still under investigation for various reasons. One of the most significant features of electricity market is that the commodity of electricity should be transferred through the network and the transmission line capacity limits should be considered at all times. The complicated issues of congestion management arise, which is relevant to both system operation security and market economic efficiency [1].

Mathematically, congestion management problem can be generally described as a centralized optimal power flow (COPF) problem with the objective function of maximizing social welfare and the constraints of load flow equations and operation

limitations. However, the COPF approach has some apparent drawbacks in market environment. First, COPF requires the submission of detailed private information of market participants to the ISO that may include their cost/benefit functions. In the market environment, such sensitive information is a commercial secret that market participants are unwilling to disclose to the ISO. Secondly, the COPF by the ISO will be a “black box” that lacks transparency to market participants, since the congestion price is more likely “set” by the ISO, but not “discovered” through market mechanisms. As a centralized authority, the ISO has “super power,” which is inconsistent with the principles of competitive markets. Moreover, for the congestion management of inter-regional trades, which can share the common resources (transmission lines, generation units, etc.) across regions efficiently and increase the scale of economy (e.g., in [2] and [3]), the decentralized optimal power flow (DOPF) approach is becoming more attractive for its possibility of using one agent for coordination and even without the existence of a real ISO for the entire system. Based on the above discussion, it is desirable to develop tools to address the issues of decentralized congestion management without sacrificing market efficiency.

In the study of decentralized congestion management, the decomposition techniques of a global OPF, which solve the problem iteratively by necessary information exchange and coordination strategies, are very attractive. In [2], the coordinated multilateral trade is proposed which separates the economy and reliability issues of system operation. It proves that the coordination result can achieve the maximal social welfare. In [5], the brokers of forward market arrange their contracts under the price signals of the congested paths updated by the public coordinator. This coordination process can converge to the optimal solution of original COPF. In [6], the Lagrange Relaxation (LR) decomposition technique is applied with the coordination of power exchanges of regional markets, and three algorithms for updating the Lagrange multipliers are compared. In [7] and [8], the concept of “dummy bus” is applied to separate the decoupled constraints by auxiliary-problem principle (APP) in [9], and hence to decompose global OPF by areas. The distributed optimization software in [10] based on the APP technique can calculate distributed optimal power flow with arbitrary division of the system. The decentralized dc load flow problem is converted to an optimization problem in [11], which targets at minimizing the difference between the variables of the decoupled constraints on borders and is solved by the APP technique. The calculation of decentralized power transfer distribution factors (PTDFs) and available transfer

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capacity (ATC) are also proposed. Until now, the most conventional decentralized approaches can be viewed as price-based methods. The coordinated signals are primarily relevant to Lagrange multiplier, the shadow price of congested paths in the congestion management problem. During the iterations, if any Lagrange multiplier is less than the real value in optimal solution, the infeasible intermediate solution appears and it is desired to be avoided.

This paper proposes the model of decentralized congestion management based on the new direction of the Optimal Resource Allocation (ORA) [12] of the transmission line capacity. The relevant concept was once applied to power markets in [13]. In this paper, we provide a more complete ORA-based model with relevant proof and propose two algorithms to solve the problem for the decentralized congestion management of forward markets.

The paper is organized as follows. In Section II, the mathematical model of COPF for the congestion management is converted to the decentralized model based on the ORA problem for transmission line capacity. The two suggested algorithms are presented in Section III. In Section IV, the test results on IEEE 30 bus system clearly show that the suggested approaches are effective. The proof for equivalence of proposed ORA-based decentralized model and COPF model is given in the Appendix.

II. MATHEMATICAL FORMULATION

A. Outline of COPF Model for Congestion Management in Forward Markets

We mainly focus on forward contract market for real power, where dc load flow is popular to use. The PTDFs are available to the market participants (transactions). In the perfect competitive market, the ISO adjusts the contracts to maximize the social welfare to achieve efficient operation with all constraints satisfied. This COPF is given by

$$\max_{\mathbf{x}, \mathbf{y}} \sum_{k \in T} \sum_{j \in D^{(k)}} B_j^{(k)}(y_j^{(k)}) - \sum_{k \in T} \sum_{i \in G^{(k)}} C_i^{(k)}(x_i^{(k)}) \quad (1)$$

$$\text{s.t. : } \sum_{j \in D^{(k)}} y_j^{(k)} - \sum_{i \in G^{(k)}} x_i^{(k)} = 0 \quad (k \in T) \quad (2)$$

$$x_{i, \min}^{(k)} \leq x_i^{(k)} \leq x_{i, \max}^{(k)} \quad (i \in G^{(k)}, k \in T) \quad (3)$$

$$y_{j, \min}^{(k)} \leq y_j^{(k)} \leq y_{j, \max}^{(k)} \quad (j \in D^{(k)}, k \in T) \quad (4)$$

$$\sum_{k \in T} g_k^{(m)}(\mathbf{x}_k, \mathbf{y}_k) \leq P_{\max}^{(m)} \quad (m \in I) \quad (5)$$

where

- T set of transactions in the market
 $T = \{1, 2, \dots, K\}$;
- k index of each transaction, for all $k \in T$;
- I set of transmission lines involved in congestion management $I = \{1, 2, \dots, M\}$;
- m index of transmission lines involved in congestion management, for all $m \in I$;

- $G^{(k)}$ set of generators in transaction k , for all $k \in T$;
- $D^{(k)}$ set of consumers in transaction k , for all $k \in T$;
- $x_i^{(k)}$ output of generator i of transaction k , also an element of generator output vector \mathbf{x}_k of transaction k , for all $i \in G^{(k)}$ and $k \in T$;
- $y_j^{(k)}$ demand of consumer j of transaction k , an element of demand vector \mathbf{y}_k of transaction k , for all $j \in D^{(k)}$ and $k \in T$;
- $C_i^{(k)}$ cost function of generator i of transaction k , for all $i \in G^{(k)}$ and $k \in T$;
- $B_j^{(k)}$ benefit function of consumer j of transaction k , for all $j \in D^{(k)}$ and $k \in T$;
- $g_k^{(m)}$ Load flow caused by transaction k on line m , in which PTDFs are used, for all $k \in T$ and $m \in I$;
- $P_{\max}^{(m)}$ transfer limit of line m in MW, for all $m \in I$.

The objective in (1) is to maximize total social welfare. The equality constraints in (2) denote the power balance for each transaction. The inequality constraints in (3) [or (4)] denote the output (or input) limits of generators (or consumers). Inequality constraints in (5) denote the line capacity limits. For simplicity the transmission lines is supposed to congest only in one direction. However, there is no difficulty to consider the limits for both directions by adding the opposite direction line capacity constraints into (5).

B. Mathematical Model for ORA-Based Decentralized Approach

The COPF model (1)–(5) is converted into ORA-based DOPF model below. Firstly we rewrite (1)–(5) to a simpler form. The decision variables of contract k are defined as

$$\mathbf{u}_k^T \stackrel{\text{def.}}{=} (\mathbf{x}_k^T, \mathbf{y}_k^T) \quad (k \in T).$$

The social welfare related to transaction k is rewritten as

$$W_k(\mathbf{u}_k) \stackrel{\text{def.}}{=} \sum_{j \in D^{(k)}} B_j^{(k)}(y_j^{(k)}) - \sum_{i \in G^{(k)}} C_i^{(k)}(x_i^{(k)}) \quad (k \in T).$$

Equations (2)–(4) are local constraints of individual contracts that are not interactive among transactions. For transaction k , its local constraints are denoted as the feasible domain Dom_k . The original COPF model is written to a simpler form as

$$\begin{aligned} & \max \sum_{k \in T} W_k(\mathbf{u}_k) \\ & \text{s.t. : } \sum_{k \in T} g_k^{(m)}(\mathbf{u}_k) \leq P_{\max}^{(m)} \quad (m \in I) \\ & \quad \text{– global constraints} \\ & \quad \mathbf{u}_k \in \text{Dom}_k \quad (k \in T) \text{ – local constraints.} \end{aligned} \quad (6)$$

In the ORA-based method, the ISO tries to optimally allocate the capacity resource of M lines to K transactions. We denote that the ISO allocates the capacity of line m to transaction k

by the amount of $P_{k,\max}^{(m)}$, and define the Resource Allocation Weighing Matrix (RAWM) as α , an $M \times K$ matrix with its m th row and k th column element $\alpha_k^{(m)}$ denoting the proportion of the capacity of line m allocated to the transaction k , then

$$\begin{aligned} P_{k,\max}^{(m)} &= P_{\max}^{(m)} \alpha_k^{(m)} \quad (m \in I, k \in T) \quad (7) \\ \sum_{k \in T} \alpha_k^{(m)} &= 1 \quad (m \in I). \quad (8) \end{aligned}$$

It is noticed that the index $\alpha_k^{(m)}$ in (8) can be negative caused by counter flow. By introducing α , the COPF problem in (6) can be rewritten as

$$\begin{aligned} &\max \sum_{k \in T} W_k(\mathbf{u}_k) \\ \text{s.t. : } &g_k^{(m)}(\mathbf{u}_k) \leq P_{\max}^{(m)} \alpha_k^{(m)} \quad (k \in T, m \in I) \\ &\sum_{k \in T} \alpha_k^{(m)} = 1 \quad (m \in I) \\ &\mathbf{u}_k \in \text{Dom}_k \quad (k \in T). \quad (9) \end{aligned}$$

The problem in (9) can be decomposed into K independent sub-OPF problems corresponding to K transactions under the fixed α assigned by ISO. Thus, (9) is redefined as the optimization problem with decision variables of α and the objective $\theta(\alpha)$ is

$$\theta(\alpha) \stackrel{\text{def.}}{=} \max_{\mathbf{u}} \left\{ \sum_{k \in T} W_k(\mathbf{u}_k) \left| \begin{array}{l} g_k^{(m)}(\mathbf{u}_k) \leq P_{\max}^{(m)} \alpha_k^{(m)} \quad (k \in T, m \in I) \\ \mathbf{u}_k \in \text{Dom}_k \quad (k \in T) \end{array} \right. \right\} \quad (10)$$

which is equivalent to

$$\begin{aligned} \theta(\alpha) &= \sum_{k=1}^K \theta_k(\alpha) \\ \theta_k(\alpha) &\stackrel{\text{def.}}{=} \max_{\mathbf{u}_k} \left\{ W_k(\mathbf{u}_k) \left| \begin{array}{l} g_k^{(m)}(\mathbf{u}_k) \leq P_{\max}^{(m)} \alpha_k^{(m)} \quad (m \in I) \\ \mathbf{u}_k \in \text{Dom}_k \end{array} \right. \right\}. \quad (11) \end{aligned}$$

By (8), the feasible domain A of α is

$$A : \alpha \in A = \left\{ \alpha \left| \sum_{k \in T} \alpha_k^{(m)} = 1 \quad (m \in I) \right. \right\}. \quad (12)$$

The feasible domain A can be further decomposed into M independent subdomains $A^{(m)}$ based on (8), and we have

$$\begin{aligned} A &= A^{(1)} \times \dots \times A^{(m)} \times \dots \times A^{(M)} \\ A^{(m)} : \alpha^{(m)} \in A^{(m)} &= \left\{ \alpha^{(m)} \left| \sum_{k \in T} \alpha_k^{(m)} = 1 \right. \right\}. \quad (13) \end{aligned}$$

Based on (6) and (11), the master α -opt. problem for ISO based on ORA model should be

$$\max_{\alpha} \left\{ \theta(\alpha) = \sum_{k \in T} \theta_k(\alpha) \mid \alpha \in A \right\}. \quad (14)$$

To solve the ORA problem for ISO in (14), we try to derive its optimality condition. From (8), the subdomain $A^{(m)}$ is a hyperplane. According to Karush Kuhn Tucker (KKT) necessary conditions, the optimal solution of (14) satisfies

$$\frac{\partial(\theta(\alpha))}{\partial \alpha_1^{(m)}} = \dots = \frac{\partial(\theta(\alpha))}{\partial \alpha_k^{(m)}} = \dots = \frac{\partial(\theta(\alpha))}{\partial \alpha_K^{(m)}} \quad (m \in I). \quad (15)$$

The explicit expression of $\theta(\alpha)$ is very difficult to derive. However, we can derive its first-order derivative (the sensitive information) $\partial(\theta(\alpha))/\partial \alpha_k^{(m)}$ can be derived according to [15]

$$\begin{aligned} \frac{\partial(\theta(\alpha))}{\partial \alpha_k^{(m)}} &= \frac{\partial(\theta_k(\alpha))}{\partial \alpha_k^{(m)}} \\ \frac{\partial \theta_k(\alpha)}{\partial P_{k,\max}^{(m)}} &= \lambda_k^{(m)}, \quad \frac{\partial \theta_k(\alpha)}{\partial \alpha_k^{(m)}} = P_{\max}^{(m)} \lambda_k^{(m)} \quad (k \in T, m \in I). \quad (16) \end{aligned}$$

In (16), $\lambda_k^{(m)}$ is the corresponding Lagrange multiplier of line m limits in the sub-OPF problem of transaction k in (11). The strict mathematical derivation is clearly given in [14, Ch. 12]. Understood from economic theory, (16) holds intuitively since Lagrange multipliers (shadow prices) denote the change rate of objective with respect to the constrained limited resource in optimization problem. If $\lambda_k^{(m)}$ is zero, the corresponding transfer limits is inactive. Meanwhile, adding a tiny capacity resource of line m cannot cause any increment of profit for transaction k (also for the social welfare); the change is zero. Thus, formula (16) holds according to whether or not $\lambda_k^{(m)}$ is zero. And the gradient matrix $\nabla \theta(\alpha)$ can be written as

$$\begin{aligned} \nabla \theta(\alpha) &= \begin{bmatrix} \frac{\partial \theta(\alpha)}{\partial \alpha_1^{(1)}} & \dots & \frac{\partial \theta(\alpha)}{\partial \alpha_K^{(1)}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \theta(\alpha)}{\partial \alpha_1^{(M)}} & \dots & \frac{\partial \theta(\alpha)}{\partial \alpha_K^{(M)}} \end{bmatrix} = \begin{bmatrix} (\frac{\partial \theta(\alpha)}{\partial \alpha^{(1)}})^T \\ \vdots \\ (\frac{\partial \theta(\alpha)}{\partial \alpha^{(M)}})^T \end{bmatrix} \\ &= \begin{bmatrix} \lambda_1^{(1)} P_{\max}^{(1)} & \dots & \lambda_K^{(1)} P_{\max}^{(1)} \\ \vdots & \ddots & \vdots \\ \lambda_1^{(M)} P_{\max}^{(M)} & \dots & \lambda_K^{(M)} P_{\max}^{(M)} \end{bmatrix} \\ &= \begin{bmatrix} P_{\max}^{(1)} & \dots & P_{\max}^{(M)} \end{bmatrix} \begin{bmatrix} \lambda_k^{(m)} \end{bmatrix}_{M \times K}. \quad (17) \end{aligned}$$

Equation (15) becomes

$$\lambda_1^{(m)} = \dots = \lambda_k^{(m)} = \dots = \lambda_K^{(m)} \quad (m \in I). \quad (18)$$

Equations (18) and (8) are the optimality necessary condition of (6) for ISO. The optimality necessary condition of master α -opt. problem by ISO and the KKT necessary conditions of transactions are equivalent to the KKT necessary conditions of COPF. The proof is given in the Appendix. Generally, from the economic theory, if maximal social welfare is reached and no further resource reallocation is necessary, each line has the same economic efficiency for all transactions.

Equations (11), (12), and (14) form the ORA-based congestion management model, with its optimality condition for ISO in (18) and (6). The master α -opt. problem by ISO and slave (sub) \mathbf{u}_k -opt. problem by each transaction will be solved iteratively to search the right α^* yielding \mathbf{u}_k ($k \in T$) in transactions

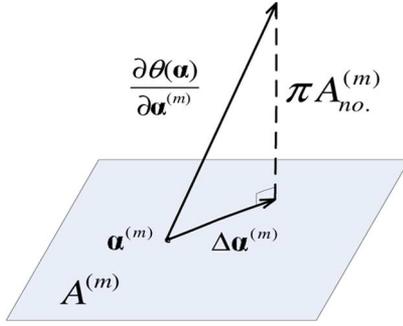


Fig. 1. Relation of $\partial\theta(\alpha)/\partial\alpha^{(m)}$, $\Delta\alpha^{(m)}$ and $A^{(m)}$.

equal to the solution of COPF. The detailed solution method and procedure will be given in next section.

III. RAWM UPDATING ALGORITHM FOR α - OPT. AND OVERALL SOLUTION PROCEDURE

A. Gradient-Projection Method (GP)

In the master α - opt. problem by ISO, the explicit expression of $\theta(\alpha)$ in (10) cannot be attained and only the sensitive information matrix $\nabla\theta(\alpha)$ and feasible domain A are available. The gradient projection approach will be proposed to solve such problems. The idea of the gradient-projection method is to project $\nabla\theta(\alpha)$ to its feasible domain A to get feasible α - opt. search direction $\Delta\alpha$ to realize $\theta(\alpha) \rightarrow \max$. During each iteration, $\nabla\theta(\alpha)$ is calculated according to (16) with $[\lambda_k^{*m}]_{M \times K}$ submitted from each transaction $k \in T$, after its \mathbf{u}_k - opt. problem (11) is solved $\Delta\alpha$ is denoted as

$$\Delta\alpha^T = [\Delta\alpha^{(1)}, \dots, \Delta\alpha^{(m)}, \dots, \Delta\alpha^{(M)}]$$

where $\Delta\alpha^{(m)} \in R^K$ ($m \in I$) is subsearch direction vector of line m . The relation of vectors $\partial\theta(\alpha)/\partial\alpha^{(m)}$, $\Delta\alpha^{(m)}$ and domain $A^{(m)}$ is shown in Fig. 1, where $\Delta\alpha^{(m)}$ is computed by the projecting $\partial\theta(\alpha)/\partial\alpha^{(m)}$ into hyperplane $A^{(m)}$.

The unit normal vector of $A^{(m)}$ is denoted as $A_{no}^{(m)}$. The relation in Fig. 1 is given by (π is length coefficient)

$$\frac{\partial\theta(\alpha)}{\partial\alpha^{(m)}} - \Delta\alpha^{(m)} = \pi A_{no}^{(m)}. \quad (19a)$$

$$(\Delta\alpha^{(m)} + \alpha^{(m)}) \in A^{(m)}. \quad (19b)$$

In the case of an optimal solution of (14), $\partial\theta(\alpha)/\partial\alpha^{(m)}$ parallels $A_{no}^{(m)}$ (which is $\partial\theta(\alpha)/\partial\alpha^{(m)} = \pi A_{no}^{(m)}$) and projection vector $\Delta\alpha^{(m)}$ is zero for all $m \in I$. The ‘‘parallel’’ means that all elements of vector $\partial\theta(\alpha)/\partial\alpha^{(m)}$ are the same, since $A_{no}^{(m)}$ is vector $[1]_{K \times 1}$ from (8) and (12). This analysis is consistent to optimality condition (15) and (18). To solve $\Delta\alpha^{(m)}$, there are

K equations from (19a) and one implicit ($\sum_{i=1}^K \Delta\alpha_i^{(m)} = 0$) from (19b) available and given by

$$\begin{bmatrix} 1 & 0 & \dots & 0 & 1 \\ 0 & 1 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ 0 & & & 1 & 1 \\ 1 & \dots & \dots & 1 & 0 \end{bmatrix} \times \begin{bmatrix} \Delta\alpha^{(m)} \\ \pi \end{bmatrix} = \begin{bmatrix} \lambda_1^{(m)} \\ \vdots \\ \vdots \\ \lambda_K^{(m)} \end{bmatrix} P_{\max}^{(m)}, \quad (m \in I). \quad (20)$$

With (20), $\Delta\alpha$ is attained from the independent calculation of $\Delta\alpha^{(m)}$ ($m \in I$). The coefficient matrix in (20) is fixed and sparse, making the computation of (20) very fast. The RAWM α can be updated in the direction of $\Delta\alpha$ with suitable step-length s

$$\alpha^{\text{new}} = \alpha^{\text{old}} + s\Delta\alpha. \quad (21)$$

The α^{new} will be forwarded to each transaction again to conduct the slave \mathbf{u}_k - opt. problems and new $[\lambda_k^{*m}]_{M \times K}$ will be feedback to the ISO, forming one round of iteration. The coordination of master-slave optimization process will stop if the condition (18) can be satisfied. This gradient projection method has the first-order convergence quality.

B. Trust Region and Quasi-Newton Method (TRQN)

The well-known trust region and BFGS quasi-Newton (TRQN) optimization methods in 0 are implemented here as an alternative way to solve the problem at better convergence around the optimum, which is outlined below. For round n with known ${}^n\alpha$, the trust-region problem for ISO in (14) is

$$\begin{aligned} \max_{\Delta\alpha} \theta({}^{n+1}\alpha) &= \theta({}^n\alpha + \Delta\alpha) \\ &= \nabla\theta({}^n\alpha)\Delta\alpha + 0.5(\Delta\alpha)^T \nabla^2\theta({}^n\alpha)\Delta\alpha \\ \text{s.t. : } {}^{n+1}\alpha &= \Delta\alpha + {}^n\alpha \in A \\ \|\Delta\alpha\| &\leq r \end{aligned} \quad (22)$$

where the objective is the local quadratic approximation of $\theta({}^{n+1}\alpha)$. The optimum of $\theta({}^{n+1}\alpha)$ is searched in the region with radius r . The local quadratic model requires the Hessian $\nabla^2\theta(\alpha)$. Since the exact Hessian is unknown, a positive definite approximation of $\nabla^2\theta(\alpha)$ is computed by the BFGS quasi-Newton algorithm as follows:

$$\begin{aligned} s_n &= {}^n\alpha - {}^{n-1}\alpha \\ y_n &= \nabla\theta({}^n\alpha) - \nabla\theta({}^{n-1}\alpha) \\ \nabla^2\theta({}^{n+1}\alpha) &= \nabla^2\theta({}^n\alpha) \\ &\quad - \frac{\nabla^2\theta({}^n\alpha)s_n s_n^T \nabla^2\theta({}^n\alpha)}{s_n^T \nabla^2\theta({}^n\alpha)s_n} + \frac{y_n y_n^T}{y_n^T s_n}. \end{aligned} \quad (23)$$

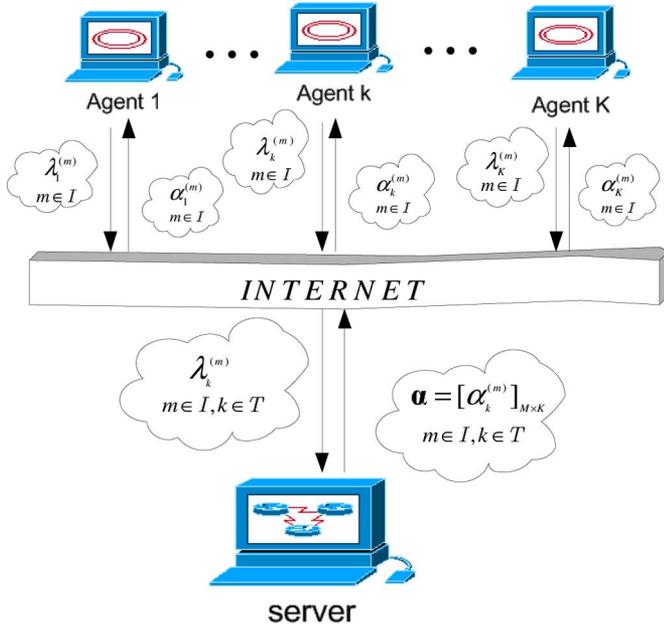


Fig. 2. Infrastructure of ORA-based decentralized congestion management by multi-agent system.

The second-order Hessian in (23) is based on gradient and Hessian information in the previous iterations. It provides more information of the problem to improve the convergence rate. The computation burden of (22) in the master α - opt. problem by ISO is greater than (20). However, it takes less iteration to converge. We will discuss this point in next section.

C. Scheme of Decentralized Congestion Management

Based on the mathematical model and proposed RAWM updating algorithms, the general scheme for ORA-based decentralized congestion management for forward market is as follows.

Step 0: The ISO determines the possible congested transmission lines and give the initial RWAM α . If inexperienced, the ISO can uniformly assign line capacity to each transaction, i.e., $\alpha_k^{(m)} = 1/K (m \in I)$.

Step 1: ISO sends resource allocation signal $\alpha_k^{(m)}$ ($m \in I, k \in T$) to the corresponding transaction.

Step 2: Each transaction solves \mathbf{u}_k for (11) to pursue the maximal profit under limits $\alpha_k^{(m)}$ ($m \in I$) allocated by ISO, and then sends its $\lambda_k^{(m)}$ ($m \in I$) back to the ISO.

Step 3: The ISO check whether the received Lagrange multipliers from each transaction can be close enough for all lines according to (18). If not, ISO updates α by GP method in (20) or TRQN method in (22) and (23), and go to step 2.

Nowadays, the implementation of coordinated algorithm suggested above is possible with the fast development of information technology, such as World-Wide-Web and multi-agent technology in 0. A schematic diagram (see Fig. 2) shows the brief implementation of the algorithm through a multi-agent system (MAS). In the MAS, agent 1 to K will solve (11) on transactions participants' behalves, while the server will solve the ISO resource allocation problem with necessary information exchanged over the Internet.

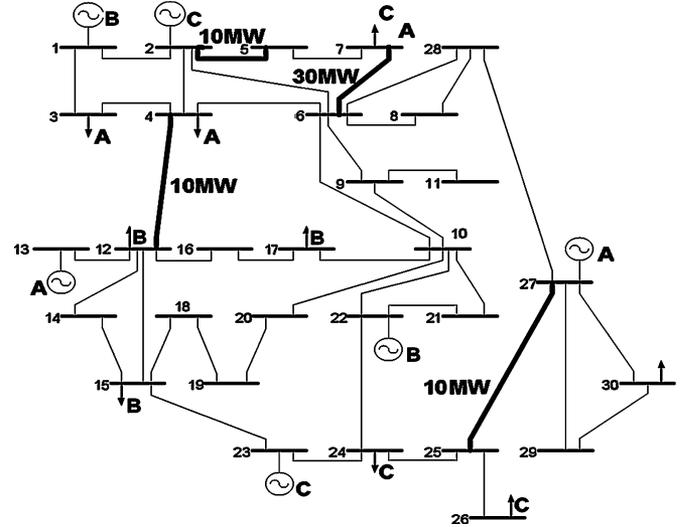


Fig. 3. IEEE 30-bus system.

TABLE I
COST AND BENEFIT FUNCTION OF TEST CASE

	Gen. bus	Marginal cost (10\$/MWh)	Con. Bus	Marginal benefit (10\$/MWh)
A	13	$2+0.02q$	3	$3.88-0.03q$
	27	$1.75+0.0175q$	4	$3.8-0.05q$
			7	$3.8-0.02q$
B	1	$3+0.025q$	12	$3.8-0.04q$
	22	$3+0.025q$	15	$3.95-0.02q$
			17	$3.7-0.02q$
C	23	$2.8+0.0625q$	24	$3.6-0.03q$
	2	$2.3+0.0283q$	26	$3.8-0.04q$
			7	$3.7-0.02q$

TABLE II
RESULTS OF COPF

		Max	A	B	C	Sum	Lagrange multiplier
Capacity used (MW)	Line 2-5	10	2.35	1.85	5.80	10.00	1.68
	Line 6-7	30	19.00	-1.85	8.78	25.93	0
	Line 4-12	10	26.82	-15.38	-1.44	10.00	0.82
	Line 25-27	10	7.84	0.75	1.41	10.00	2.07
Trans. profit (M\$)			86.15	15.70	21.05	122.90	

*Only lines 2-5, 4-12 and 25-27 are congested, and line 6-7 is not congested.

IV. TEST RESULT

The IEEE 30-bus test system (see Fig. 3) is used for testing, which is conducted by one computer with series computation. We consider the case that there are three multilateral transactions (A, B, C) and each has five participants of two generators and three consumers with their marginal cost function of generators and marginal benefit function of consumers listed in Table I. In total, four transmission lines are involved in the congestion management and shown as bold lines in Fig. 2.

Table II shows the results of COPF with negative values for counter flow. Lines 2-5, 4-12, and 25-27 are congested and the corresponding Lagrange Multipliers are greater than 0, and Line 4-12 will be further analyzed to show the validity of the suggested methods.

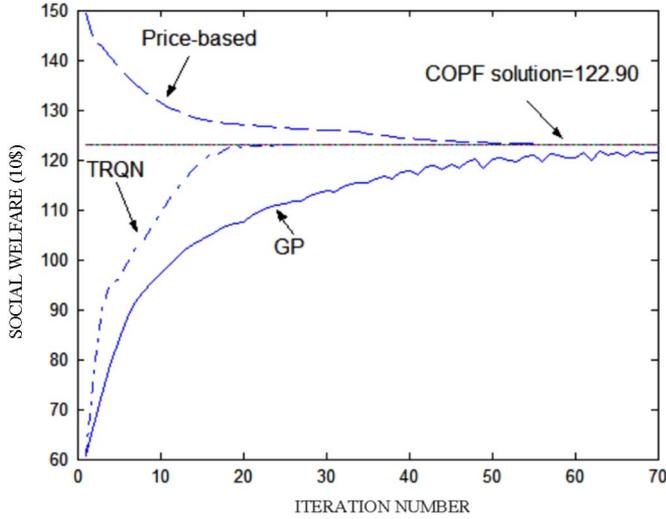


Fig. 4. Social welfare during the iterations.

TABLE III
COMPARISON OF CPU TIME BY GP AND TRQN METHODS

Methods	Total CPU time (series compute)	Average ISO CPU time.	Optimal obj. value (%COPF[122.90])
GP (n=25)	4.82s	10 ⁻⁶ s	121.68 (99.01%)
TRQN (n=70)	3.76s	0.1086s	122.89 (99.99%)

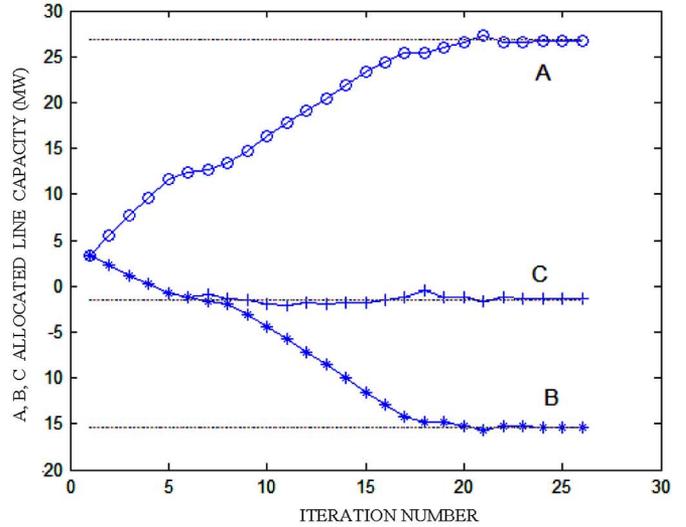
In Fig. 4, the dynamic process of social welfare during iteration by different decentralized approaches is compared with the COPF solution. The price-based method 0, 0 converges to the solution of COPF from the infeasible domain, since it underestimates the initial congested price. The two suggested ORA-based methods reach the COPF solution effectively with their intermediate solutions keeping feasible. Thus, the ORA-based method has the advantage over price-based method for that it can stop at any iteration with a feasible overall solution.

Table III compares the two suggested α -opt. methods. Since series computation is used, there is much room for reducing the total CPU time when parallel computation is used for individual transaction. In Table III, it can be seen that TRQN method takes fewer rounds and lower overall CPU time to converge to the optimal solution than GP method does, but at the cost of longer CPU time of ISO.

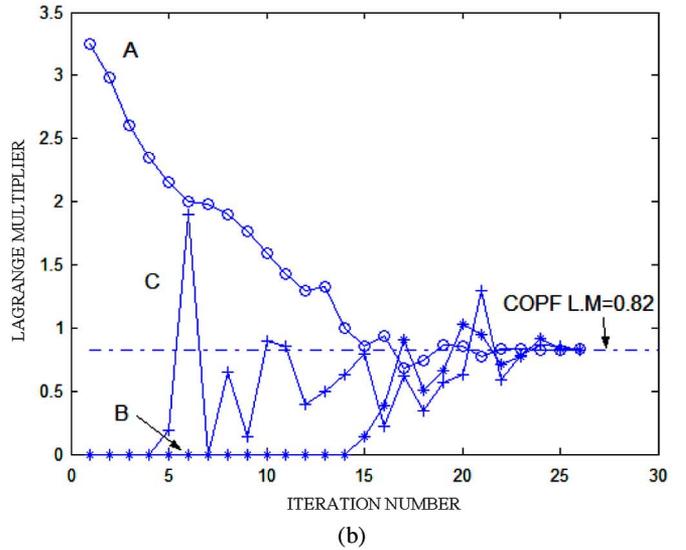
Fig. 5(a) shows the dynamic process of allocated capacities of congested line 4–12 to transactions A, B, C by the TRQN method. The allocated capacity of line 4–12 for transaction A is increasing; while for transactions B and C, it decreases and converges to negative value as counter flow. Jointly, three transactions make full and optimal utilization of the capacity of line 4–12 to yield maximal social welfare. Fig. 5(b) shows dynamic process of Lagrange multipliers of transactions A, B, C for line 4–12. They tend to be identical at the final optimization solution and satisfy (18). Shown in Fig. 4, both allocated capacities and Lagrange multipliers tend to converge to that in COPF results.

V. CONCLUSION

A novel ORA-based decentralized approach is proposed for congestion management in the electricity forward market. The capacity of possible congested transmission lines is optimally



(a)



(b)

Fig. 5. (a) Allocated capacities and (b) Lagrange multipliers of line 4–12.

allocated to individual transactions for maximizing the social welfare. The derivation of mathematical model and relevant proof show the equivalence of the proposed decentralized model with COPF. Testing on the IEEE 30-bus system shows the proposed model and solution algorithms are effective. The distinct feature of ORA-based method is that it can always keep the intermediate solution feasible during iterations. To use the decentralized model, the impacts on limited resources by individuals are required to be able to rewrite in the form of (5). The ORA decentralized model is promising to apply to some other issues of electricity market. Further research work will be conducted to extend the method to ac load flow based on OPF with reactive power and voltage level impacts included.

APPENDIX

The Appendix gives the proof for the equivalence of the KKT condition of COPF and ORA-based decentralized model.

The local constraints $\mathbf{u}_k \in \text{Dom}_k (k \in T)$ are generally described as $d_l^{(k)}(\mathbf{u}_k) \leq L_l^{(k)}$ ($l \in V_k, k \in T$) where V_k is the set

of constraints equations in $\mathbf{u}_k \in \text{Dom}_k$. The KKT conditions of two models are shown in P1 and P2, respectively.

P1: The KKT condition of COPF in (6).

Lagrange function of (6) is

$$\begin{aligned} L(\mathbf{u}, \lambda, \beta) &= \sum_{k \in T} W_k(\mathbf{u}_k) + \sum_{m \in I} \lambda^{(m)} (P_{\max}^{(m)} - \sum_{k \in T} g_k^{(m)}(\mathbf{u}_k)) \\ &\quad + \sum_{k \in T} \sum_{l \in V_k} \beta_l^{(k)} (L_l^{(k)} - d_l^{(k)}(\mathbf{u}_k)). \end{aligned}$$

The partial derivative equations of the Lagrange function are

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{u}} &= \left(\frac{\partial L}{\partial \mathbf{u}_1}; \dots; \frac{\partial L}{\partial \mathbf{u}_k} \dots; \frac{\partial L}{\partial \mathbf{u}_K} \right) = \mathbf{0} \quad (k \in T) \\ \frac{\partial L}{\partial \mathbf{u}_k} &= \frac{\partial (\sum_{k \in T} W_k(\mathbf{u}_k))}{\partial \mathbf{u}_k} \\ &\quad - \frac{\partial (\sum_{m \in I} \lambda^{(m)} \sum_{k \in T} g_k^{(m)}(\mathbf{u}_k))}{\partial \mathbf{u}_k} \\ &\quad - \frac{\partial (\sum_{k \in T} \sum_{l \in V_k} (\beta_l^{(k)} d_l^{(k)}(\mathbf{u}_k)))}{\partial \mathbf{u}_k} \\ &= \frac{\partial W_k(\mathbf{u}_k)}{\partial \mathbf{u}_k} - \sum_{m \in I} \lambda^{(m)} \frac{\partial g_k^{(m)}(\mathbf{u}_k)}{\partial \mathbf{u}_k} \\ &\quad - \sum_{l \in V_k} \beta_l^{(k)} \frac{\partial d_l^{(k)}(\mathbf{u}_k)}{\partial \mathbf{u}_k}. \end{aligned}$$

Suppose that \mathbf{u}_k ($k \in T$) is the solution of (6). Then there are Lagrange Multipliers $\lambda^{(m)}$ for line transfer limits and $\beta_l^{(k)}$ for local constraints ($m \in I; k \in T; l \in V_k$), satisfying the KKT conditions given by

$$\begin{aligned} \frac{\partial W_k(\mathbf{u}_k)}{\partial \mathbf{u}_k} - \sum_{m \in I} \lambda^{(m)} \frac{\partial g_k^{(m)}(\mathbf{u}_k)}{\partial \mathbf{u}_k} \\ - \sum_{l \in V_k} \beta_l^{(k)} \frac{\partial d_l^{(k)}(\mathbf{u}_k)}{\partial \mathbf{u}_k} = \mathbf{0} \quad (k \in T) \end{aligned} \quad (\text{f1})$$

$$P_{\max}^{(m)} - \sum_{k \in T} g_k^{(m)}(\mathbf{u}_k) \geq 0 \quad (m \in I) \quad (\text{f-2-1})$$

$$\lambda^{(m)} \left(P_{\max}^{(m)} - \sum_{k \in T} g_k^{(m)}(\mathbf{u}_k) \right) = 0 \quad (m \in I) \quad (\text{f-2-2})$$

$$\lambda^{(m)} \geq 0 \quad (m \in I) \quad (\text{f-2-3})$$

$$d_l^{(k)}(\mathbf{u}_k) \leq L_l^{(k)} \quad (l \in V_k, k \in T) \quad (\text{f-3-1})$$

$$\beta_l^{(k)} (L_l^{(k)} - d_l^{(k)}(\mathbf{u}_k)) = 0 \quad (l \in V_k, k \in T) \quad (\text{f-3-2})$$

$$\beta_l^{(k)} \geq 0 \quad (l \in V_k, k \in T). \quad (\text{f-3-3})$$

P2: The KKT conditions for transactions and ISO of DOPF supposing that \mathbf{u}_k ($k \in T$) is the solution of (11) for each transaction, there are $\lambda_k^{(m)}, \alpha_k^{(m)}, \beta_l^{(k)}$ ($m \in I; k \in T; l \in V_k$) sat-

isfying the KKT conditions of transaction given by (f-4)–(f-6) and optimality condition of ISO given by (f-7)

$$\begin{aligned} \frac{\partial W_k(\mathbf{u}_k)}{\partial \mathbf{u}_k} - \sum_{m \in I} \lambda_k^{(m)} \frac{\partial g_k^{(m)}(\mathbf{u}_k)}{\partial \mathbf{u}_k} \\ - \sum_{l \in V_k} \beta_l^{(k)} \frac{\partial d_l^{(k)}(\mathbf{u}_k)}{\partial \mathbf{u}_k} = \mathbf{0} \quad (k \in T) \end{aligned} \quad (\text{f-4})$$

$$P_{\max}^{(m)} \alpha_k^{(m)} - g_k^{(m)}(\mathbf{u}_k) \geq 0 \quad (m \in I, k \in T) \quad (\text{f-5-1})$$

$$\lambda_k^{(m)} (P_{\max}^{(m)} \alpha_k^{(m)} - g_k^{(m)}(\mathbf{u}_k)) = 0 \quad (m \in I, k \in T) \quad (\text{f-5-2})$$

$$\lambda_k^{(m)} \geq 0 \quad (m \in I, k \in T) \quad (\text{f-5-3})$$

$$d_l^{(k)}(\mathbf{u}_k) \leq L_l^{(k)} \quad (l \in V_k, k \in T) \quad (\text{f-6-1})$$

$$\beta_l^{(k)} (L_l^{(k)} - d_l^{(k)}(\mathbf{u}_k)) = 0 \quad (l \in V_k, k \in T) \quad (\text{f-6-2})$$

$$\beta_l^{(k)} \geq 0 \quad (l \in V_k, k \in T) \quad (\text{f-6-3})$$

$$\lambda_1^{(m)} = \dots = \lambda_k^{(m)} = \dots = \lambda_K^{(m)} \quad (m \in I) \quad (\text{f-7-1})$$

$$\sum_{k \in T} \alpha_k^{(m)} = 1 \quad (m \in I). \quad (\text{f-7-2})$$

The procedure to prove P1 \Leftrightarrow P2 is: if P1 holds, according to P1, we can find $\lambda_k^{(m)}, \alpha_k^{(m)}, \beta_l^{(k)}$ ($m \in I, k \in T, l \in V_k$) satisfying equations for P2 from (f-4) to (f-7) with \mathbf{u}_k ($k \in T$) unchanged as in P1 (P1 \Rightarrow P2); and *vice versa* (P2 \Rightarrow P1). (Here, uniqueness of the Lagrange multipliers is not required.)

Proof of P1 \Rightarrow P2: We can set $\alpha_k^{(m)}$ satisfying (f-7-2) as

$$\alpha_k^{(m)} = \begin{cases} g_k^{(m)}(\mathbf{u}_k) / P_{\max}^{(m)}, & \text{for } k \neq K \quad (m \in I) \\ 1 - \sum_{k=1}^{K-1} \alpha_k^{(m)}, & \text{for } k = K. \end{cases}$$

By setting $\lambda_k^{(m)} = \lambda^{(m)}$, (f-7-1) and (f-5-3) holds. Also, fixing \mathbf{u}_k and $\beta_l^{(k)}$ unchanged in P1, (f-4) and (f-6) holds.

When $k \neq K$ then

$$P_{\max}^{(m)} \alpha_k^{(m)} - g_k^{(m)}(\mathbf{u}_k) = 0$$

when $k = K$, by (f-2-1) then

$$\begin{aligned} P_{\max}^{(m)} \left(1 - \sum_{k=1}^{K-1} \alpha_k^{(m)} \right) - g_K^{(m)}(\lambda_k^{(m)}) \\ = P_{\max}^{(m)} - \sum_{k=1}^K g_k^{(m)}(\mathbf{u}_k) \geq 0. \end{aligned}$$

Therefore, (f-5-1) holds. When $\lambda^{(m)} = 0$, (f-5-2) holds. When $\lambda^{(m)} \neq 0$, if $k \neq K$ then

$$\lambda_k^{(m)} (P_{\max}^{(m)} \alpha_k^{(m)} - g_k^{(m)}(\mathbf{u}_k)) = 0$$

and if $k = K$, then by (f-2-2) there is

$$\begin{aligned} \lambda_k^{(m)} (P_{\max}^{(m)} \alpha_k^{(m)} - g_k^{(m)}(\mathbf{u}_k)) \\ = \lambda_k^{(m)} \left(P_{\max}^{(m)} - \sum_{k=1}^K g_k^{(m)}(\mathbf{u}_k) \right) = 0. \end{aligned}$$

Therefore, (f-5-2) still holds.

Proof P2 \Rightarrow *P1*: By setting $\lambda^{(m)} = \lambda_k^{(m)}$ corresponding to (f-7-1), fixing \mathbf{u}_k and $\beta_t^{(k)}$ unchanged in *P2*, (f-5-3), (f-3) and (f-1) hold.

By (f-7) and (f-5-1), then

$$\begin{aligned} P_{\max}^{(m)} - \sum_{k=1}^K g_k^{(m)}(\mathbf{u}_k) \\ &= P_{\max}^{(m)} \sum_{k=1}^K \alpha_k^m - \sum_{k=1}^K g_k^{(m)}(\mathbf{u}_k) \\ &= \sum_{k=1}^K \left(P_{\max}^{(m)} \alpha_k^m - g_k^{(m)}(\mathbf{u}_k) \right) \geq 0. \end{aligned}$$

Therefore, (f-2-1) holds. By (f-7) and (f-5-2), it has

$$\begin{aligned} \lambda^{(m)} \left(P_{\max}^{(m)} - \sum_{k=1}^K g_k^{(m)}(\mathbf{u}_k) \right) \\ &= \lambda^{(m)} \left(\sum_{k=1}^K P_{\max}^{(m)} \alpha_k^{(m)} - \sum_{k=1}^K g_k^{(m)}(\mathbf{u}_k) \right) \\ &= \sum_{k=1}^K \lambda_k^{(m)} \left(P_{\max}^{(m)} \alpha_k^{(m)} - g_k^{(m)}(\mathbf{u}_k) \right) = 0. \end{aligned}$$

Therefore, (f-2-2) holds.

Thus from the proof presented above, P1 and P2 are equivalent. The KKT conditions of the COPF and ORA-based DOPF model are consistent to each other.

It should be noticed that in some special cases when a local limit and a global limit become active at the same point for transaction k , the relevant $\lambda_k^{(m)}$ may sharply change (or say, jump) from one value denoted as $-\lambda_k^{(m)+}$ to the other value denoted as $\lambda_k^{(m)-}$

— right at that point. Actually in this situation, $\lambda_k^{(m)}$ has multiple solutions which vary from $\lambda_k^{(m)+}$ to $\lambda_k^{(m)-}$ — satisfying the KKT conditions. According to the above explanation, the equivalence of KKT conditions of COPF and DOPF models still holds, and, for this special case, the ISO in DOPF model can still select proper $\lambda_k^{(m)} \in [\lambda_k^{(m)-}, \lambda_k^{(m)+}]$ to satisfy (18). But computer program should be developed properly to handle such a special case.

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